Pitfalls of Back-Analyses

Les Pièges des Analyses à Rebours

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SYNOPSIS With the development of field instrumentation and numerical analyses, the back-analysis of case histories has become the prefered method of improving the geotechnical knowledge. It has been generally usefull, but on many occasions it has proved unreliable or even misleading. Five examples are taken from practice to illustrate some major pitfalls of back-analyses. Guidelines are consequently proposed to avoid these pitfalls and their embarrassing consequences on the development of geotechnique.

INTRODUCTION

In an engineering science dealing with a material so complex as soil, development can be achieved only by constant reference to field observations (Terzaghi 1936). In the last 30 years, field instrumentation and full scale testing techniques have been used increasingly to gather evidence on the in situ behaviour of soils. The interpretation of this often complex evidence to support existing theories or to develop new ones has been made easier with the advent of the computer and of numerical methods such as the FEM. The numerical back-analysis of field observations has now become a prefered method of validating theories, or, more frequently, of evaluating soil parameters to be used in "accepted methods".

The principle of back analysis can be expressed in either of the modified forms of Lambe's (1973) equation:

Observation + soil parameter → validated theory (1)
Observation + theory → empirical soil parameter (2)

The operator in these equations, i.e. the back-analysis, must be handled rigourously if the outcome is to have any reliability, in particular in equation (1). Unfortunately this condition has often been ignored, leading to the validation of wrong theories or to the use of irrelevant soil parameters. Important waste of money, time and research effort has resulted.

THE CLASSICAL APPROACH TO BACK-ANALYSES

The back analysis of any case history involves a series of assumptions and procedures, each of which is a potential source of errors.

The first step in each analysis is to simplify the geometry of the problem and the soil stratigraphy; important features may then be cancelled out. In most cases the second step is to assume an idealized soil's response, viz. drained or undrained, so as to select the type of analytical method. The third step is to postulate that the soil behaviour corresponds to the analytical model at hand: linear or non linear elasticity, isotropy or anisotropy, Mohr Coulomb criterion or plastic flow rules; a large degree of idealization is unavoidable here. Laboratory tests are carried out to obtain orders of magnitude for the relevant input soil parameters. Then the actual back analysis is performed and the results obtained are compared to the observations. In most cases the first comparison is not too successfull and some of the earlier assumptions on boundary conditions, model or input parameters are modified until a satisfactory fit is achieved between computations and observations. The common practice in this process is to adjust only a few key input parameters and to try and obtain a good fit only for one or eventually two parameters of the field behaviour: for example, analysing the response of a clay foundation to an embankment loading, specific back analyses may be developed for evaluating settlements but not necessarily lateral displacements, construction pore pressures but not deformations, or strength parameters but not the stress-strain response up to failure. Once a good fit is obtained for that particular aspect of the field behaviour which the back-analysis had intended to investigate, the theory or model used or the techniques for determining the relevant soil parameters are declared valid.

This methodology can be questioned in many aspects. One of the most serious shortcomings is that basic soil mechanics principles are frequently not satisfied by the results: for example, many cases are reported in the literature of "correct" settlements computed together with erroneous pore pressures and effective stresses. When dealing with the results of back analyses, we suffer from the same weaknesses as identified by Terzaghi (1936): our conclusions are founded on *unbalanced evidence*. As soon as a good fit is obtained for that parameter under investigation, no serious attention is paid to the quality of the fit for other parameters and a serious investigation of the causes of eventual mispredictions is replaced by a statement such as: "my model is good only for ...".

Nevertheless, in the last 10 years, major progress has been achieved on a variety of problems, thanks to the use of back analyses. On the other hand, many situations have occurred, where the approach has proved unreliable or even misleading as a result of abusive assumptions, inaccurate input parameters or unsufficient understanding of the problem under consideration. From these negative experiences a series of pitfalls of back-analyses can be identified. It is the purpose of this paper to illustrate some of the common pitfalls and to suggest some guidelines in order to avoid them in future back-analyses.

ILLUSTRATION OF COMMON PITFALLS

Response of clay foundations to embankment loading

In the last 30 years, a great number of embankments have been built on well instrumented clay foundations to gain an understanding of the response of clay deposits during construction. In view of the low permeability of clays and of the short duration of construction, and in the absence of field evidence, it was originally assumed that this response would be undrained. Since then, this assumption has turned into one of the "laws" of geotechnical engineering. No serious effort has been made to check its validity against the accumulating field evidence. Rather, the undrained analysis has been the standard method of interpreting pore pressures, deformations and failures observed in situ.

The back-analysis of settlements and lateral displacements developed in the presumably undrained response of clay foundations to embankment construction has formed a large part of the research on this problem. Finite element studies have reportedly been quite successful at back-analising settlements on the basis of undrained moduli E_u and $\nu = 0{,}5$ (Fig.1a). However, in most cases, these back-analyses have failed to produce a good simultaneous fit on the lateral displacements, as indicated in figure 1b. Even though systematic, this problem was not considered seriously until Poulos (1972) proposed tentative explanations and corrections related mainly to the anisotropy and the non-linearity of the undrained stress-strain behaviour of clays. Since then, these factors have been incorporated into the back-analyses, but the quality of the results has not really improved.

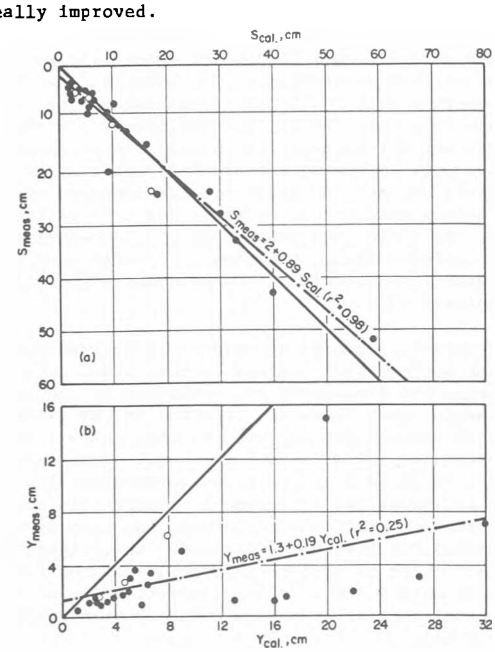

Fig.1 Predictions and observations of settlements and lateral displacements Y (from Tavenas et al. 1979a).

Now, compiling pore pressure observations from 30 case histories, Leroueil et al. (1978) have shown that the assumption of an undrained response throughout construction is not consistent with the field evidence. Rather, a significant consolidation developes initially in the overconsolidated clay; it is only after the clay has become normally consolidated that an undrained response is practically established. This sequential behaviour should strongly reflect on the development of settlements and lateral deformations. This is indeed what has been observed from the field data compiled by Tavenas et al. (1979a). Typical results are shown in figure 2: initially the maximum lateral displacement y_m represents less than 30% of the settlement s; $\Delta y_m = \Delta s$ only after the vertical effective stress has reached σ'_p as indicated by pore pressure observations.

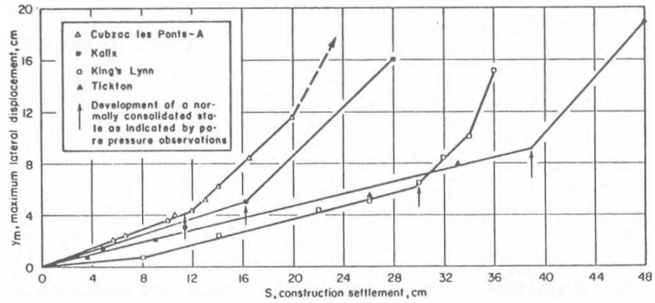

Fig.2 Lateral displacement vs settlement during construction of 4 test fills (from Tavenas et al. 1979a).

The lack of success of past back-analyses was thus entirely due to the wrong assumption of an undrained clay response. This fundamental error, which has led to a significant waste of research effort, could have been identified earlier if the entire field evidence had been looked at on its own merits before attempting to fit numerical models to parts of it and if more attention had been paid to the inconsistency of the results.

Terzaghi consolidation theory for interpreting settlements

The one-dimensional consolidation theory using a constant coefficient of consolidation C_v is routinely used for evaluating the variations with time of effective stresses and settlements in clay foundations. One practical difficulty in using this theory is the determination of the proper value for C_v. Back-analyses of pore pressure and settlement observations have often been carried out to obtain "field values" of C_v and thus empirical correction factors to apply to oedometer test results. This approach has been successfully applied to cases where the clay foundation was overconsolidated. On the other hand it is associated with major problems when applied to normally consolidated clays.

When building embankments on clay, it is a common situation that the applied vertical stress exceeds the preconsolidation pressure σ'_p. In this case, the assumption of $C_v = C^{st}$ is a gross oversimplification of the actual behaviour, but it has different consequences depending on the

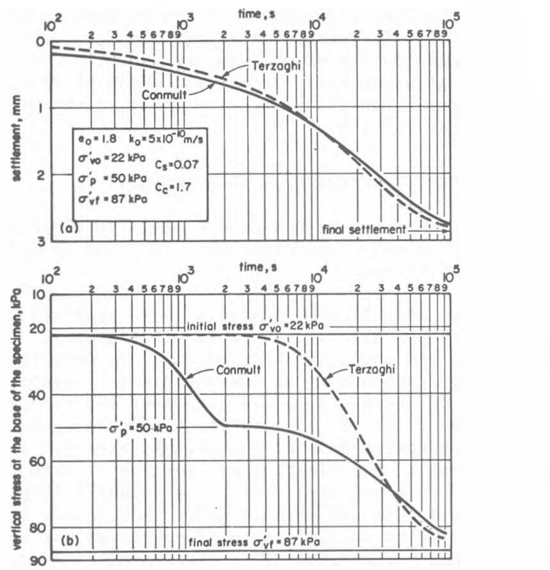

Fig.3 Total settlement and vertical stress at the base of the specimen as computed by Terzaghi and CONMULT

parameter under investigation. Tavenas et al. (1979b), have used a numerical model (CONMULT) accounting for the variations of σ', e, k, C_c and C_v to evidence the shortcomings of Terzaghi's solution in the case of an oedometer specimen, with the properties indicated on figure 3a, the total settlements computed, from Terzaghi using a C_v value corresponding to the normally consolidated state, and from CONMULT are in good agreement; stopping at this point, one could thus conclude to the validity of Terzaghi's solution. A quite different conclusion is reached if other parameters of the problem are considered. As shown in figure 3b, the variations with time of the vertical effective stress at the non-draining base of the specimen are totally different, the CONMULT solution evidencing the strong effect of σ'_p on the consolidation process. The comparisons of the distributions, within the specimen, of the effective stress (Fig.4a) and of the settlement (Fig.4b), when the total settlement computed in both methods was 0,7 mm, clearly indicate that the "good prediction" of the total settlement in Terzaghi's solution is the result of a natural error cancellation process.

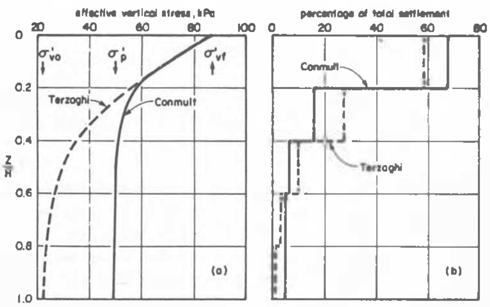

Fig.4 Distribution of effective stresses and settlement in the specimen as computed by Terzaghi and CONMULT at a total settlement of 0,7 mm (25% consolidation).

This example first demonstrates one of the frequent problems with analytical solutions, where an apparently correct result proves to be the sum of compensating errors when the solution is analysed in detail. From a more practical point of view, the results presented herein indicate that Terzaghi's consolidation theory with $C_v = C^{st}$ is just not representative of the clay behaviour, so that it cannot serve as a basis for the back-analysis of field observations of settlements or pore pressures when σ'_p is exceeded during consolidation. While a good fit could conceivably be obtained for the total settlement in cases with simple boundary conditions, this theory is necessarily misleading when analysing settlements in layered deposits or when predicting effective stress variations, for example for the analysis of stability and the monitoring of stage construction of high embankments.

CONMULT analysis of long term settlements

It was just shown that Terzaghi's consolidation theory with $C_v = C^{st}$ cannot be used in back-analyses. On the other hand, the CONMULT model which accounts for the actual variations of all parameters during consolidation and which handles complex boundary conditions is an excellent tool for back-analysing consolidation field data. However, its use is not exempt of problems, which generally arise from the possible variability and error of the numerous input parameters.

The CONMULT model was used to back analyse the field observations on the St-Alban test embankments (Tavenas et al. 1974) for the first five years after construction. The computed total settlements is compared to the field data for test section D in figure 5. Aside from a limited scatter of the measurements, there is essentially an excellent agreement between observation and prediction throughout

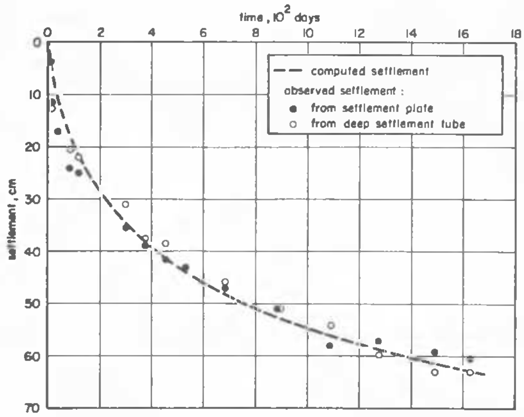

Fig.5 Observed and computed settlements under test embankment D at St-Alban.

the entire period of observation. Since the total settlement is usually the objective of consolidation analyses in designs one could have stoped here, concluding that both the model and the numerical values of the input parameters were perfectly conform to the field evidence.However,again here, the detailed analysis of all parameters leads to a quite different conclusion. Figure 6a shows the distributions with depth of the observed and computed settlements 1600 days after construction: the CONMULT analysis has underestimated the settlement in the crust by about 40% and overestimated the compression of the soft second layer by more than 20%. Thus the good total settlement prediction is the result of compensating errors. Figure 6b presents the vertical effective stress profiles obtained from the observed and the computed pore pressures after 1600 days. The differences are significant: according to CONMULT, consolidation is nearly completed, while the field data suggest that more than half of the effective stress increase beyond σ'_p, i.e. in the range where large strains are generated, still has to occur. Consequently, large settlements should be expected to develop in the future, leading to a strong deviation between predictions and observations. The only possible explanation for the diverging conclusions arising from figures 5 and 6 is that the input parameters k, σ'_p, C_c, C_α, etc.. assigned to the different strata were erroneous, but in such a manner that the errors were partly self cancellating.

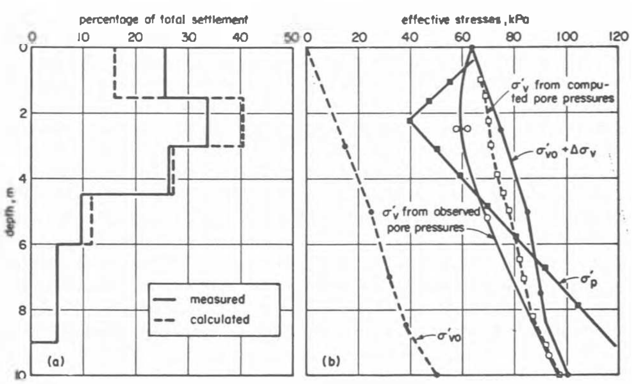

Fig.6 Observed and computed distributions of settlement and effective stresses under test embankment D at St-Alban

This case is a good illustration of the fact that a good analytical model, allowing the successfull back-analysis of one parameter of a given problem, cannot necessarily serve as a basis for validating methods of measuring or evaluating the necessary input soil parameters. To achieve such a validation of great practical importance, it is absolutely necessary that all parameters of the field behaviour be successfully back analysed. A resulting cardinal rule for peoples dealing in back-analyses should be that a 50% good prediction is 100% wrong and it cannot lead to any meaningfull conclusion.

Stability of cuttings in London clay

Back analyses have been the prefered method for determining the nature and the magnitude of the strength parameters relevant to the failure of natural or cut slopes. However, the method is most difficult to handle properly in this case where many of the key parameters are not known. Case histories, i.e. failed slopes, are frequently detected and investigated quite some time after the failure. As a result the original slope geometry must be approximated from maps, air photos or neighbouring slopes; above all the pore pressure regime at the time of failure and the detailed failure process are totally unknown and must be assumed. Consequently, the results of back-analyses are highly influenced by the initial assumptions, up to the point where contradictory conclusions may be reached from the same case histories back-analysed with different sets of "reasonable assumptions". A good illustration of the problem is provided by the analysis of the stability of cuttings in London clay.

The stiff fissured brown London clay is sufficiently uniform in properties, that slope failures from a variety of locations can be combined into a single data set. Ten cuttings, excavated between 1838 and 1931, and which have failed between 1841 and 1966, form the experimental evidence used to develop an understanding of the processes and strength parameters involved in cut slope failures. In all but one of these slopes reliable pore pressure data was not available and had to be assumed.

In the original back-analysis of these case histories, Skempton (1964) made the tacit assumption that the permeability of the fissured clay is sufficiently high to allow a quick equilibration of the negative pore pressures generated during excavation. In all cases the pore pressures at the time of failure were thus assumed stable. The observed time-dependence of failures was consequently interpreted as the result of the time dependent decrease of both the effective cohesion c' and the effective friction angle ϕ', from peak values, available initially, to residual values, achieved on the very long term. The concept of progressive failure was proposed to explain this significant strength reduction in fissured clays.

A few years later, Skempton (1969, 1970) realized that the reduction of ϕ' to residual values requires very large displacement along a failure surface. Since such displacement cannot be developed in a first-time slide at the initiation of failure the reduction of ϕ' must be very limited. Accounting for the concept of critical or fully softened state, which corresponds to a constant value of $\phi' = 20°$, Skempton (1969) developed a new back analysis of the available case histories. Still assuming a permanent pore pressure regime, it was concluded that the effective cohesion c' was time dependent as indicated in figure 7a. The process of softening of the clay, due to dilatancy and the opening of fissures, was proposed to explain this reduction.

In the following years, pore pressure observations in 3 stable cuttings (Vaughan & Walbancke 1973) indicated that more than 20 years could be necessary for the full pore pressure equilibration. Skempton (1977) was thus led to carry out a new series of back analyses, in which the strength parameters of the clay were now assumed constant and equal to the fully softened values c'=1 kPa, ϕ = 20°. The conclusion was that the time dependence of the observed failures reflected the time dependence of the pore pressure (Fig.7b).

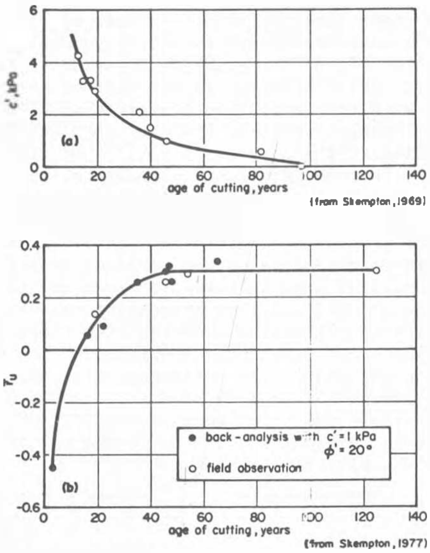

Fig.7 Time dependence of the effective cohesion c' or of the $\overline{r}_u$ parameter obtained from back analyses of cuttings in London clay.

Leonards (1979) questioned this new explanation on the basis that drainage schemes in London clay are fully effective in less than 10 years. He then suggested that the slow increase of $\overline{r}_u$ with time results from the continued generation of negative pore pressures due to the structural breakdown of the clay. Analysing this process qualitatively, Leonards (1979) concluded that it would imply a relationship between the slope inclination and the time to failure. The available data set indeed confirmed such a relationship.

Most recently Tavenas & Leroueil (1980) have used the same series of case histories to develop yet another explanation. Based on a general model of the creep behaviour of overconsolidated clays, they suggest that the delayed failure of cuttings results from the combined reduction of the effective stresses during pore pressure equalization and of the peak strength envelope produced by creep (Fig.8). The strength mobilized at failure in this model corresponds to the critical state friction angle $\phi' = 20°$ and to an empirical cohesive component. In this concept, as with Leonards (1979) but for different theoretical reasons the time to failure should be related to the slope inclination and height, as indeed confirmed by the available field data.

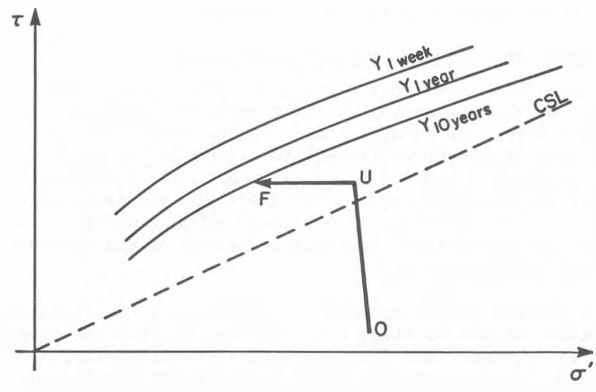

Fig.8 Schematic effective stress path and time dependence of strength in the development of failure in cut slopes.

Five contradictory theories have thus been successively validated by back-analysing the very same series of case histories and there is no reason to believe that other theories could not be validated in the future. The lesson from this experience is that back analyses which require assumptions on key input parameters just cannot be used reliably to validate theories or to develop new concepts, since their conclusions directly reflect the initial assumptions.

Empirical strength parameters for regional slope stability

In spite of the difficulties discussed in the previous example as well as of the fundamental problems associated with slope stability analyses (Tavenas et al. 1980), the back-analysis of failed natural slopes in a given geological setting is increasingly used to determine empirical "regional strength parameters" of the local soils. These parameters are then used in design analyses to check the stability of existing slopes or to define the geometry of permanent man-made slopes. The definition of these parameters is associated with some difficulties, as in their subsequent use.

In the Champlain clay deposits of Eastern Canada the method was first used on a case by case basis. For any individual slope an infinite number of couples of c',ϕ' values can lead to F = 1,0 in the back-analyses. Indeed the reported "relevant" values vary by as much as 0 to 14 kPa for c' and 27 to 55° for ϕ', bearing no relationship to the range of values which can be obtained from laboratory tests. In order to obtain a consistent picture of the problem a series of failed slopes with different geometries must be analysed, to cover a range of effective stress conditions. In this case, as shown in figure 9, the strength conditions at failure correspond to a much narrower range of c' = 7 to 12 kPa and ϕ' = 28 to 33°. This range is by no means negligible but its lower limit could be used to define a safe design strength envelope.

However, a major problem in developing such an approach is the accuracy of the back analyses and their results. More specifically, the exact pore pressures at the instant of failure are generally unknown and must therefore be assumed. Now, an error of ±0,5 m on the position of the water table, or Δu = ±5 kPa, results in an uncertainty on the position of the resistance envelope representative of the field situation. As shown in figure 10, this error translates directly into an error on the effective cohesion, $\Delta c' = \Delta u \tan \phi'$, i.e. $\Delta c'$ ±2,5 kPa. The magnitude of this error is apparently small, and yet unacceptable in practice. This is evidenced in analysing a typical slope with

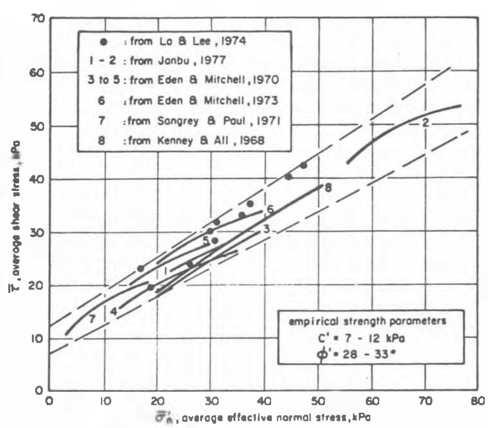

Fig.9 Resistance envelopes and empirical strength parameters for natural slopes in Canadian marine clays (from Tavenas & Leroueil 1980).

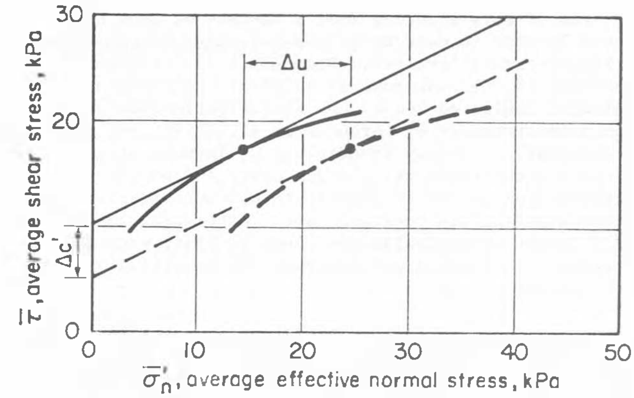

Fig.10 Effect of pore pressure variations on resistance envelope and back-analysed strength parameters.

a height of 8 m, an inclination of 23° and a water table at 0,5 m depth, in a clay with γ = 16 kN/m^3, ϕ' = 30° and c' = 7,5 ± 2,5 kPa; the computed factor of safety with the lower limit of c' would be 0,99 indicating instability, but with the higher limit of c'= 10 kPa it would be 1,33 which is generally considered representative of a stable natural slope. The reliability of stability analyses based on empirical strength parameters developed from back analyses is thus questionable.

This example shows that back-analyses cannot be used to determine reliable numerical values of soil parameters whenever some of the key input data to the back-analyses have to be assumed. Indeed in this case the results of the analyses directly reflect the initial assumptions.

GUIDELINES FOR THE PROPER USE OF BACK-ANALYSES

The experience shows that back-analyses are rather difficult to use correctly and efficiently in the development of geotechnical engineering. Costly errors can be avoided only by making a complete and unbiased use of all available field evidence and by satisfying fundamental soil mechanics principles with the utmost rigor. More specific guidelines for the proper use of back-analyses may now be established.

1. All aspects of the behaviour of soils are actually integrated into one effective stress dependent response pattern. It is improper to split this response into separate limited phenomena such as pore pressure response, strain development, strength mobilization. Rather, all parameters of the soil's response must be looked at simultaneously and understood in a consistent manner.

2. Soil behaviour is many orders of magnitude more complex than any of our theories. Consequently, the experience should always be given precedence. Also, theories and concepts which provide a framework for the qualitative interpretation of observations, such as the limit state concept for clays (Tavenas & Leroueil 1979), are preferable to complex analytical solutions which necessarily involve assumptions and simplifications and which can be evaluated only by comparing numerical values.

3. In spite of the development of field observations, our present knowledge on the in situ behaviour of soil deposits is still limited. Therefore the absolute first priority of geotechnicians should be the unbiased field observation of all conceivable parameters pertinent to a given problem before attaching too much importance to numerical analyses.

4. Once such observations are available, a qualitative

analysis of this evidence from a variety of case histories should be made to develop an understanding of the nature of the processes involved. Frequently such a qualitative knowledge of the phenomena is sufficient to develop good empirical design methods: the investigations on construction pore pressures by Leroueil et al. (1978) and on lateral deformation in clay foundations by Tavenas et al. (1979 a) are typical examples. In any case, a good qualitative understanding of the problem is an absolute requirement before starting any back analysis. Otherwise, there is a great danger of analysing the wrong problem, such as carrying out an undrained analysis for a partially drained field problem.

5. Implicit and explicit assumptions in theories or analytical methods must be clearly identified and checked against the actual soil behaviour as obtained from laboratory and field investigations. If the assumptions are not in conformity with the evidence, back-analyses using them must be considered only as very crude empirical tools and they cannot be used for developing new concepts or validating theories.

6. All input parameters which are naturally variable or which must be assumed, must be identified. In most cases, assumptions on key input parameters will lead to non reliable conclusions of the back-analyses.

7. Soil mechanics principles must apply to the results of back-analyses. This means that all interrelated parameters must be back-analysed with equal success... or with a consistent error to allow a reliable empirical correction. It is often considered that this condition may be neglected if some of the back-analysed parameters can be calibrated agains field data. It must be realized that such an approach leads to problems when dealing with complex field boundary conditions or when using a method calibrated on one parameter to analyse another parameter of the same problem. A back analysis successfull on only 50% of the parameters is nothing but 100% wrong.

8. Time is a systematic parameter in all geotechnical problems. The validity of back-analyses should therefore be checked at various stages of the process under investigation before drawing any conclusion.

9. Finally, in soil mechanics as in any science, the sum of many errors cannot be anything but an error ... even if it fits! Users of back analyses should always be looking for self cancellating errors which have so many possibilities to occur in complex field situations.

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