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Soil/Structure Interaction and Soils Heterogeneity

Interaction Sol/Structure et Hétérogénéité du Sol

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SYNOPSIS In this paper the influence of spatial variability of deformation characteristics on contact pressure distribution and settlement predictions in a soil-structure interaction problem is studied. The computation of contact pressures and settlements is based on a modified version of dynamic relaxation method presented in the second part of the paper. In the third part a probabilistic formulation is presented. In the last part the proposed process is applied to the study of a rigid isolated foundation. The results are shown in figures and they introduce interesting commnents and conclusions.

INTRODUCTION

The accuracy of predictions in Soil Mechanics computations generally depends on the degree to which the simplified mathematical model reflects a field situation and on the accuracy of input data. (Lambe , 1973). The spatial scatte - ring of the engineering soil properties is especially one of the main factors which affects the accuracy of prediction. This natural hetero - geneity is caused by variation in mineral compo- sition and stress history. Generally, the process governing the soil formation imposes a random pattern of variation onto an overall trend. Thus, resulting soil properties can be regarded as being controlled by a random process.

Statistical methods with refined numerical methods of analysis and the use of computers can be of great value for the improvement of prediction, especially when the value of spatial variability of soil properties is significant.

PRESENTATION OF THE APPLIED NUMERICAL METHOD

A revised form of Dynamic Relaxation method is applied in the present work, based on a Finite Difference Iterative and explicit scheme.

If we consider the D.R. as mathematicians we can classify it as a matrix iterative method for the solution of Simultaneous linear equations. Further- more D.R. is an explicit method and as pointed out by OTTER et. al. (1966), it is what Frankel (1950) refers to as the second order Richardson method. From the engineer's point of view, on the the other hand, when the method is applied in the field of structural mechanics, it consists es- sentially in solving the static stress equili - brium equations. This is accomplished by inte - grating with a step by step technique the damped wave equation, due to a constant load function, until a steady state strain response is achieved. The constitutive law is satisfied at each step of the integration, insuring compatibility. The wave equation is integrated using the standard central finite difference form in time and space, until the velocity and acceleration terms reach acceptably small values, thereby providing static equilibrium. The simultaneous satisfaction of the constitutive equation quarantees compatibility.

The initial conditions, at time t = 0, are the displacements u, v, w and the stresses σx, σy, σz, Txy, Tyz, Txz are zero everywhere except at the boundary, where the boundary contitions are applied.

Each computational iteration in the applied mo- dified version is composed of the following 8 stages in finite difference form.

1. Calculation of the gradients of stresses from the stresses e.g. for the xx directions :

$$\frac{\partial \sigma x_x^{r+1}}{\partial X} = \frac{\sigma x_{x+1}^{r} - \sigma x_{x-1}^{r}}{X_{x+1} - X_{x-1}} \quad *$$

2. Application of the differential equation of motion with a critical viscous damping term. Evaluation of the nodal accelerations. e.g. for the xx direction :

$$\rho \frac{\partial^2 u_x^{r+1}}{\partial t^2} = \frac{\partial \sigma x_x^{r+1}}{\partial X} + \frac{\partial \tau_{XYx}^{r+1}}{\partial Y} + \frac{\partial \tau_{XZx}^{r+1}}{\partial Z} + \underbrace{\left(\frac{\partial u_x^{r}}{\partial t} - \text{average of the velocities of surrounding nodes}\right) W_x}_{\text{damping term}}$$

(where, ρ=density of soil, Wx = coefficient)

3. Calculation of the nodal velocities.

$$\frac{\partial u_x^{r+1}}{\partial t} = \frac{\partial u_x^{r}}{\partial t} + \frac{\partial^2 u_x^{r+1}}{\partial t^2} \cdot \Delta t$$

4. Calculation of the nodal displacements.

$$u_x^{r+1} = u_x^{r} + \frac{\partial u_x^{r+1}}{\partial t} \cdot \Delta t$$

5. Calculation of the nodal strains.

$$\varepsilon x_x^{r+1} = \frac{u_{x+1}^{r+1} - u_{x-1}^{r+1}}{X_{x+1} - X_{x-1}}$$

* the superscripts refer to successive iterations and the subscripts to the label of the nodal points

6. Calculation of the increments of strains.

$$D\varepsilon_{x_x}^{r+1} = \varepsilon_{x_x}^{r+1} - \varepsilon_{x_x}^{r}$$

7. Application of the constitutive law to calculate increment stresses from the evaluated into the stage 6 increments strains. A new constitutive law was considered based on the Duncan and Chang law. (Duncan and Chang, 1970).

$$E_t = (1-Rf.S)^{nd} E_i \quad (1) \quad \text{in which :}$$

E_t = The tangent Young Modulus.
E_i = The initial tangent Young Modulus.
Rf = The failure ratio. (The value of Rf has been found to be between 0,75 and 1,00)
S = The stress level, or fraction of strength mobilized, $(\sigma_1-\sigma_3)/(\sigma_1-\sigma_3)_{fail}$
nd = Exponent (According to Duncan and Chang nd take the value 2 and the strees-strain curve has a hyperbolic form).

In our consideration the essential innovation is that the initial modulus Ei and the exponent nd were assumed to be random variables.

8. Calculation of the stresses : $\sigma_{x_x}^{r+1} = \sigma_{x_x}^{r} + D\sigma_{x_x}^{r+1}$
These stresses will be used into the next computational iteration.

In the present procedure the load is applied gradually. In fact, the rate is controlled automatically by the developed acceleration's mean, so that during the whole process the system remains close to the equilibrium.

This condition is satisfied because the damping term continuously digests the motion, which is due to the gradually applied load, and thus we can apply a non-linear pseudoelastic law in an economical incremental form, without additional labour or time. The above mentioned control technique is applied by a relative operation in which every increase of the mean acceleration has as consequence the brake of the load application's rate. The basic stimulation is a gradual uniform downward displacement of the foundation interface. In each computational iteration the deformation of the interface is calculated from the contact pressure distribution. This deformation is taken into account for the next iteration.

PROBABILISTIC FORMULATION

For the statistical analysis of contact pressure distribution and settlements using the dynamic relaxation method in the two-dimensional case, we must compute from the data of soil profile mean values and standard deviations of spatial averages and also correlation coefficients between spatial averages. For this reasons, we use the theoretical results on the reduction factor, presented by Vanmarcke (1977). The probability density functions of contact pressure distribution and settlements are obtained by a stimulation technique.

The steps of the formulations in more detail are:

- A soil property q is considered as random variable. The value of q at a point with coordinates xi, yi, is the consequence of a trend component ti = fτ(xi,yi) and of a random component pi with variance V(pi) = fv(xi,yi).
- Estimation of the trend component and of the variance of the random component: A polynomial trend component

$$t = \sum_{j=0}^{k} \beta j \cdot xj \qquad (2)$$

can be estimated using the method of Least Squares from the test results qi at n locations (xi,yi) i = 1,n. The variance V(pi) of the random component can be also estimated from the results of the tests.

- Estimation of the autocorrelation function : The correlation coefficient between the values of p at two points A(x_o,y_o) and B (x_o+x, y_o+y) is expressed in this paper for the two-dimensional case by a quadratic exponential decay correlation model :

$$\rho(X,Y) = \exp\left(-\left[\left(\frac{X}{bx}\right)^2 + \left(\frac{Y}{b_y}\right)^2\right]\right) \qquad (3) \qquad \text{where}$$

b_x, b_y decay parameters

- The soil continuum is separated in n elements with dimensions Δx, Δy and area F = Δx·Δy. The vector M of the mean values of n spatial averages is :

$$M = (t_1^F, t_2^F, \ldots, t_n^F) \qquad (4)$$

where t_i^F is the average of the considered soil property over the i-element which can be easily estimated by the trend function.

- The variances of spatial averages $V(p_i^F)$, i = 1,n can be estimated from an appropriate variance function which measures the reduction in the variance as the size of the average area increases. Applying the results derived by Vanmarcke it can be shown that if the correlation function has the form of equation (3), then the variance function can be represented by the equation :

$$\Gamma(F) = 1, \quad \text{for} \quad \Delta x<\delta x \quad \text{and} \quad \Delta y<\delta y$$
$$\Gamma(F) = \frac{\delta x \cdot \delta y}{F}, \text{for } \Delta x>\delta x \quad \text{and} \quad \Delta y>\delta y \qquad (5)$$

in which δx, δy are the scales of fluctuation in x and y directions where

$$\delta x = \sqrt{\pi}.b_x, \quad \delta y = \sqrt{\pi}.b_y \qquad (6)$$

- Estimation of the covariance matrix C : In the cases examined in this paper the area F of the elements is greater than the elementary area δx.δy. Thus, the coefficient of correlation between two areal averages is :

$$\rho(p_i^F, p_j^F) = 1, \quad \text{for } i = j \quad (i,j = 1,n)$$
$$\rho(p_i^F, p_j^F) = 0, \quad \text{for } i \neq j \quad (i,j = 1,n) \qquad (7)$$

Consequently the elements of the covariance matrix are :

$$Cov(p_i^F, p_j^F) = V(p_i^F), \text{for } i=j \quad (i,j = 1,n)$$
$$Cov(p_i^F, p_j^F) = 0, \quad \text{for } i \neq j \quad (i,j = 1,n) \qquad (8)$$

- Application of a simulation techique to obtain the probability functions of contact pressure distribution and settlements. Starting from the stochastic vector x = (x_1,x_2,...x_n) with components normally distributed, means zero, standard deviations one, and all independent, we generate L-stochastic vectors :

$$R_L = (q_1^F, q_2^F, \ldots, q_n^F) \qquad (9)$$

These vectors are characterized by normal distribution and parameters M (from (4)) and C (from (8)), according to :

$$R'_L = M' + S \cdot X', \qquad (10)$$

where S = C (since the covariance matrix is non-negative definite we can form one of the square roots of C).

- The probability functions of contact pressure distribution, settlements, and vertical and horizontal stresses are obtained by a simple statictical analysis of the results of the L computer runs.

APPLICATION

A typical two - dimensional isolated foudation applied on the surface is considered (fig. 1). The value of Gorbunov parameter t=0,10,(Gorbunov-Possadov, Malicova, 1973) shows that foundation is rigid.

Table I

	Random Variable	Trend Component	St. Dev. of Rand. Comp	Scales of fluct. in X & Y direct.	Results are shown in
1	The initial tangent Young's Modulus E_i	$f_t(X,Y)=1000$	250	$\delta_y=0.8$ $\delta_x=0.8$	Figure 1
2	The initial tangent Young's Modulus E_i	$f_t(X,Y)=500+\frac{1000}{4.5a}Y$	250	$\delta_y=0.8$ $\delta_x=0.8$	Figure 2
3	The exponent nd in equation 1	$f_t(X,Y)=2,0$	0,5	$\delta_y=0.8$ $\delta_x=0.8$	Figure 3

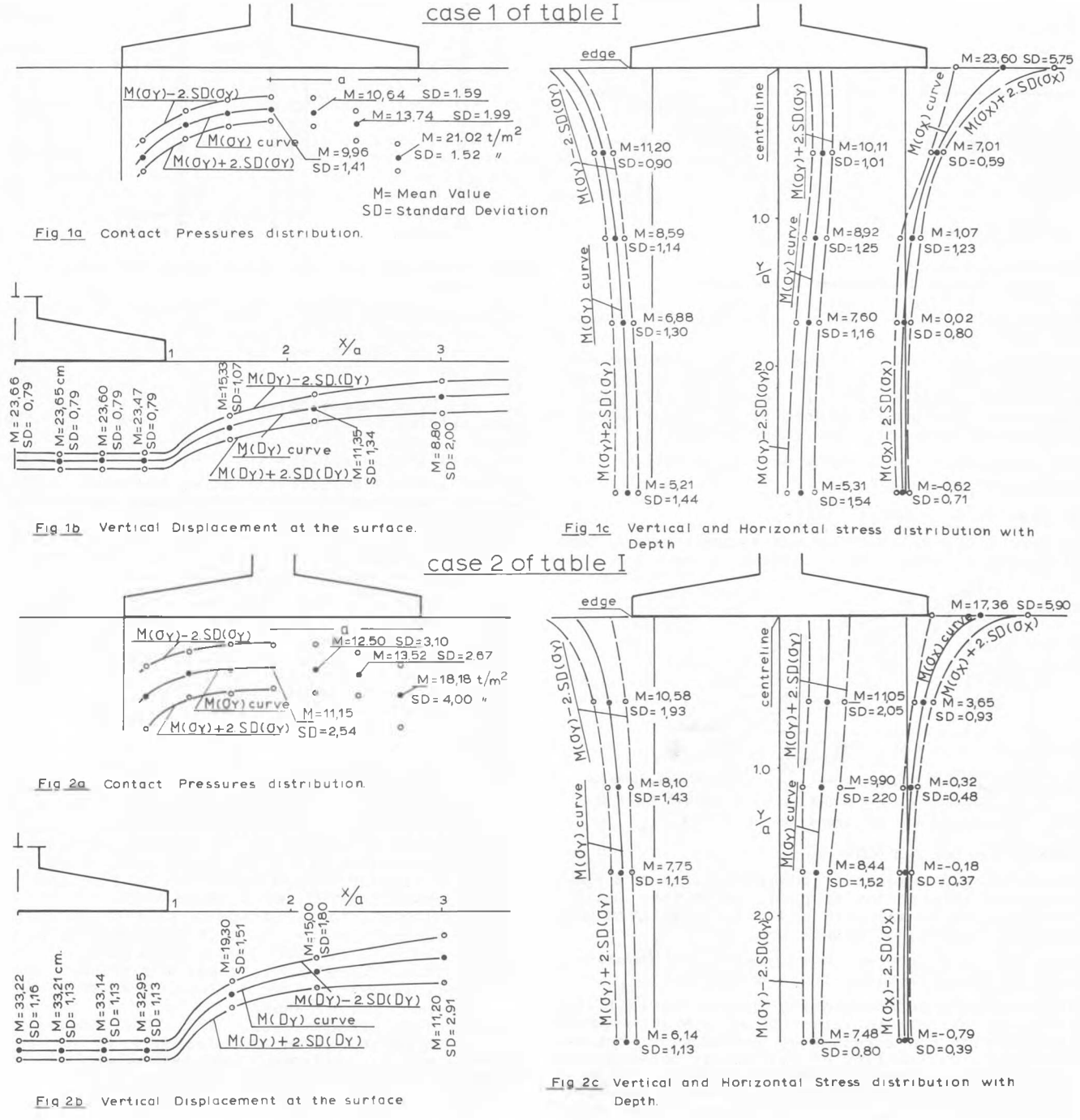

Fig 1a Contact Pressures distribution.

Fig 1b Vertical Displacement at the surface.

Fig 1c Vertical and Horizontal stress distribution with Depth

Fig 2a Contact Pressures distribution.

Fig 2b Vertical Displacement at the surface

Fig 2c Vertical and Horizontal Stress distribution with Depth.

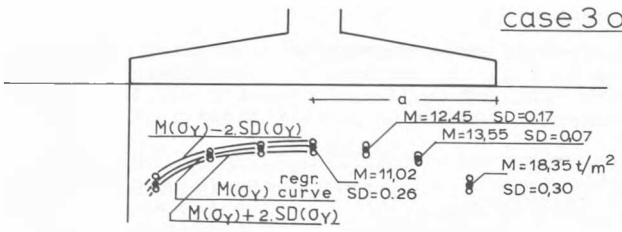

Fig 3a Contact Pressures distribution. (maximum value of S = 0,23)

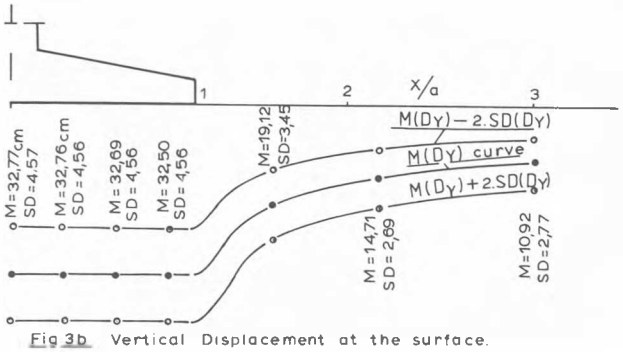

Fig 3b Vertical Displacement at the surface.

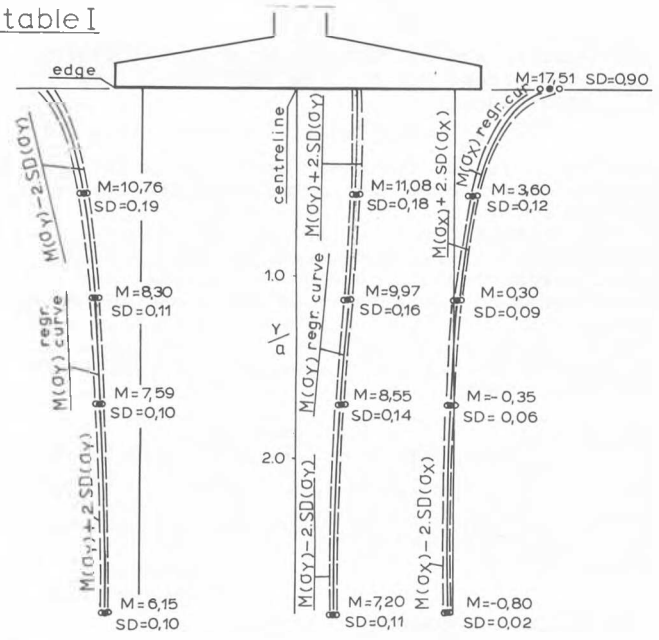

Fig 3c Vertical and Horizontal Stress distribution with Depth.

On the reffered in table I, the following remarks are worth mentioning :

The values of columns 2 and 3 correspond to a given soil profile in an area of Thessaloniki. The estimated coefficients of variation are in agreement with some representative values given by lumb and with the data shown in fig. 4.

The dimensions of elements are: $\Delta x = \Delta y = 1m$.

in case 1 and 2, nd = 2.

in case 3, $E_i = 500 + \frac{1000}{4,5a} \cdot y$

in case 3 the value of nd was assumed independent of x_i and y_i. This value changes in each run.

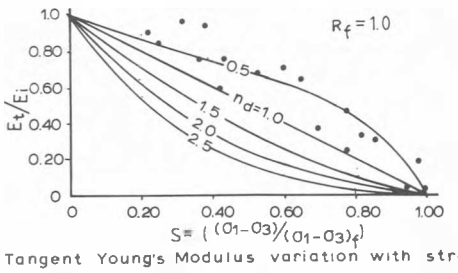

Fig.4 Tangent Young's Modulus variation with stress level. (Laboratory results after Parton & Pender. Plots of equation 1 for different values of nd)

COMMENTS - CONCLUSIONS

The developed numerical process applied on a soil/structure interaction problem, takes into consideration the variability and non linearity of soil and the rigidity of foundation.

The results presented in figures 1,2 and 3 combined with the data of table I has shown that :

1) Assuming a representative spatial variability of E_i, the calculated variances of contact presure distribution and stresses σy and σx were rather small. The variability of horizontal stresses σx seems to be larger than the variability of vertical stresses. The existence of a trend component with the linear type of case 2 affects mean values of stresses and deforms contact pressure distribution, but it does'nt essentially affect the mean value of variances. However, considering the distribution with depth of Cv(σy) and cv(σx) from figures 1 and 2, (where cv=coef. of variation), we can see by a comparison a difference worth mentioning related with the decrease of $cv(E_i)$ with the depth in case 2. The latter case is in general a more realistic approach concerning site conditions.

2) The spatial variability of E_i had small influence on settlements. This observation should be related to the dimensions of foundation and the assumed values of scales of fluctuations in x direction. In the studied cases the half of foundation width a, was about 4,5 times δx.

3) The assumption that the value of nd varies had a negligible effect on contact pressure and stress distributions, but it had an essential effect on settlements. This last effect depends on the mobilized fraction of strength, increasing with it. Once a spatial variability of nd had been assumed, then settlements variation was essentialy reduced.

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