Prediction of Time-Dependent Behaviour of Clay

Prédiction des Mouvements dans le Temps des Argiles

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SYNOPSIS This paper is concerned with the constitutive equation of normally consolidated clay and the prediction of time-dependent behavior of clay. The constitutive equation is derived based on the Cambridge theory and Perzyna's elastic/viscoplastic theory. The derived equation can universally explain not only such time-dependent behaviors as creep, stress relaxation and strain rate effect, but also as secondary consolidation and one-dimensional consolidation including the secondary compression. The proposed theory has a feature that secondary consolidation rate can be determined by the strain rate controlled undrained triaxial compression test.

INTRODUCTION

Since normally consolidated clays are regarded as strain hardening plastic and rate sensitive materials with dilatancy, the constitutive model must be the one which can describe the behaviors due to those properties. The researches concerned with the constitutive equations for the materials may be divided into two categories, namely, (i) the deformation characteristics at the equilibrium state and (ii) the rate sensitive properties. The aim of this paper is to construct more general realistic constitutive equations for fully saturated normally consolidated clays by unifying the results of above two major approaches, based on Perzyna's elastic/viscoplastic theory(1963) and Cambridge theory(1963), so-called Cam-clay model.

DERIVATION OF CONSTITUTIVE EQUATIONS

In this paper, we will discuss the infinitesimal strain field. The total strain rate tensor ε_{ij} is given by

$$\dot{\varepsilon}_{ij}=\dot{\varepsilon}^{e}_{ij}+\dot{\varepsilon}^{vp}_{ij} \tag{1}$$

where $\dot{\varepsilon}^{vp}_{ij}$ is the viscoplastic strain rate tensor and $\dot{\varepsilon}^{e}_{ij}$ is the elastic strain rate tensor.

The following three fundamental assumptions are set up to derive the constitutive equations.

(1) Adachi and Okano(1974) firstly tried to extend the critical energy theory for the clays so that it can explain the time-dependent behaviors by using the modified Perzyna's theory and some assumptions. In their theory, clays are assumed to be attained to their static equilibrium state after completion of the primary consolidation. Arulanandan et al.(1971) made clear, however, that the increase of pore water pressure took place when giving back to undrained condition after the end of primary consolidation. Febres-Cordero and Mesri(1974) also pointed out the secondary consolidation occurred even in the case of isotropic consolidation. These secondary consolidation phenomena are deeply related to Bjerrum's concept of delayed compression(1967). Taking into account above experimental evidences, it is natural to assume that normally consolidated clays never reach their static equilibrium state at the end of primary consolidation.

(2) We assume that the mechanical behavior of clays at the static equilibrium state can be described by the critical state energy theory which is called Cambridge theory(1963). So, the static yield function is defined as follows.

$$f_s=\kappa_s \tag{2}$$

$$f_s=\sqrt{2J_2}/M^*\sigma'_m+\ln\sigma'_m \tag{3}$$

$$\kappa_s=\ln\sigma'^{(s)}_{my} \tag{4}$$

where $J_2=\frac{1}{2}s_{ij}s_{ij}$ is a second invariant of deviatoric stress tensor s_{ij}, $\sigma'_m=\sigma'_{kk}/3$ is the mean effective stress, M^* is the value of $\sqrt{2J_2}/\sigma'_m$ at the critical state, and κ_s and $\ln\sigma'^{(s)}_{my}$ are hardening parameters. The superscript (s) denotes the value at static state.

In the Cambridge theory, the strain hardening parameter $\ln\sigma'^{(s)}_{my}$ is related to the plastic volumetric strain v^p.

$$v^p-v^p_i=\frac{\lambda-\kappa}{(1+e)}\ln\sigma'^{(s)}_{my}/\sigma'^{(s)}_{myi} \tag{5}$$

where $\sigma'^{(s)}_{myi}$ and v^p_i deonte the initial values of $\sigma'^{(s)}_{my}$ and v^p respectively, and e is a void ratio.

(3) The viscoplastic strain rate tensor is determined by Perzyna's elasto/viscoplastic theory(1963).

$$\dot{\varepsilon}^{vp}_{ij}=\langle\Phi(F)\rangle\frac{\partial f}{\partial\sigma_{ij}} \tag{6}$$

$$F=\frac{f-\kappa_s}{\kappa_s} \tag{7}$$

$$\langle\Phi(F)\rangle = \Phi(F) \quad F>0 \;, \quad = 0 \quad F\leq 0 \tag{8}$$

where σ_{ij} is a stress tensor.

Since F=0 expresses the statical yield condition,the function f for dynamic state in Eq.(6) takes the same form as Eq.(3). From Eq.(7), we have $f=(1+F)\kappa_s = \kappa_d$, and the dynamic yield function is expressed as follows.

$$f=\sqrt{2J_2}/M^*\sigma'_m + \ln\sigma'_m = \ln\sigma'^{(d)}_{my} = \kappa_d \tag{9}$$

Fig.1 shows the static and dynamic yield surfaces and effective stress path.

Taking into account the elastic stress-strain relation and from Eqs.(1)-(4),(6) and (9), the following constitutive equations for normally consolidated clays are obtained.

$$\dot{\varepsilon}_{ij} = \frac{1}{2G}\dot{s}_{ij} + \frac{\kappa}{(1+e)}\frac{\dot{\sigma}'_m}{\sigma'_m}\frac{1}{3}\delta_{ij} + \frac{1}{M^*\sigma'_m}\langle\Phi(F)\rangle\frac{s_{ij}}{\sqrt{2J_2}} + \frac{1}{M^*\sigma'_m}\langle\Phi(F)\rangle\left(M^* - \frac{\sqrt{2J_2}}{\sigma'_m}\right)\frac{1}{3}\delta_{ij} \tag{10}$$

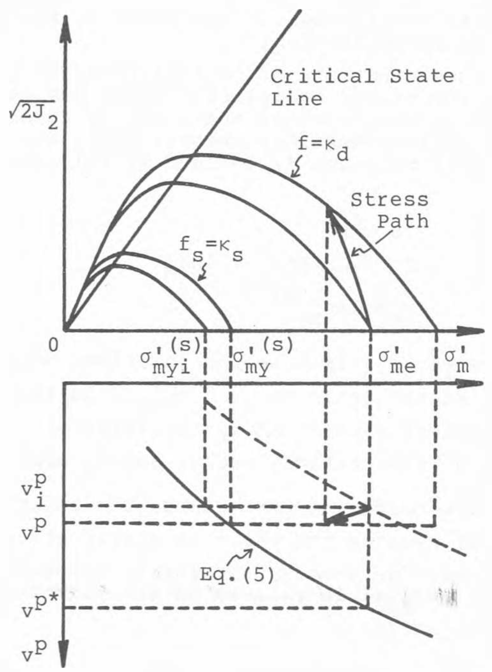

Fig.1 Yield Surfaces and Undrained Stress Path

Table I Values of Material Constants of Clay

λ	κ	M	m'	G	C_2
				(kPa)	(1/sec)
0.1	0.02	1.5	28.8	363	1.44×10^{-8}

where e is a void ratio, λ is the slope of e-lnσ'_m line for consolidation test,κ is the swelling index, G is the elastic shear modulus.

DERIVATION OF FUNCTIONAL FORM $\Phi(F)$

The functional form of $\Phi(F)$ is originally assumed to be

$$\Phi(F)=c_0\exp\{m'\ln(\sigma'^{(d)}_{my}/\sigma'^{(s)}_{my})\} \tag{11}$$

Now, let's examine Eq.(11) and discuss the way to determine the parameter m' by using the experimental evidences. Substituting Eqs.(2)-(4) and (9) into Eq.(11), an expression is found.

$$\Phi(F)=C_0\exp\{m'(\frac{\sqrt{2J_2}}{\sigma'_m} + \ln\sigma'_m - \frac{\sqrt{2J_2}^{(s)}}{\sigma'^{(s)}_m} - \ln\sigma'^{(s)}_m)\} \tag{12}$$

in which the superscript (s) means that the stress value at a static state where having the same value of plastic volumetric strain for the corresponding dynamic state. First of all, the validity of Eq.(12) will be examined by constant strain rate test results. Since the volume change is assumed to be negligible under undrained conditions, i.e. $\varepsilon_{kk}=0$, we have the relation of effective stress and plastic volumetric strain.

$$v^p = -\frac{\kappa}{(1+e)}\ln(\sigma'_m/\sigma'_{me}) \tag{13}$$

with the condition $v^p=0$ at $\sigma'_m=\sigma'_{me}$.
where σ'_{me} denotes the effective stress state reached after arbitrary time duration of consolidation.

When assuming $\dot{\varepsilon}_{11}=\dot{\varepsilon}^{vp}_{11}$, the following relation is found for two points having the same value of vP on the differnt stress path obtained by the test with different strain rate.

$$\ln(\dot{\varepsilon}^{(1)}_{11}/\dot{\varepsilon}^{(2)}_{11}) = \frac{m'}{M^*}(\sqrt{2J_2}^{(1)}/\sigma'_m - \sqrt{2J_2}^{(2)}/\sigma'_m) = \frac{m'}{M}(q^{(1)}/\sigma'_m - q^{(2)}/\sigma'_m) \tag{14}$$

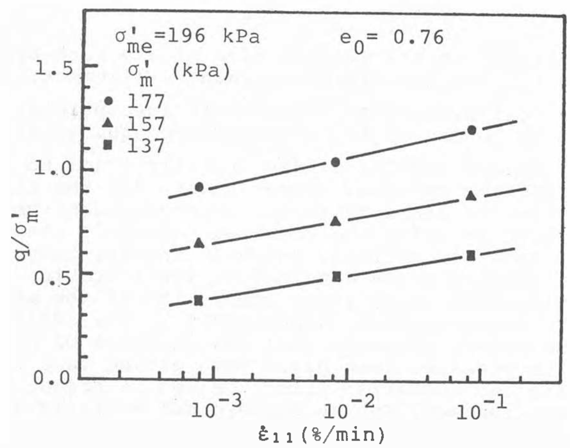

Fig.2 Relationships between Stress Ratio and Strain Rate in Undrained Compression Test

where $M=\sqrt{\frac{3}{2}}M^*$, $q^{(1)}=(\sigma_{11}-\sigma_{33})^{(1)}$, $q^{(2)}=(\sigma_{11}-\sigma_{33})^{(2)}$.

Fig.2 shows the relation between stress ratio q/σ'_m and strain rate $\dot{\varepsilon}_{11}$ of the undrained constant strain rate triaxial compression test on normally consolidated clays. The material constants are listed in Table I. It is clearly shown that there is a linear relation between $\ln\dot{\varepsilon}_{11}$ and q/σ'_m and Eq.(14) is satisfied. The value of m' is determined from the linear relation in the figure.

Now let's examine the time-dependent behaviors under isotropic stress state mentioned in Assumption (1)and discuss the relation between the secondary consolidation rate α and the strain rate parameter m' of shear deformation process. $\ln(\sigma'_m/\sigma'^{(s)}_m)$ has the following relation to v^P.

$$\ln(\sigma'_m/\sigma'^{(s)}_m)=\frac{1+e}{\lambda-\kappa}(v^{P*}-v^P) \tag{15}$$

in which v^{P*} is the value of v^P attained by the completion of secondary consolidation under isotropic stress state σ'_m.

In the case of $\sqrt{2J_2}=0$, we have the following relation from Eqs.(10),(12) and (15),

$$\dot{\varepsilon}^{vp}_{kk}=C_1M^*\exp\{m'\frac{1+e}{\lambda-\kappa}(v^{P*}-v^P)\} \tag{16}$$

where $C_1=C_0/M^*\sigma'_m$.

Furthermore, taking $C=C_1M^*\exp(m'\frac{1+e}{\lambda-\kappa}v^{P*})$ (17)

$$\text{and} \quad \alpha=\frac{\lambda-\kappa}{(1+e)m'} \tag{18}$$

the following relation is obtained by integrating Eq.(16) with the condition $v^P=v^P_0$ at $t=t_0$ and $C_1=$ const.

$$v^P=\alpha\ln(t/t_0)+v^P_0 \quad (v^P_0=-\alpha\ln(\alpha/Ct_0)) \tag{19}$$

The parameters λ,κ and e_0(initial void ratio) are given from the consolidation and soil test. Therefore, the next task is to discuss the determination of c_0 and $\sigma'^{(s)}_{myi}$ (initial value of $\sigma'^{(s)}_{my}$)

From Eqs.(5) and (10), we obtain

$$\dot{\varepsilon}^{vp}_{11}=\sqrt{\frac{2}{3}}C_2\exp\{m'(\frac{\sqrt{2J_2}}{M^*\sigma'_m}+\ln\sigma'_m/\sigma'_{me}-\frac{1+e}{\lambda-\kappa}v^P)\} \tag{20}$$

$$C_2=\frac{C_0}{M^*\sigma'_m}\exp\{-m'\ln(\sigma'^{(s)}_{myi}/\sigma'_{me})\} \tag{21}$$

Because Eq.(20) is valid under the undrained conditions, for instance, C_2 can be determined from Eq.(20), by substituting the value of $\dot{\varepsilon}^{vp}_{11}$ $(=\dot{\varepsilon}_{11})$ and $q/\sigma'_m=(\sqrt{3/2}\sqrt{2J_2}/\sigma'_m)$which are given as the corresponding value of v^P or σ'_m in Fig.2. Also, one can determine C_2 from v^P by Eqs.(10) and (21). Thus the constitutive equations (10) are completed.

It has been mentioned that the way to determine C_0 and $\ln\sigma'^{(s)}_{myi}$ would be discussed. For the completion of the constitutive equations, however, it is enough to know the value of C_2 which is expressed by those two parameters, C_0 and $\ln\sigma'^{(s)}_{myi}$.

So far six parameters C_2, m',λ,κ,e_0 and σ'_{me} are already determined. The other parameters M^* and G are got from the shear test.

EFFECT OF SECONDARY CONSOLIDATION

In order to evaluate the effect of secondary consolidation, the experimental results are compared with the calculated results obtained by using the proposed constitutive equations. The physical properties of clays used are as follows; specific gravity=2.61, liquid limit= 48.5% and plastic limit= 26.7 %. The material constants are shown in Table I. Fig.3 shows the theoretical and experimental stress paths of 1 day consolidated and 7 day consolidated samples, and Fig.4 shows the effect of secondary consolidation on the stress-strain relations. The clay sample consolidated for a long period increases in the undrained strength and the initial tangent modulus.

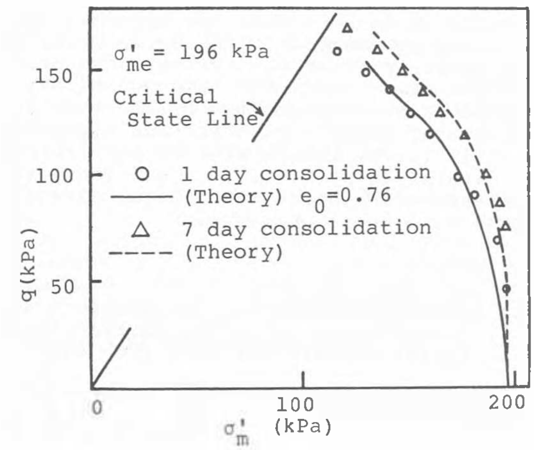

Fig.3 Effective Stress Path in Undrained Compression Test

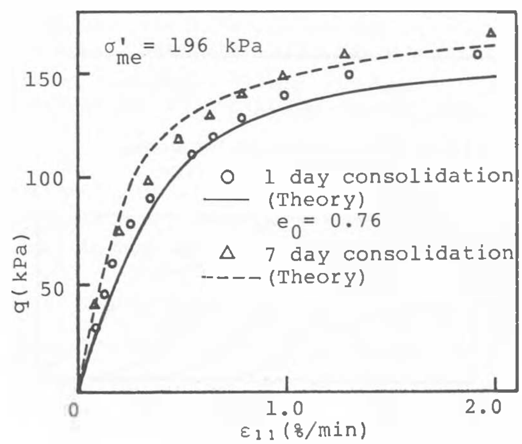

Fig.4 Stress-Strain Curve in Undrained Compression Test

These results correspond to Shen et al.'s(1973) findings. The experimental results fairly agree-with the results predicted by the proposed theory.

ONE-DIMENSIONAL CONSOLIDATION

One-dimensional consolidation analysis was carried out by using the Eq.(10) and the equation of motion of two phase media. The numerical solution was obtained by the finite difference method. During the one-dimensional consolidation process, it is assumed that the effective stress ratio $K_0(=\sigma_h'/\sigma_v')$ is constant. σ_h' is the horizontal effective stress and σ_v' is the vertical effective stress. In Fig.5 and 6, $\bar{U}_m$ is the value of $u/\Delta\sigma_v$ at the impervious boundary, u is the excess pore water pressure, $\Delta\sigma_v$ is the increment of vertical stress, $\psi=\Delta\sigma_v/\sigma'_{v(0)}$, $\sigma'_{v(0)}$ is the initial value of vertical stress, ε is the settlement per a unit of height and H is the maximum drainage distance. The parameters used for computation are as follows: $e_0=1.184$(initial void ratio), $k=1.0\times10^{-9}$m/sec(permeability), $\sigma'_{v(0)}=392.2$kPa, $\psi=0.5$, $\lambda=0.1$, $\kappa=0.02$, $M^*=1.225$, $m'=28.8$, $C_2=2.497\times10^{-4}$ 1/sec, $K_0=0.5$.

From Fig.5, it is seen that the vertical settlement ε tends to decrease with the increase of H and the latter part of ε-t curves are nearly paralell to each other. The tendency of the ε-t curve appears to be supported by the experimental results of Aboshi(1973) and the hypothesis of Ladd(1977). The final value of settlement can be determined by Eq.(15). Fig.6 shows that the increase of H leads to the later dissipation of excess pore water pressure.

CONCLUSIONS

In this study, the constitutive equations for normally consolidated clays are derived that can universally explain not only such time

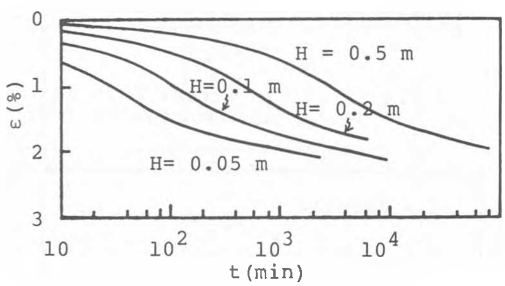

Fig.5 Relationships between Settlement and Time

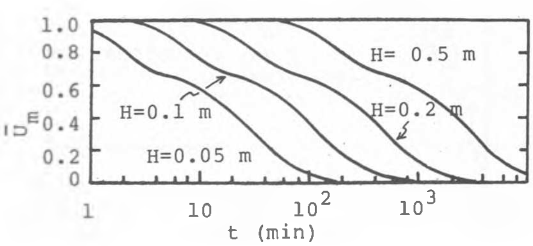

Fig.6 Relationships between Excess Pore Water Pressure and Time

dependent behaviors as creep, stress relaxation and strain rate effect, but also as secondary consolidation(or delayed compression) and one-dimensional consolidation including the secondary compression. It must be emphasized that the proposed theory has a feature to be able to determine the secondary consolidation rate from the results of strain rate controlled undrained triaxial compression test. The derived equations have eight material parameters which can be determined by conducting two undrained triaxial compression tests with different constant rate in addition to usual consolidation test. The proposed theory is applicable to the analysis of settlement of clay deposit.

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