Formulation and Prediction of Sand Behaviour

Mise en Formule et Prévision du Comportement des Sables

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SYNOPSIS In combination with Hooke's law the hyperbolic shear strain relationship has been proven useful for sand deformation problems. This combination has limitations, however, and for this reason several elastoplastic models have been proposed. Indeed, plasticity theory provides the possibility to link empirical relationships in a realistic and theoretically sound manner. However, instead of linking well-established relationships, a tendency has developed to introduce new, formerly unknown, relationships. The model proposed here is the conjunction of three well-known relationships. For brevity, it is only described for conditions of triaxial compression. It is shown that the model can be used in successful computer predictions of the behaviour of sand under load.

INTRODUCTION

In the early sixties some valuable ideas on the deformation of sand were published. For increasing stress ratios, Kondner (1963) showed that the stress-strain curves may be approximated by hyperbolae with good accuracy. For constant stress ratios, Chaplin (1961) and Janbu (1963) showed that the stress-strain curves may be described by a power law. Hansen (1965) combined the power law and the hyperbolic function to describe the shear strain measured in triaxial tests. Later, this idea was put forward and validated by Duncan and Chang (1970). Rowe (1962, 1971) showed both theoretically and experimentally that the total volume strain, v, may be subdivided such that $v = v_c + v_d$. Here, v_c is caused by compression and the dilatancy v_d is caused by shear stress. In the course of fifteen years several researchers have validated the above ideas.

In recent years a wide range of elastoplastic models have become available, and in each particular model a number of empirical relationships is assembled. With the exception of the power law, Lade and Duncan (1975) introduced new relationships and arrived at a nine-constant model. Later, sophistications have led to a model with fourteen material constants (Lade, 1977). Nova and Wood (1979) assembled quite different relationships to obtain a seven-constant model. Here, the number of material constants is comparable to the pseudo-elastic model by Duncan and Chang (1970), which involves six constants in the case of a cohesionless material. These authors used both the shear strain equation as well as the power law mentioned above. They would have arrive at seven material constants if dilatancy had not been neglected. Such a seven-constant model was advocated by the writer (1977, 1980), but then in an elastoplastic form.

The latter model is considered attractive. In the first place since it unites previously isolated but well-established ideas for the strains induced upon primary loading. In the second place since the available data on unloading and reloading is approximated in a thermodynamically sound manner.

In section 2 we shall treat five constants by reviewing the relationships for γ, v_c and v_d respectively. We shall only consider primary loading in triaxial compression, i.e. $\sigma_1' \geqslant \sigma_2' = \sigma_3'$, and we shall use

$$p = \frac{1}{3}(\sigma_1' + \sigma_2' + \sigma_3') \;,\; q = \sigma_1' - \sigma_3'$$

$$v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \;,\; \gamma = \varepsilon_1 - \varepsilon_3$$

Compressive stresses and contractive strains will be considered positive.

In section 3 we shall treat unloading-reloading behaviour on the basis of two additional constants. In the subsequent sections the attention will be focussed on predictions and performances. Many of the details omitted here for lack of space are reported in the last reference.

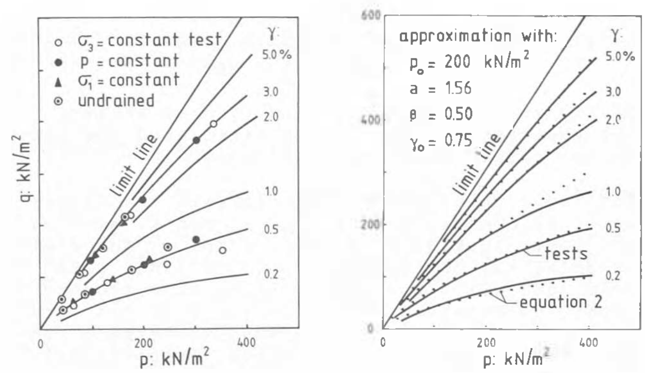

Fig. 1 (a) data from Tatsuoka & Ishihara for a loose sand
(b) experimental and predicted shear strain contours

2. TRIAXIAL COMPRESSION-FIRST LOADING

Kondner and Zelasko (1963) performed drained tests for various constant values of the mean stress p, and they showed that the results could be well approximated by the hyperbolic law $\gamma = q(A + B\gamma)$ where A and B depend on p. Meanwhile, a power law had been proposed by authors who studied the compressibility of sand under constant ratios of q/p. Using A/B = C and Bp = 1/a the above laws can be written as

$$\gamma = C\frac{q/p}{a-q/p} \quad , \quad v_c = v_o p^{\beta}/p_o^{\beta} \qquad (1)$$

for a sand at a given initial density. Here, C depends on p, p_o is some reference stress and a, v_o, β are true constants. The constant a stands for the ultimate value of q/p and it can be derived that $a = 6\sin\phi'/(3-\sin\phi')$. The value chosen for p_o influences v_o, since v_o represents the volume strain in isotropic compression up to $p = p_o$. Most published values for β are in the range between 1/3 and 1/2.
Hansen (1965) expressed C analogous to v_c to obtain

$$\gamma = \gamma_o \frac{p^{\beta}}{p_o^{\beta}} \frac{q/p}{a-q/p} \qquad (2)$$

where the constant γ_o represents the shear strain for $p = p_o$ and $q/p = a/2$. Virtually the same equation became well-known by the work of Duncan and Chang (1970). In the latter's formulation p_o is chosen to be the atmospheric pressure and in the first numerator p is replaced by the minor principal stress,but these differences are not essential. Other work has shown (see for instance section 3) that the above shear strain relationship is valid for all kinds of triaxial compression paths.

Shearing of sand involves contraction or dilation, even if the mean stress p is kept constant. This volume strain induced by shear stress is referred to as dilatancy. In this paper it will be denoted as v_d. Most authors agree that the ratio $dv_d/d\gamma$ depends on the existing stress ratio in the specimen considered. Here, dv_d and $d\gamma$ stand for small increments of v_d and γ respectively. The most convincing theory stems from Rowe (1962, 1971). This theory involves the constant k and yields

$$v = v_c + v_d \quad , \; dv_d = \frac{k\sigma_3' - \sigma_1'}{k\sigma_3' + \frac{1}{2}\sigma_1'} d\gamma \qquad (3)$$

Having formulated γ as a function of stress, the equation for dv_d can be integrated for any stress path, at least numerically.

The above relationships incorporate the dimensionless constants a, β, γ_o, v_o and k. We need the same constant β both for γ and v_c in order to model the following feature of sand. Data from constant stress ratio tests show proportional deformation (Rowe, 1971), i.e. the strain increases but the ratio between the strain components is constant. Vice versa, proportional deformation in truly strain controlled tests yields constant stress ratios (Gudehus, 1980).

3. UNLOADING-RELOADING

It is recalled that the above relationships are only valid for first loading. For the shear strain γ this means that equation (2) can be used as long as it predicts an increase of γ. For a better understanding of this criterion we consider shear strain contours as shown in Fig.1. Here, data from Tatsuoka and Ishihara (1975) are fitted by equation (2); the appropriate curvature in the contours was obtained for $\beta = 0.5$. We have first loading as long as the stress path intersects subsequent shear strain contours. In terms of plasticity theory this means that the (shear) yield locus is a shear strain contour. This statement is validated by the work of Stroud (1971) and Tatsuoka and Ishihara (1974). Thus, instead of introducing a new mathematical expression, we can simply describe the shear yield locus by means of the well-known equation (2).
Analogous to the shear strain relationship, the equation for v_c is only valid as long as it predicts an increase of v_c. This criterion leads to a volumetric yield locus (a cap) perpendicular to the p-axis in p,q-plane.

The elastic strains, which are recoverable upon unloading, are calculated from Hooke's law, e.g.

$$\varepsilon_1^e = \frac{1}{E}(\sigma_1' - \nu\sigma_2' - \nu\sigma_3') \quad , \; E = E_o(\frac{p}{p_o})^{1-\beta}$$

where ν is Poisson's ratio and E Young's modulus. Similar to Duncan-Chang (1970) and Lade (1977), ν and E_o are considered to be constants. However, in those studies Hooke's law is applied to increments of stress and strain, and this implies that the elastic strains are only recovered upon exact reversal of the loading path. For the secant approach, however, the strains are recovered even when the unloading path deviates from the loading path. Although the above equation is correct from the viewpoint of recovery and available test results (Vermeer, 1978), it is thermodynamically not sound. Therefore, the expression for E must be modified such that a strain energy function exists. Such a modification is achieved by the equations (Vermeer, 1980):

$$E = E_o(\sigma/p_o)^{1-\beta}$$

$$\sigma^2 = \frac{1}{3}\,\frac{1}{1-2\nu}(\sigma_1'^2 + \sigma_2'^2 + \sigma_3'^2 - 2\nu\sigma_1'\sigma_2' - 2\nu\sigma_2'\sigma_3' - 2\nu\sigma_3'\sigma_1')$$

For triaxial compression we have $\sigma_2' = \sigma_3'$, and σ may be written as

$$\sigma = p(1 + \frac{2}{9}\,\frac{1+\nu}{1-2\nu}\,\frac{q^2}{p^2})^{\frac{1}{2}}$$

Thus, σ is proportional to p in constant stress ratio paths, and the formulation complies with the experimental finding that the Young's moduli vary with the magnitude of p.
The energy function W with property $\varepsilon_i^e = \partial W/\partial\sigma_i'$ for i = 1,2,3 is

$$W = W_o\sigma^{1+\beta} \quad , \quad W_o = \frac{3}{1+\beta}\,\frac{1}{E_o}\,p_o^{1-\beta}$$

4. ON THE USE OF SCALE MODELS

In geomechanics prototype displacements are sometimes predicted from the results of scale model tests on the same sand at gravity scale 1. Let the model bed be carefully prepared such that the initial stress field is similar to the one for the prototype, i.e. $\sigma_m = \sigma_p/\lambda$, where σ_m and σ_p represent the stress fields for the model and the prototype respectively. The factor λ stands for the ratio of the prototype dimensions to the model dimensions. For simplicity, it is assumed that the loading programme does not induce pore pressures nor inertia effects nor very large strains. Furthermore, the ratio of external prototype loads to external model loads is exactly λ^2. Under these conditions the above stress-strain relationships can be used to derive that

$$s_p = \lambda^{1+\beta} s_m$$

where s_p, s_m stand for the prototype and scale model displacements respectively. The above law corresponds to the one used by Hettler and Gudehus (1980). Comparing the results of many model tests, they found $\beta \simeq 0.3$ for static loading and $\beta \simeq 0.4$ for repeated loading. Thus the present elastoplastic stress-strain law complies with some results of model tests.

5. A POWDER COMPACTION PROBLEM

A computer program for large strain problems has been written by Kloosterman and Lissenburg from Philips Research Laboratories at Eindhoven, The Netherlands, while the writer assisted in implementing the constitutive model. Practical use of such a program requires confidence in the numerical results obtained, and this has partly been achieved by consideration of a problem with a known solution. Such a problem is the computation of boundary tractions occurring during one-sided compaction of a fine ferric oxide powder in a cylindrical die, since experimental results exist to verify the computational results (see Strijbos and Vermeer, 1977). Some results for the cylindrical compact are presented here, since it indicates that realistic results can be obtained on the basis of the constitutive model considered.

Fig. 2a shows a cross section over the cylindrical die. It resembles the oedometer that is used in soil testing. However, the ratio of height to diameter of the powder sample is such that one-sided compression yields a strongly non-uniform stress and strain field in the interior of the sample. A segment of the cylindrical sample was therefore divided into 48 eight-noded isoparametric finite elements, while special interface elements were used to model the sliding of the powder along the wall. The dashed load-displacement curve in Fig. 2b was calculated using a great number of small loading steps. The dashed curves in Fig. 3 indicate the magnitude of the computed normal stresses at the boundary when the height of the sample had decreased from 32 mm to 24 mm. Comparison with the experimental findings shows that the numerical hindsight is accurate.

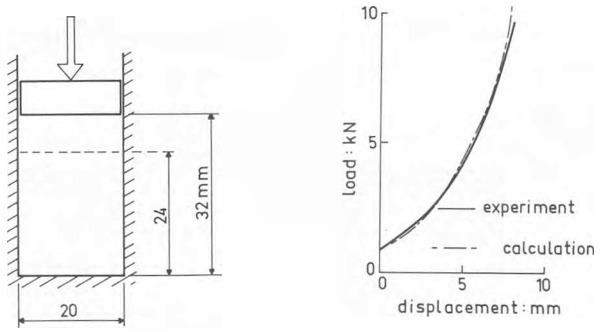

Fig.2. Die for powder compaction; Load-displacement curves.

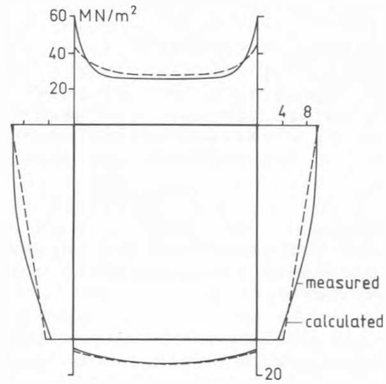

Fig.3. Normal stresses at the wall of the die.

The material constants used in the computation were obtained from a series of standard triaxial tests at very high confining stresses. Chosing the reference stress $p_o = 10$ MN/m^2, the elastic constants ν, E_o and the volume strain constants v_o, k were found to be

$$\nu = 0.0 \ , \ E_o = 300 \text{ MN/m}^2 \ , \ v_o = 0.4 \ , \ k = 4.8$$

respectively. At the high confining pressures considered only contractive strains v_d were measured. Nevertheless, it could be described by equation (3). The shear strain constants in equation (2) were found to be

$$\beta = 0.3 \ , \ \gamma_o = 0.083 \ , \ a = 1.5$$

Considering large strains, the logarithmic strain measure was used.

6. PREDICTED AND OBSERVED PORE PRESSURES

In this section genuine predictions (given before the event) will be considered. In contrast to the previous problem, the attention is focussed on water saturated sand. When a foundation on such a soil is rapidly loaded, excess pore pressures develop. The pore water then flows from regions of higher excess pore pressures to regions of lower excess pore pressures.

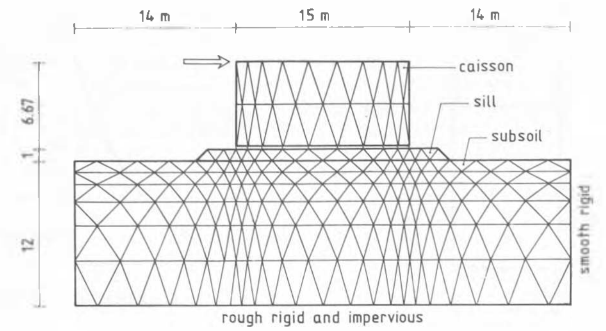

Fig.4. Finite element mesh for caisson on densified sand.

Equilibrium conditions apply to the stress field while conservation of mass and Darcy's law apply to the pore fluid. Such problems are referred to as consolidation problems. For elastoplastic consolidation of soil in plane state of strain, a computer program named "Elplast" was developed. The program was successfully used to predict the behaviour of a caisson in the Oosterschelde estuary of the Netherlands, and also in the case of a model caisson in a large wave tank. In the present paper the pore pressures induced by cyclic loading of the field caisson will be considered. For the predicted and observed displacements the reader is referred to a previous paper (Vermeer, 1978)

The finite element mesh used in the calculations is shown in Fig. 4. The caisson was assumed to be linear elastic, but very rigid with respect to the subsoil. The stress-strain properties of the coarse sill material were simply taken identical to the sand. In the coarse sill material, however, the excess pore pressures were assumed equal to zero. In situ measurements in the subsoil indicated 10^{-4}m/s for the coefficient of horizontal permeability and 0.4 x 10^{-4}m/s in the vertical direction. The programme of loading consisted of several parts, each with a specific variation of the horizontal load. A particular part of the programme is indicated in Fig. 5. The ratio of the arrow lengths for q_x and q_y indicates the moderate nature of the cyclic loading; the cyclic horizontal load, with a period of 3 seconds, is small with respect to the caisson weight. This implies that the loading induced relatively small stress reversals in the elements of the subsoil. Therefore, we expected little accumulation of strain in the subsequent cycles. Furthermore, we did not expect a gradual build up of pore pressures, since the medium dense soil could drain towards the coarse sill. It was thus expected that after some load repetitions the system would reach a cyclic steady state during which the soil would behave more or less elastically. An elastoplastic model with isotropic hardening, is suited for such a problem. As a rule it produces a short cyclic transient state and then a cyclic steady state (shakedown). Indeed, for the in situ test considered, the calculations showed a cyclic steady state after a few load repetitions. The pore pressure variations within such an "elastic" cycle are shown in Fig. 5. Here, the elastic behaviour was modelled by means of the constants

$\beta = 0.5$, $\nu = 1/3$, $E_o = 40000 kN/m^2$ for $p_o = 25 kN/m^2$

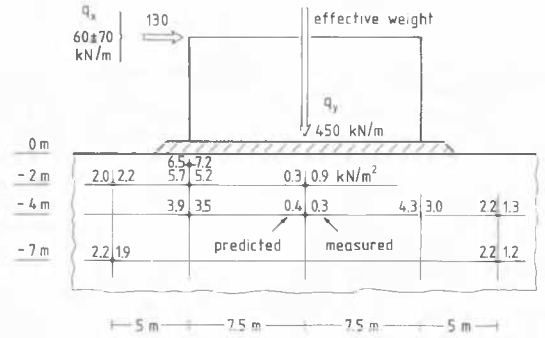

Fig.5. Pore pressure amplitudes for a cyclic steady state.

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