Constitutive Laws of Normally Consolidated Clay

Lois Constitutives de l'Argile Normalement Consolidée

R.L. WEI Vice Director of Geotechnical Eng. Div., Nanjing Hydraulic Research Institute, Nanjing, China

SYNOPSIS The same expression of yield locus as that previously (Wei 1964) obtained from test data is derived from the energy equation. Then by means of associated flow rule the general formula of the elasto-plastic stress-strain relationship and its simplified forms for some special cases are derived. It is shown that the prediction by the proposed model agrees with the tests better than the modified Cam-clay model. In fact, the latter is just a special case of the proposed model.

INTRODUCTION

Soon after Roscoe et al. (1963) had developed the Cam-clay model in which the yield surface takes bullet-shape, Wei(1964) obtained from the data of stress path of undrained triaxial compression tests that the yield locus of normally consolidated clay in the stress space (p,q) was generally elliptical. Later on, yield loci of elliptical shapes were also proposed by Roscoe and Burland (1968), Khosla and Wu (1976) and others. In fact, the former was just a special case of the general expression proposed by Wei, while the latter was identical with Wei's.

According to the definition of neutral loading that the plastic strain equals to zero as the stress path moves along the yield surface, the following expression of the yield locus was obtained (Wei, 1964):

$$\left(\frac{p-\gamma p_o}{\alpha}\right)^2 + \left(\frac{q}{\beta}\right)^2 = p_o^2 \qquad (1)$$

where p and q are the mean normal effective stress and deviatoric stress respectively; p_o is the equivalent isotropic consolidation pressure of the given stress state (p,q); α, β and γ are parameters which determine the geometry of the yield surface, and they are not all independent. From the relative position of the yield surface with reference to the critical state line (whose slope is M) and q-axis, it is evident that (Fig. 1):

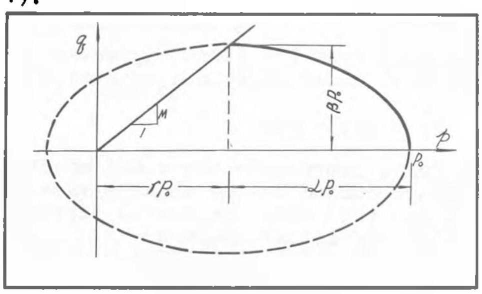

Fig. 1 Yield Surface

$$\alpha = 1-\gamma; \quad \beta = M\gamma \qquad (2)$$

When $\alpha = \gamma = 1/2$, eq. (1) reduces to the expression of yield surface of the modified Cam-clay model.

ENERGY EQUATION

The increment of the applied plastic work can be written as:

$$\delta W = p\,\delta v^p + q\,\delta\varepsilon^p \qquad (3)$$

where δv^p and $\delta\varepsilon^p$ are plastic volumetric and distortional strain increments respectively. It is assumed that during the deformation of clay, the rate of internal energy dissipation with respect to some generalized plastic strain not only depends upon the mean normal stress p but also is a function of the stress ratio η:

$$\frac{\delta E}{\delta\bar{\varepsilon}^p} = pR(\eta)$$

Where $\delta\bar{\varepsilon}^p = \sqrt{(\alpha\,\delta v^p)^2 + (\beta\,\delta\varepsilon^p)^2}$ is defined as the generalized plastic strain increment which is a measure of the total plastic strain increment tensor $\delta\varepsilon_{ij}^p$; α and β are the parameters identical with those in formula (1); $R(\eta)$ is called the factor of internal energy dissipation. Therefore the energy dissipated during the deformation of clay per unit volume is

$$\delta E = pR(\eta)\,\delta\bar{\varepsilon}^p \qquad (4)$$

Put

$$R(\eta) = \sqrt{\frac{1}{\alpha^2} + \left(1-\frac{\gamma^2}{\alpha^2}\right)\frac{\eta^2}{\beta^2}} \qquad (5)$$

then eq. (4) satisfies the following two extreme conditions:

In isotropic compression,

$$(\delta E)_{\eta=0} = p\,\delta v^p$$

In critical state,

$$(\delta E)_{\eta=M} = Mp\,\delta\varepsilon^p$$

Hence it is suggested that eq. (4) would be a reasonable generalization of the above two particular conditions. According to the balance between the applied work and the internal energy, i.e. $\delta W=\delta E$, a new energy equation different from those in the Cam-clay and its modified model may be proposed:

$$p\delta v^p + q\,\delta\varepsilon^p = pR\sqrt{(\alpha\,\delta v^p)^2+(\beta\,\delta\varepsilon^p)^2}$$

Applying the normality condition the yield locus as shown by eq. (1) can also be obtained from the above equation.

ELASTO-PLASTIC STRESS-STRAIN RELATIONSHIP

As mentioned above, the equation of yield locus of normally consolidated clay may be written in the form

$$f=\sqrt{\left(\frac{p-\gamma p_o}{\alpha}\right)^2+\left(\frac{q}{\beta}\right)^2}=p_o \qquad (1')$$

or

$$\frac{p_o}{p}=\frac{\alpha^2R-\gamma}{\alpha^2-\gamma^2} \qquad (7)$$

According to the associated flow rule the plastic strss-strain relationships are

$$\delta v^p=\frac{\partial p_o}{\partial p}\,d\omega=\frac{1-\gamma R}{(\alpha^2-\gamma^2)R}\,d\omega \qquad (8)$$

$$\delta\varepsilon^p=\frac{\partial p_o}{\partial q}\,d\omega=\frac{\eta}{\beta^2R}\,d\omega \qquad (9)$$

Moreover it is obtained from the anisotropic compression and swelling curves that

$$\delta v^p=(\lambda-\varkappa)\frac{\delta p_o}{p_o}=\frac{\lambda-\varkappa}{(\alpha^2R-\gamma)R_p}\cdot\left[(1-\gamma R)\delta p+\frac{\eta(\alpha^2-\gamma^2)}{\beta^2}\delta q\right] \qquad (10)$$

where λ and $\varkappa$ are the slope of compression and swelling curves on the v-ln p diagram respectively. By comparing eq. (5) with eq. (10), it is obtained

$$d\omega=\frac{(\lambda-\varkappa)(\alpha^2-\gamma^2)}{(\alpha^2R-\gamma)p}\left[\delta p+\frac{(\alpha^2-\gamma^2)\eta}{\beta^2(1-\gamma R)}\delta q\right]$$

Substituting it into eq. (9), the latter becomes

$$\delta\varepsilon^p=\frac{(\lambda-\varkappa)(\alpha^2-\gamma^2)\eta}{\beta^2(\alpha^2R-\gamma)\,Rp}\left[\delta p+\frac{(\alpha^2-\gamma^2)}{\beta^2(1-\gamma R)}\delta q\right] \qquad (11)$$

Combining eqs. (10) and (11), the plastic stress-strain relationship may be written in matrix form

$$\begin{Bmatrix}\delta v^p\\ \delta\varepsilon^p\end{Bmatrix}=\frac{\eta(\lambda-\varkappa)}{(\alpha^2R-\gamma)Rp}\begin{bmatrix}\frac{1}{\eta}(1-\gamma R) & \frac{\alpha^2-\gamma^2}{\beta^2}\\ \frac{\alpha^2-\gamma^2}{\beta^2} & \frac{\eta(\alpha^2-\gamma^2)^2}{\beta^4(1-\gamma R)}\end{bmatrix}\begin{Bmatrix}\delta p\\ \delta q\end{Bmatrix} \qquad (12)$$

The elastic strain increments are

$$\delta v^e=\varkappa\,\delta p/p$$

$$\delta\varepsilon^e=\delta q/G_o$$

where G_o is the elastic shear modulus. Adding the elastic stress-strain relationship to eq. (12) the following elasto-plastic stress-strain equation may be obtained

$$\begin{Bmatrix}\delta v\\ \delta\varepsilon\end{Bmatrix}=M_d\begin{bmatrix}M_k & 1\\ 1 & M_g\end{bmatrix}\begin{Bmatrix}\delta p\\ \delta q\end{Bmatrix} \qquad (13)$$

where $M_d=D/p$, $M_k=1/\mu$, $M_g=p/(DG)$,

$$\mu=\frac{(\lambda-\varkappa)(\alpha^2-\gamma^2)\eta}{\beta^2[\lambda(1-\gamma R)-\varkappa(1-\alpha^2R^2)]},$$

$$D=\frac{(\lambda-\varkappa)(\alpha^2-\gamma^2)}{\beta^2(\alpha^2R-\gamma)R}, \qquad (14)$$

$$1/G=1/G^p+1/G_o,$$

$$G^p=\frac{\beta^4(1-\gamma R)(\alpha^2R-\gamma)Rp}{(\lambda-\varkappa)(\alpha^2-\gamma^2)^2\eta^2}$$

The inverse of eq. (13) is

$$\begin{Bmatrix}\delta p\\ \delta q\end{Bmatrix}=\frac{1}{M_d(M_kM_g-1)}\begin{bmatrix}M_g & -1\\ -1 & M_k\end{bmatrix}\begin{Bmatrix}\delta v\\ \delta\varepsilon\end{Bmatrix} \qquad (15)$$

This is the general form of the elasto-plastic stress-strain relationship of the normally consolidated clay which can be used in finite element analysis of displacement method, where the number of parameters needed to be determined experimentally is altogether five, i.e. λ, $\varkappa$, M, G_o and α (or γ).

Some special cases are discussed in the following.

Undrained Test

It is apparent from the first expression of eq. (13) that $\delta v=0$ under undrained condition, then

$$\delta u=-\delta p=\mu\,\delta q \qquad (16)$$

Hence $\mu=\delta u/\delta q$ represents the coefficient of pore pressure caused by the shear stress. Substituting eq. (16) into the second expression of eq. (13), the latter becomes

$$\delta\varepsilon_u=\delta q/G_u \qquad (17)$$

where $G_u = \delta q/\delta\varepsilon_u$ is called the undrained shear modulus, and $1/G_u = 1/G_u^p + 1/G_o$, while

$$G_u^p = \frac{\beta^4(1-\gamma R)[\lambda(1-\gamma R)-\varkappa(1-\alpha^2R^2)]}{\varkappa(\lambda-\varkappa)(\alpha^2-\gamma^2)^2\eta^2} \tag{18}$$

is the plastic undrained shear modulus. Because

$$\delta q/p = \delta\eta + \eta\delta p/p$$

substituting the above equation together with eq. (16) into eq. (17), the undrained shear stress-strain relationship may be obtained as following

$$\delta\varepsilon_u = \frac{p}{(1+\mu\eta)G_u}\delta\eta \tag{19}$$

Drained Test

In the pure shear tests, the mean normal stress is constant, i.e. $\delta p=0$. Then from eq. (13), it is obtained

$$\begin{aligned} \delta v &= D\,\delta\eta \\ \delta\varepsilon &= p\,\delta\eta/G \end{aligned} \tag{20}$$

Hence $D=\delta v/\delta\eta$ is called the coefficient of dilatancy, and $G = \delta q/\delta\varepsilon$ is the drained shear modulus, G^p [cf. eq. (14)] is the plastic drained shear modulus.

In the conventional drained triaxial compression tests $\delta p = \delta q/3$, hence $\delta q/p = 3\delta\eta/(3-\eta)$. Substituting into the second expression of eq. (13), it is obtained

$$\begin{aligned} \delta v &= (\frac{1}{\mu}+3)\frac{D}{3-\eta}\delta\eta \\ \delta\varepsilon &= (D+\frac{3p}{G})\frac{\delta\eta}{3-\eta} \end{aligned} \tag{21}$$

WHen $\alpha = \gamma$

For some soils, such as the undisturbed Ninpo clay, it is obtained by tests $\alpha=\gamma$. Then the equation of the yield locus is reduced to

$$\frac{p_o}{p} = \frac{M^2+\eta^2}{M^2} \tag{22}$$

While the equations of stress-strain relations [eqs. (13), (15), (19), (20), (21)] still remain unchanged, the expressions of some parameters and moduli have been changed. For example,

$$\begin{aligned} \mu &= \frac{2(\lambda-\varkappa)\eta}{\lambda(M^2-\eta^2)+2\varkappa\eta^2} \\ D &= \frac{2(\lambda-\varkappa)\eta}{M^2+\eta^2} \\ G^p &= \frac{(M^4-\eta^4)p}{4(\lambda-\varkappa)\eta^2} \\ G_u^p &= \frac{(M^2-\eta^2)[\lambda(M^2-\eta^2)+2\varkappa\eta^2]p}{4\varkappa(\lambda-\varkappa)\eta^2} \end{aligned} \tag{23}$$

Eq. (22) is identical with the eq. of yield locus of modified Cam-clay, therefore the latter is just a special case of the proposed model when $\alpha=\gamma$. Moreover the proposed stress-strain relationships take the elastic shear strain into account, this is its another feather different from the modified Cam-clay model.

COMPARISON OF PREDICTED AND EXPERIMENTAL VALUES

In order to certify tentatively the proposed model, the results calculated by eqs. (19) and (21) are compared with the available data of the conventional drained and undrained triaxial compression tests. The predicted stress-strain curves coincide fairly well with the test data as shown in Figs. 2, 3 and 4, and the agreement is better than that of the modified Cam-clay model. The pore pressure or effective stress path in the undrained test may be calculated from the following equation

$$\delta u = \mu p\,\delta\eta/(1+\mu\eta) \tag{24}$$

When $\alpha=\gamma$, this equation reduces to

$$\delta u = Dp\,\delta\eta/\lambda \tag{25}$$

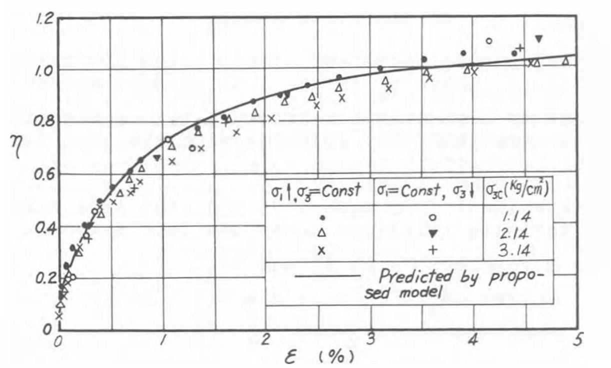

Fig. 2 Undrained Stress-Strain Curve of Undisturbed Ninpo Clay

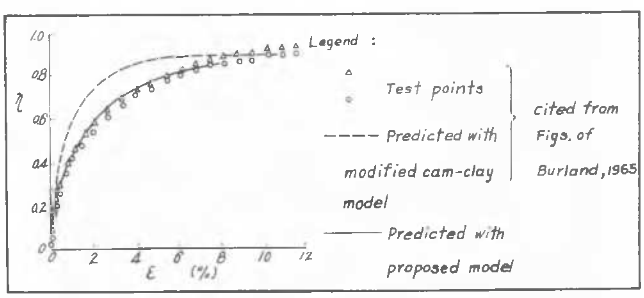

Fig. 3 Undrained Stress-Strain Curve of Remoulded Kaolin

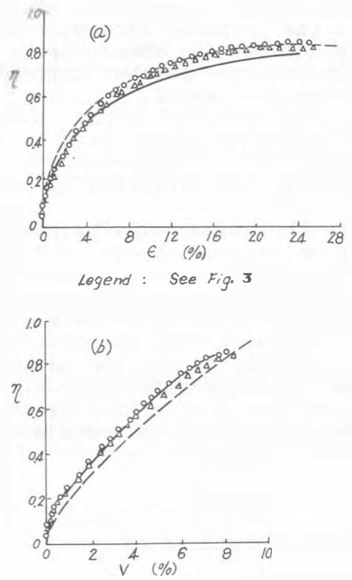

Fig. 4 Drained Stress-Strain Curves of Remoulded Kaolin

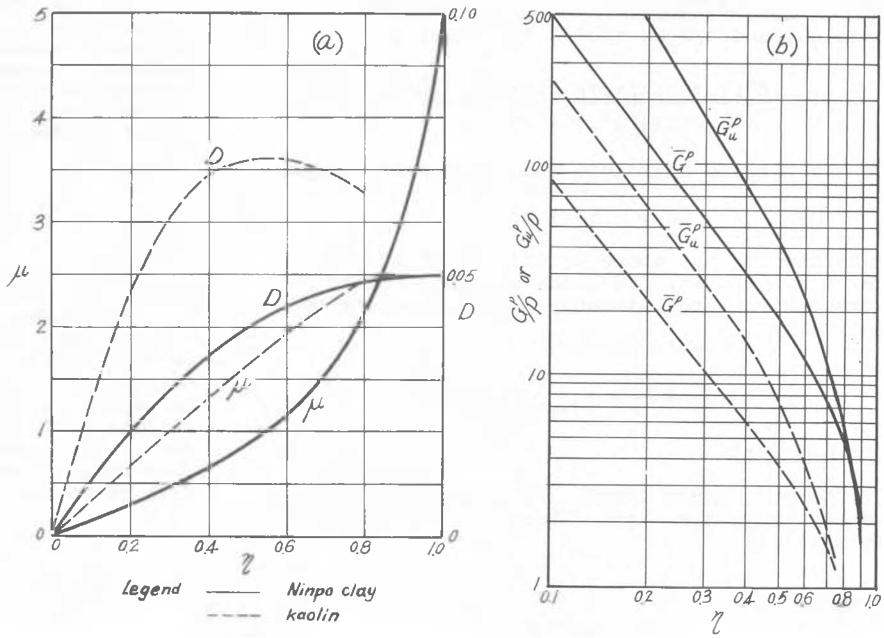

Fig. 6 Deformation Parameters of Ninpo Clay and Kaolin

The calculated undrained stress paths are shown in Fig. 5, where p_c is the initial mean normal stress or consolidation pressure before shearing. It is seen that they coincide with the test data too.

It is evident from eqs. (14) and (18) that there are definite relations among the four deformation parameters μ, D, G^p_u and G^p:

$$D/\mu = \varkappa G^p_u/G^p; \qquad 1/\mu = \varkappa/D + DG^p/p$$

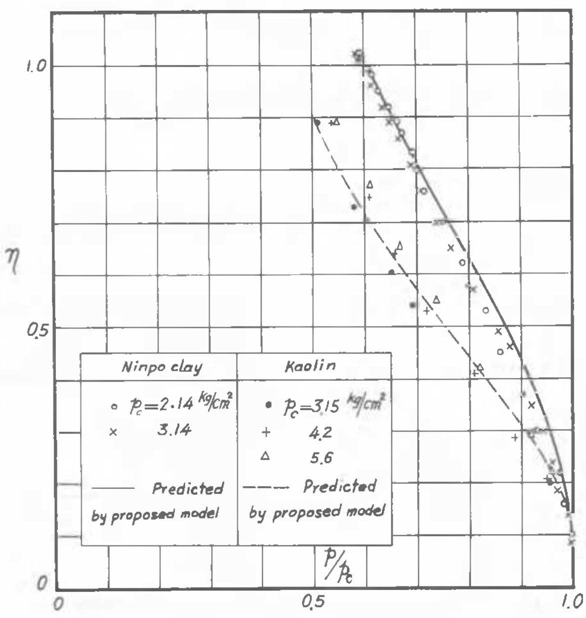

Fig. 5 Strss Paths in Undrained Tests of Ninpo Clay and Kaolin

In Fig. 6, the deformation parameters of the undisturbed Ninpo clay and the remoulded kaolin are plotted as functions of stress ratio η. Consequently the variation of deformation parameters and their influence may be unifiedly examined by means of proposed model. This would be significant for further studying of the deformation behaviour of soils and its mechanism.

REFERENCES

Burland, J.B. (1965). The yielding and dilation of clay. Geotechnique, (15), 2, 211-214.

Khosla, V. K. and Wu, T. H. (1976). Stress-strain behaviour of sand. ASCE J. GT (102), 4, 303-321.

Roscoe, K. H. and Burland, J. B. (1968). On the generalized stress-strain behaviour of 'wet' clay. "Engineering Plasiticity" Cambridge Univ. Press, 535-609.

Roscoe, K. H., Schofield, A. N. and Thurairajah, A. (1963). Yielding of clay in states wetter than critical. Geotechnique, (13), 3, 211-240.

Wei. R. L. (1964). The plastic potential of normally consolidated clay. Journal of Hydraulic Engineering, (in Chinese) Beijing, 6, 9-20.