INTERNATIONAL SOCIETY FOR SOIL MECHANICS AND GEOTECHNICAL ENGINEERING

Computation of Land Subsidence in Shanghai, China

Calcul d'Affaissement de Surface à Shanghai, Chine

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SYNOPSIS Computations of land subsidence in Shanghai are carried out by employing the basic one-dimensional consolidation equation and the Laplace transformation. They are based on the inferences: (1) validity of the classical consolidation theory; (2) validity of superposition theorem of boundary water-level conditions in aquifers; and (3) adaptability of bi-linear model under cyclic loading. Analysis shows that the phenomena of land subsidence can be well interpreted by stress redistribution and its consequent deformation occurred in the soil layer as a result of subterranean pumping. The comparisons from 1965-1978 between the computed and actually measured data on hydrostatic excess pressures and deformations are good. This analysis also provides a sound basis for suggesting groundwater recharging as an effective means to control the subsidence.

INTRODUCTION

The long-term subsidence which hampers the industrial development and impairs the public life in the city of Shanghai, necessitates to investigate mechanism of subsidence, its prediction methods and effective control measures. Through the long collaboration with the Shanghai Geological Department, a comprehensive study was carried out both in the laboratory and in the field (Tsien, 1968a). This paper presents the work recently completed at the Institute.

HISTORICAL BACKGROUND

Land subsidence in Shanghai was first reported in 1921 (Shi et al 1979). Accompanying the industrial development, the subsidence has been continuously extending in its scale and forming dish-like depressions in the areas of major pumpage. During 1921-1965, the maximum accumulated subsidence had reached 2.63 m and the maximum subsiding rate amounted to 200 mm per year. In 1963, control measures such as restriction or rationalization of using groundwater began to take effect and subsidence lessened. The average subsidence was 23mm in 1965. In the period of 1966-1971, the subsidence was generally brought under control. By adopting artificial groundwater recharging into the aquifers, setting-up annual water-consumption plan (e.g., "winter-recharging summer-consuming", "summer-recharging winter-consuming" etc.) together with adjusting the exploited aquifers throughout the city, large-scale minor rebound of the land surface emerged during this time. Since 1972, however, the land has again undergone minor subsidence and the yearly average figure in the urban districts was about 3.6 mm (Fig. 1). Thus, it may be stated that the land surface has been undergoing the following stages —— evident subsiding-rebounding-minor subsiding.

GEOLOGICAL FEATURES

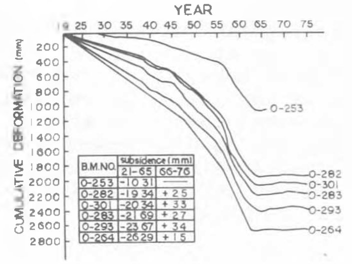


Fig. 1 Typical Cumulative Deformation (Shi et al 1979)
+ ——rebound - ——subsidence

In the city area, the Quaternary System sediments overlying the baserock are about 300 m thick. From the stratification benchmark data, the major compressible strata are located within 70 m depth below the ground surface, The total thickness of the compressible layers is very much smaller than the size of the overall pumping area of the city. Although the depressed water-levels in the exploited aquifers are funnel-shaped, the hydraulic gradient of the funnel line is very small. It is, therefore, reasonable to assume that both seepage and deformation are one-dimensional. In this paper, a typical subsiding district of the city, Lao-Dong Park is selected for analysis.

A comprehensive and precise survey for deformations occurred in each layer has been made by the Shanghai Geological Department (Su, 1979). The accuracy of the observation error of the stratification benchmark is in the order of o.1 mm. Three observation wells were installed to check the water-level variations. Besides, changes of pore-water pressures within the compressible layers have been successfully measured by the Casagrande-type piezometers. All these field data are indispensable for analysis. In this paper, the two major compressible layers were studied. The sandy clay layer (42.3-72.8 m deep, interbedding with clay and sandy clay layers, $W_L=26\text{-}40\%$, $W_p=18.5\text{-}24.7\%$, $w=35\%$, $\gamma=18.2$ kN/m^3) is directly affected by pumping in its underlying aquifer II. The other silty clay layer (3.5-19.5 m deep, $W_L=41.5\%$, $W_p=25.7\%$, $w=52\%$, $\gamma=17.2$ kN/m^3) is less influenced. Nevertheless, due to its high compressibility, the latter has been becoming the principal subsiding stratum. Its deformation is caused by the water-level changes in aquifer I directly underneath the stratum.

LABORATORY TESTING

Consolidation Test under Pumping

Experiment conducted on silty clay samples by Gu (1965) showed that the consolidation process of routine test and test under pumping were similar in nature. Within primary consolidation stage, the experimental results agreed, in general, with the Terzaghi's theory (Terzaghi, 1943). But, following the completion of the stage, there showed a conspicuous secondary compression, especially for small load-increment ratios.

Repeated-Loading Test

Due to the yearly periodic variations of water-level present in the aquifers under pumping, investigation of soil behavior in response to repeated loading is important. By selecting appropriate load-increment ratios to simulate the field condition, two loading schemes were adopted: i.e. loading-unloading and varied boundary water-level condition. The apparatus for the latter scheme is shown in Fig. 2.

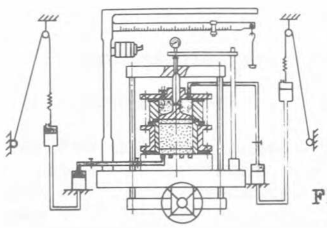


Fig.2 Repeated-loading Consolidometer (Fu et al 1968)

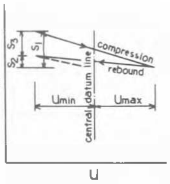


Fig.3 A Typical Compression Rebound Cycle

It was shown from the tests that the soil underwent basically primary consolidation under repeated loading. This gives the basis for ignoring secondary compression in the analysis. It is also conceivable that after a certain number of load repetitions, an average value between the coefficients of compression and rebound may be used. Fig. 3 shows a typical compression-rebound cycle which is approximated by a bi-linear model for analysis.

Validity of Superposition Theorem

The tests (Fu, 1965) showed that with different lowerings of water-heads at ends of a sample so as to establish a trapezoidal distribution of effective stress increments within the sample, the deformation-time curve was similar to one obtained by superposing a rectangular and a triangular effective stress increments.

METHOD OF COMPUTING DEFORMATION

Equation

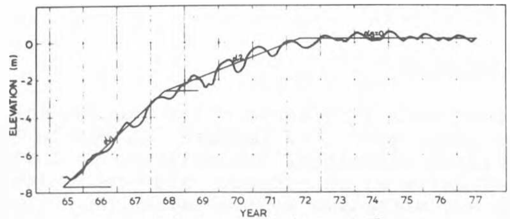
The one dimensional basic consolidation equation is adopted in the analysis.

$$\frac{\partial u}{\partial t} = C_{vc}\frac{\partial^2 u}{\partial z^2} \quad \text{or} \quad \frac{\partial u}{\partial t} = C_{vs}\frac{\partial^2 u}{\partial z^2} \qquad (1)$$

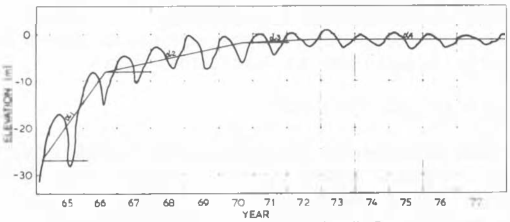
where, the subscripts c and s indicate compression and rebound respectively.

Solutions of the Basic Equation for Various Boundary Conditions

In computing the compression, it is assumed that the upper boundary water level remains unchanged, i.e. $z=0$, $u(0,t)=0$; whereas the lower boundary varies in accordance with the water-level changes in respective underlying aquifers. The continuous records of the ground water-levels in aquifers I and II are shown in Fig. 4.



(a) actually measured water-level in aquifer I



(b) actually measured water-level in aquifer II

Fig. 4 Yearly Ground Water Level Fluctuations

The fluctuating water-level is resolved into three components: namely, fixed drawdown and linear upward variation of the central water-level line and cyclic changes. By taking the Laplace transformation of Eq. (1), the following expressions for each of these cases are obtained.

(1) Fixed drawdown of central water-level line

Let $A=z/H$, $B=n\pi$, $D=C_v/H^2$, $E=\sin AB\exp(-B^2Dt)$,

$$u_1(z,t)=-\gamma_w h_2\left[A+2\sum_{n=1,2,3\cdots}^{\infty}(-1)^n E_c/B\right] \qquad (2)$$

where, h_2= drawdown of lower boundary water level; and H= thickness of soil layer.

(2) Linear upward variation of central water-level line

$$u_2(z,t)=\alpha\left[At-A(1-A^2)/6D_s+2\sum_{n=1,2,3\cdots}^{\infty}(-1)^{n+1}E_s/B^3D_s\right] \qquad (3)$$

in which α = slope of linear variation, expressed as pressure difference per unit time.

(3) Cyclic water-level changes   By neglecting the transient part of the solution, it is shown

$$u_3(z,t)=P\exp\left[-\sqrt{\frac{\omega}{2C_v}}(H-z)\right]\sin\left[\omega t-\sqrt{\frac{\omega}{2C_v}}(H-z)\right] \quad (4)$$

for sinusoidal changes, $P\sin\omega t$; in which P= peak cyclic water pressure, and ω=frequency of cyclic change. In the actual cases, the cyclic water-level changes at random. The final solution can be obtained by harmonic analysis.

Initial Condition

In view of the fact that Shanghai began to subside many years ago, stress history must be taken into account as a prerequisite factor for determining u and S(deformation) following any designated intermediate year. For the sandy clay layer, the equivalent pumping time for August 1965 is determined (Tsien, 1968b) and for the silty clay layer, the initial u distribution curve during August 1965 is found as below:

$$f(z)=u(z,0)=0.97\exp\left[-0.25(20-z)\right] \quad (5)$$

which is shown in Fig. 5.

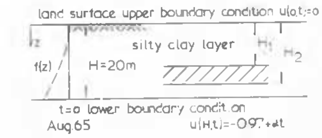


Fig.5 Initial Condition of Hydrostatic Excess Pressure,u, for the Silty Clay Layer

From Eq.(1), an expression for u under f(z) can be derived as

$$u_4(z,t)=\sum_{n=1,2\cdots}^{\infty}\frac{2}{H}\int_0^H\left[f(z)\sin AB\,dz\right]E_c \quad (6)$$

Substituting Eq.(5) into Eq.(6), then

$$u_4(z,t)=0.0965\sum_{n=1,2\cdots}^{\infty}\frac{B/H}{(0.25)^2+(\frac{B}{H})^2}\left[(-1)^n-0.0067\right]E_c \quad (7)$$

Deformation Analysis

In Fig.(5), the soil deformation, S, between H_1 and H_2 can be computed by the following equation (Terzaghi, 1943),

$$S=m_v\int_{H_1}^{H_2}u\,dz \quad (8)$$

Substituting Eqs.(2),(3) and (4) into Eq.(8) and integrating, one obtains deformation for various components of boundary water-level conditions.

(1) Deformation due to fixed drawdown case

Let $K=H_1/H_2$, $M=1-(H_1/H_2)^2$, $D'=C_v/H_2^2$, and $F=(\cos B-\cos KB)\exp(-B^2D't)$,

$$S_1=-\frac{1}{2}m_{vc}\gamma_w h_2H_2\left[M-4\sum_{n=1,2\cdots}^{\infty}(-1)^nF_c/B^2\right] \quad (9)$$

(2) Due to linear upward variation case

$$S_2=m_{vs}\alpha H_2\left[Mt/2-M^2/24D_s'+2\sum_{n=1,2\cdots}^{\infty}(-1)^nF_s/D_s'B^4\right] \quad (10)$$

93) Due to initial condition, u(z,0)

$$S_4=\frac{0.0965m_{vc}\sum_{n=1,2\cdots}^{\infty}\left[(-1)^n-0.0067\right]F_c}{(0.25)^2+(B/H_2)^2} \quad (11)$$

compression "-", rebound "+".

(4) Due to cyclic changes

From Fig.4, the maximum and minimum hydrostatic excess pressures at each depth referred to a central datum line are

$$u_{\substack{max\\min}}=\mp P\exp\left(-\sqrt{\frac{\omega}{2C_v}}\xi\right) \quad (12)$$

where, ξ=the distance from the lower boundary line to the depth-point in question=H-z.

The net deformation, S_3, after one cycle is

$$\begin{aligned}S_3&=S_1-S_2\\&=2P\exp\left(-\sqrt{\frac{\omega}{2C_v}}\xi\right)m_{vc}\left(1-\frac{m_{vs}}{m_{vc}}\right)\end{aligned} \quad (13)$$

For the entire soil layer, it can be expressed by

$$S_3=2P\sqrt{\frac{2C_v}{\omega}}\,m_{vc}\left(1-\frac{m_{vs}}{m_{vc}}\right) \quad (14)$$

Under the repeated loading-unloading condition, m_{vc} and m_{vs} gradually approach each other. It was shown from the laboratory tests that S_3 at the end of about 20 cycles was approximately equal to 1/5 of the S_3 at the end of the first cycle.

SOIL PARAMETERS

Soil parameters obtained in the laboratory are greatly affected by load-increment ratio and loading condition. It is therefore, important to select proper soil parameters a_v and C_v in the computations. Fortunately, the long-term field observation data can serve as a large-scale in-situ test results. Therefrom, the soil parameters are deduced (Gu,1968). In case the accumulated field data are inadequate, the laboratory test results are then resorted to. The soil parameters used in this computation are listed in Table I.

TABLE I   Soil Parameters

	coefficient of consolidation C_{vc}, m^2/s	coefficient of rebound C_{vs}, m^2/s	average coef. of conso. after many cycles C_v, m^2/s	coefficient of volume compression m_{vc}, $(KPa)^{-1}$	coefficient of volume rebound m_{vs}, $(KPa)^{-1}$
Silty Clay	1.5×10^{-7}	4.0×10^{-7}	4.0×10^{-6}	1.0×10^{-4}	6.0×10^{-5}
Sandy Clay	2.0×10^{-6}	6.0×10^{-6}	2.5×10^{-5}	4.0×10^{-6}	3.0×10^{-6}

COMPUTATION RESULTS

The computed distribution curves of the hydrostatic excess pressures with respect to depth and the accumulated deformations occurred in the silty clay and sandy clay layers, for each month of August from 1965-1978 are respectively shown in Figs. 6 and 7. The actually measured data are also plotted to assess the adaptability of the analytical method.

CONCLUSIONS

1. It is shown in this analysis that the pheno-

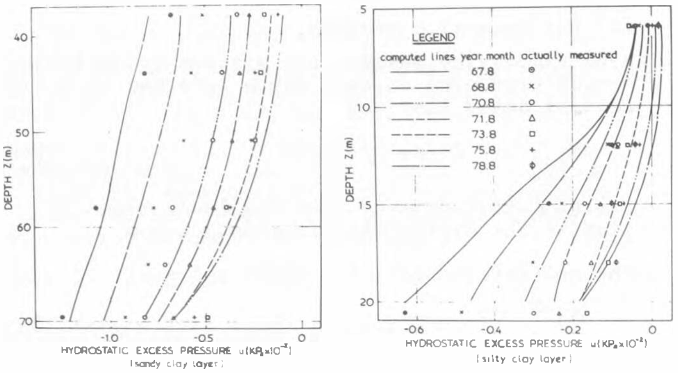


Fig.6 Hydrostatic Excess Pressures versus Depth

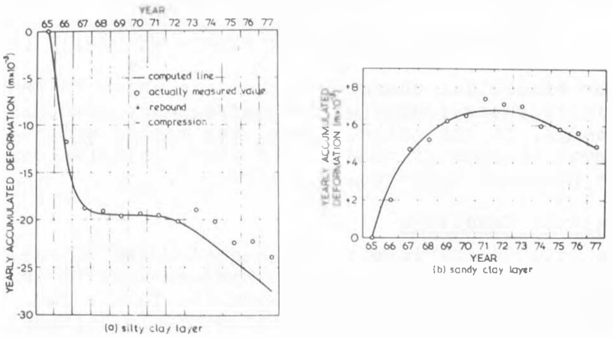


Fig.7 Yearly Accumulated Deformation

mena of land subsidence in Shanghai, China, up to now, can be well interpreted by stress redistribution and its consequent deformation occurred in the soil layers as a result of subterranean pumping. The analysis also provides a scientific basis for determining the effective control measures. The cooperation of laboratory testing and field observation is an effective way to study the mechanism of subsidence as well as to predict the soil behaviors.

2. Based on the inferences drawn from the laboratory testing and the soil parameters mainly determined from the field data, good comparisons from 1965-1978, between the computed values and the actually measured data of pore-water pressures and deformations occurred in the two most compressible soil layers in the district under consideration demonstrates the adaptability of the analytical method. The use of a set of constant equivalent soil parameters during such a long period introduces error.

3. As shown in the figures, during 1965-1978, the hydrostatic excess pressures had been steadfastly decreasing from year to year. It proves that to control the land subsidence in this city, ground-water recharging into the aquifers is effective and the computation, up to now, based on primary consolidation is acceptable. It should be noted, however, that the effectiveness of the recharging declines gradually and minor subsidence will be expected to continue for a number of years. To increase its effectiveness, field experimental research on horizontal recharging is proposed. Since the role of secondary compression will become more important in the future, further study on this behavior and the corresponding control measures is also suggested.

ACKNOWLEDGEMENT

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