Finite Strain Consolidation of Sedimenting Clay Deposits

Consolidation en Déformation Finie des Dépôts d'Argile Sédimentaire

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SYNOPSIS - This paper applies the theory of finite strain consolidation to sedimenting marine deposits. The theory is nonlinear and takes account of the variations of permeability and compressibility as consolidation proceeds. The self-weight of the sediment is an integral part of the theory. No restrictions are placed on the strains developed during consolidation. A comparison is made between finite strain consolidation of a sedimenting marine clay and a comparable linear, infinitesimal strain theory.

INTRODUCTION

The theory of one-dimensional consolidation, as originally developed (Terzaghi, 1924) is based upon two fundamental assumptions. First, the strains and flow velocities are small, creating an infinitesimal strain theory. Second, a linear, reversible (elastic) mechanism relates effective stresses and strains, and water pressures and velocities. This theory has had several extensions, all related to the examination of the effect of nonlinear effective stress-strain postulations (Schiffman and Gibson, 1964; Davis and Raymond, 1965; Mikasa, 1965). A newer theory of one-dimensional consolidation which considers finite strains, and is unrestricted as to the linearity of the effective stress-strain relationship, but still assumes Darcy's law, has been developed in recent years (McNabb, 1960; Gibson, England and Hussey, 1967; Koppula, 1970; Gibson, Schiffman and Cargill, 1980). It has been shown (Schiffman, 1980) that all the infinitesimal strain theories are special cases of finite strain theory. This paper examines the implications of the use of finite strain theory in predicting the pore water pressures and effective stresses in a sedimenting marine deposit.

INFINITESIMAL STRAIN THEORY

The classical theory of consolidation applied to problems of sedimentation is governed by (Gibson, 1958)

$$c_v \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} - \gamma' r \; , \qquad (1)$$

where (c_v) is the coefficient of consolidation, (u) is the excess pore water pressure, (γ') is the effective unit weight, (r) is the rate of sedimentation and (x) is the Eulerian coordinate measured from the base of the sediment.

FINITE STRAIN CONSOLIDATION THEORY

The process of finite strain consolidation is governed by (Gibson, England and Hussey, 1967)

$$\frac{\partial}{\partial z}\left[g(e)\,\frac{\partial e}{\partial z}\right] - b(e)\,\frac{\partial e}{\partial z} + \frac{\partial e}{\partial t} = 0 \qquad (2a)$$

where

$$g(e) = -\,\frac{k(e)}{\gamma_w(1+e)}\,\frac{d\sigma'}{de} \; , \qquad (2b)$$

$$b(e) = \left(\frac{\gamma_s}{\gamma_w} - 1\right)\frac{d}{de}\left[\frac{k(e)}{1+e}\right] , \qquad (2c)$$

in which (e) is the void ratio, (γ_s) and (γ_w) are the solid and fluid weights per unit of their own volume, respectively, (k) is the coefficient of permeability, (σ') is the effective stress, and (z) is a reduced coordinate encompassing a volume of solids in a volume of unit cross-sectional area lying between the datum plane and the Lagrangian coordinate point (McNabb, 1960). The Lagrangian (initial) coordinate (a) is related to the reduced coordinate (z) by

$$z(a) = \int_0^a \frac{da'}{1 + e(a',0)} \; . \qquad (3)$$

Equations (2) with appropriate boundary and initial conditions and constitutive properties provide the governing relationships from which a solution can be developed. The required constitutive properties are the relationship between void ratio and effective stress, and the relationship between the coefficient of permeability and the void ratio.

It is noted that the governing equation (2a), while unrestricted as to magnitude of strain and the linearity of the constitutive relationships, is not wholly unrestricted in application. This relationship is based upon the premise of homogeneity and monotonic behavior. Each element of the deposit is governed by the same constitutive relationship which is a unique function of the void ratio alone. Furthermore, the consolidation is monotonic. Load-unload-reload cycles are not permissible.

THE MARINE DEPOSIT

The data chosen for this analysis was based on a compendium of data compiled for the recent sediments deposited on the Outer Continental Shelf in the Mississippi delta complex of the Gulf of Mexico (Bryant, Hottman and Trabant, 1975; Helwick and Bryant, 1977; Shephard, Bryant and Dunlap, 1979). These Holocene sediments are generally highly plastic silty clays with moderate compressibility. Radiometric dating indicates that the rate of sedimentation varies between 0.05 and 0.4 meters per year.

Figure 1 presents a typical void ratio-effective stress plot. Figure 2 presents a statistically derived porosity-coefficient of permeability plot (Bryant, Hottman and Trabant, 1975). The relationship which was used is

$$k = \exp\,[14.3n - 26.3] \qquad (4)$$

where (n) is the porosity and the permeability (k) has units of (cm/sec).

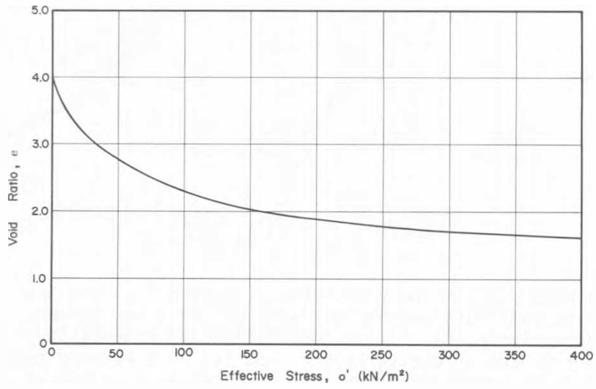

Fig. 1 VOID RATIO-EFFECTIVE STRESS RELATIONSHIP

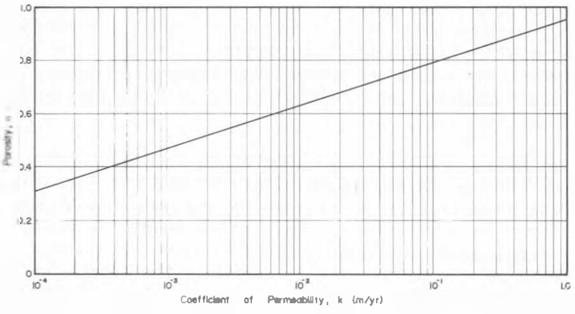

Fig. 2 POROSITY-COEFFICIENT OF PERMEABILITY RELATIONSHIP

The unit weight of solids of the sediment is taken to be 26.84 kN/m³ and the effective unit weight (γ') is 13.33 kN/m³. The unit weight of sea water (γ_w) is 10 kN/m³.

Typical oedometer parameters, compressibility (a_v) and coefficient of consolidation (c_v), are presented in Table 1 (Shephard, Bryant and Dunlap, 1979).

Table 1

OEDOMETER DATA

r (m/yr)	c_v (m²/yr)	a_v (m²/kN)
0.05	0.728	0.0154
0.1	0.807	0.0204
0.2	0.896	0.0256
0.4	0.842	0.0337

The values of (a_v) shown in Table 1 were computed from a linearization of Figure 1 using estimated maximum and minimum void ratios. The corresponding values of (c_v) were calculated using coefficients of permeability taken from Figure 2. The void ratio estimates were based upon a nominal height of sediment equal to 60 meters.

CONSOLIDATION ANALYSIS

Solutions to equations (1) and (2) were developed for a deposit sedimenting under water at a constant rate (r). The base was assumed to be impervious, while the surface of sedimentation was assumed to be subject only to hydrostatic water pressure. The solution to equation (1) was developed by a stragiht forward numerical quadrature procedure applied to the solution (Gibson, 1958). The solution to equations (2) was developed by an explicit finite difference procedure. Since equation (2a) is mildly nonlinear, care had to be exercised to determine that the solution was consistent and stable at each time step. The linear accumulation of sediment was approximated by assuming sedimentation in the form of small "slugs" of material, producing a "staircase" effect. This is a good approximation to the constant rate (r); in fact it probably mimics nature more accurately than an absolute constant rate. The soil property data presented in Figures 1 and 2 was explicitly factored into the analysis by interpolation.

The results of the computations, in which both the finite strain and linear theories are compared are shown in Figures 3, 4 and 5. These figures plot a dimensionless thickness against the effective stress (σ') and the excess pore water pressure (u). The dimensionless height is the ratio of the actual coordinate of a soil particle at a given time to the sedimented thickness of the deposit. It is noted that in the infinitesimal strain case this is always the nominal height of sedimentation which, for all cases being studied, is 60 meters.

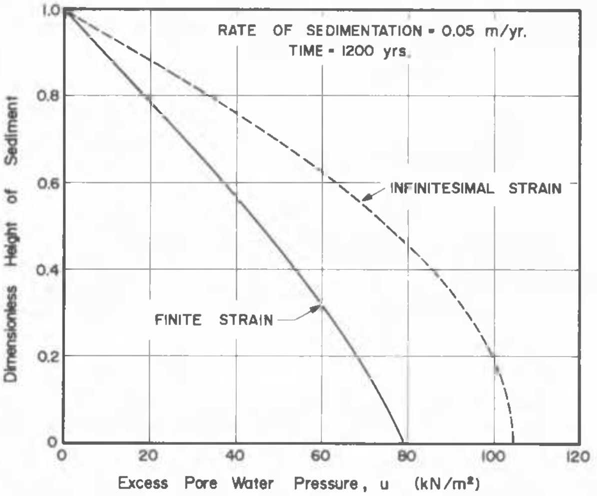

Fig. 3 EXCESS PORE WATER PRESSURE DURING SEDIMENTATION

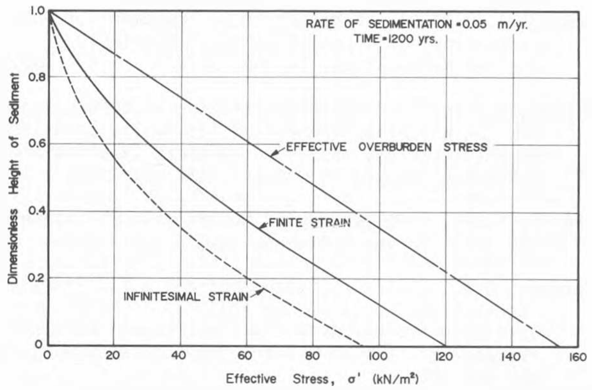

Fig. 4 EFFECTIVE STRESS DURING SEDIMENTATION

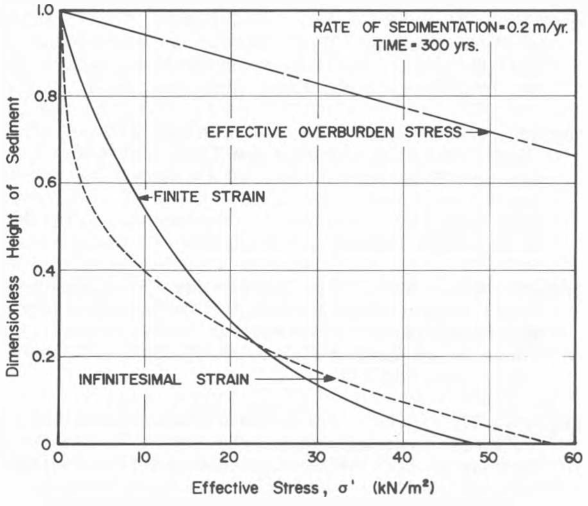

Fig. 5 EFFECTIVE STRESS DURING SEDIMENTATION

Tables 2 and 3 present comparisons of the settlement of the sediment and the degree of consolidation when the sediment thickness is again 60 meters.

Table 2

SETTLEMENT OF SEDIMENT

Rate of Sedimentation (m/yr)	Settlement (m) Finite Strain	Infinitesimal Strain
0.05	13.57	5.88
0.1	9.49	5.14
0.2	5.96	3.98
0.4	3.75	2.64

Table 3

DEGREE OF CONSOLIDATION DURING SEDIMENTATION

Rate of Sedimentation (m/yr)	Degree of Consolidation Finite Strain	Infinitesimal Strain
0.05	0.73	0.32
0.1	0.51	0.21
0.2	0.32	0.13
0.4	0.20	0.07

In the finite strain case the degree of consolidation is based upon the ultimate settlement at that nominal height of sediment. The degree in the infinitesimal strain case is based upon the dissipation of excess pore water pressure. Since the infinitesimal strain case also assumes linear constitutive relationships, both definitions are congruent.

DISCUSSION

As expected finite strain theory predicts greater magnitudes of settlement than would be calculated by infinitesimal strain theory. This is due to the use of the "true" nonlinear void ratio-effective stress relationship as opposed to the use of some "average" compressibility (a_v).

As is consistent with previous results developed for a "fixed" layer (Gibson, Schiffman and Cargill, 1980), finite strain theory provides a swifter degree of consolidation than would be indicated by conventional theory. Conventional theory assumes an "average" coefficient of consolidation (c_v) while finite strain theory permits the equivalent parameter to follow the true constitutive behavior of the sediment. This is particularly the case in early stages where the changes in properties are most pronounced. This dominates the entire sedimentation process.

The excess pore water pressure comparisons shown in Figure 3 are also consistent with the previous work cited above. Here again, consideration of the nonlinear constitutive behavior provides a prediction using finite strain theory which shows a substantially faster excess pore water pressure dissipation than would be achieved by infinitesimal strain theory.

The effective stress plots shown in Figures 4 and 5 point up the fact that even at the lowest observed rate of sedimentation the deposit is substantially underconsolidated. The change in the relationship between the theories for higher rates of sedimentation in the lower portions of the deposit is indicative of the effect of the self-weight.

It should be noted that the use of the linear theory is extremely sensitive to the parameters used in the analysis. Small changes in the values of (a_v) and (c_v) can produce large variations in the calculated isochrones. Finite strain theory is substantially less sensitive to changes in constitutive properties.

It is well established that underconsolidation plays an important role in the behavior of soft deposits (Burmister, 1942) and is particularly critical in marine sediments (Terzaghi, 1956; Sangrey, 1977; Sangrey, Clukey and Molina, 1979). An underconsolidation ratio (UCR) is defined as the ratio of the actual effective stress to the possible effective overburden stress at a point. The average UCR using finite strain theory at a sedimentation rate of 0.05 m/yr is 0.86. The comparable UCR using infinitesimal strain theory is 0.57. This tends to imply that a linear analysis will produce a substantially lower UCR than would occur in nature.

CONCLUSIONS

In this paper we have presented a comparison between a consistent theory of finite strain consolidation and conventional theory. When this theory is applied to the problem of the sedimentation of a marine deposit, it has been shown that finite strain provides for faster consolidations and lower excess pore water pressures than would be predicted by conventional (linear) theory.

It is noted that the theory presented does not provide for non-monotonic behavior due to wave action, and nonhomogeneities which will occur in the sedimentation process. These factors we leave to a later paper.

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