Identification of Earth Pressure on Tunnel Liners

Détermination de la Pression du Sol sur le Revêtemans de Tunnels

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SYNOPSIS A numerical procedure is presented for the "identification", or back calculation, of the earth pressure acting on tunnel liners on the basis of in situ measurements performed on the full scale structure. The "optimal" earth pressure distribution is back calculated by minimizing an error function defined on the basis of the inverse equations of the structural problem and of a series of additional conditions. The results obtained by applying the procedure to a significant problem are presented.

INTRODUCTION

In many practical cases it is of interest to determine the pressure exerted by the ground on tunnel liners after their installation. To this purpose, two basic approaches are commonly adopted. The first one consists in the direct measurement of the earth pressure by means of pressure cells installed on the liner. The second method consists in the back calculation of the earth pressure on the basis of displacements measured at some locations of the liner (see e.g., Kovari et al., 1977). It can be observed that both methods have advantages and shortcomings in their practical application. For instance, the measurements required by the second method are probably cheaper and more reliable than those required by the first one; on the other hand, the second method requires an analytical tool for transforming displacement data into load distributions and this may generate some problems when dealing with structures of complex shape or with non-linear behaviour.

A general approach for the identification of the earth pressure acting on structures of any shape, on the basis of in situ measurements, has been presented by Gioda and Jurina (1980). The method can be seen as an extension of the classical back analysis: in fact, other conditions can be taken into account in addition to the inverse equations of the problem and various types of in situ measurements can be considered (including displacements and rotations of points of the structure; values of the earth pressure at some locations; values of concentrated loads; etc.). The "optimal" earth pressure distribution is determined by minimizing a suitably defined error function. The procedure allows for non-linear behaviour of both structure and surrounding medium.

In this paper the application of the above procedure to the calculation of the earth pressure on tunnel liners is discussed, and the results concerning an illustrative example are presented.

In what follows, underlined capital and lower case letters denote matrices and column vectors, respectively; a superscript T means transpose.

PROBLEM FORMULATION

Consider a tunnel liner of general shape and assume, for simplicity, that after its installation a plane strain situation normal to the tunnel axis is reached. Under this condition, the liner can be discretized into a mesh of linear elements at whose nodes two displacement components and one rotation are defined. Let the finite elements be grouped in n_s sets S^i. For the i-th set, the earth pressure components normal, p_n^i, and tangent, p_t^i, to the liner can be approximated by linear combinations of suitably chosen functions.

$$p_n^i = \underline{\ell}_n^{iT}(s^i)\cdot\underline{a}_n^i \quad ; \quad p_t^i = \underline{\ell}_t^{iT}(s^i)\cdot\underline{a}_t^i \qquad (1a,b)$$

In eqs.(1), $\underline{\ell}_n^i$ and $\underline{\ell}_t^i$ are the approximating function vectors; s^i is the curvilinear abscissa along the liner for zone S^i and $\underline{a}_n^i$ and $\underline{a}_n^i$ are the unknown coefficient vectors.

Denoting with N^i the matrix of the shape functions relating the displacements of the nodes of the i-th set to the displacement functions, the principle of virtual works allows one to define the nodal force vector $\underline{f}^i$, equivalent to the earth pressure distribution, as follows:

$$\underline{f}^i = \int_{S^i} \underline{N}^{iT}\,\underline{T}\,\underline{p}^i\,ds \qquad (2)$$

In eq.(2), $\underline{T}$ is a transfer matrix relating the pressure components in the n,t reference frame to those in the global reference system and $\underline{p}^i$ is a vector having p_n^i and p_t^i as entries.

By substituting eqs.(1) into eq.(2), and by writing eq.(2) for all the element sets, the following relationship is obtained for the vector of nodal forces $\underline{f}$ equivalent to the earth pressure distribution on the entire liner:

$$\underline{f} = \underline{S}\cdot\underline{a} \qquad (3)$$

Vector $\underline{a}$ contains all the unknown coefficients of the functions approximating the earth pressure distribution and $\underline{S}$ is a matrix depending on the liner geometry and on the type of approximating functions.

If linear elastic behaviour is assumed for the liner, the relationship between nodal forces and nodal displacements $\underline{u}$ can be expressed in the following well known form:

$$\underline{K} \cdot \underline{u} = \underline{f} \tag{4}$$

where $\underline{K}$ is the stiffness matrix of the assembled finite element system.

The in situ measurements performed on the full scale structure provide a certain set of nodal displacements, $\underline{u}^*$, and, possibly, the earth pressures, $\underline{p}^*$, at some locations of the liner. These data allow one to define the values of some entries of vector $\underline{u}$ and, through eq.(2), of some entries of vector $\underline{f}$ (cf.eq.4). The identification problem, then, consists in finding the set of parameters $\underline{a}$ that fulfil eq.(4), through eq.(3), minimizing the error existing between the input data (i.e. $\underline{u}^*$ and $\underline{p}^*$) and the corresponding data obtained by the solution of eq.(4) and by eqs.(1).

In order to define the equations governing the identification problem, let vector $\underline{u}^\circ$ group the displacement components that have to be constrained in order to eliminate any rigid movement of the liner. Taking into account eq.(3), eq.(4) can be partitioned as follows:

$$\begin{bmatrix} \underline{K}_{11} & \underline{K}_{12} & \underline{K}_{13} \\ \underline{K}_{21} & \underline{K}_{22} & \underline{K}_{23} \\ \underline{K}_{31} & \underline{K}_{32} & \underline{K}_{33} \end{bmatrix} \cdot \begin{Bmatrix} \underline{u}^\circ \\ \underline{u}^* \\ \underline{u}_f \end{Bmatrix} = \begin{bmatrix} \underline{S}_1 \\ \underline{S}_2 \\ \underline{S}_3 \end{bmatrix} \cdot \underline{a} \tag{5}$$

where vector $\underline{u}_f$ collects the free (unknown) displacements.

Referring the nodal displacements to those of vector $\underline{u}^\circ$, the entries of vector $\underline{u}^\circ$ vanish. Taking into account that the matrix composed by submatrices $\underline{K}_{22}$, $\underline{K}_{23}$, $\underline{K}_{32}$ and $\underline{K}_{33}$ is not singular, because of the very nature of vector $\underline{u}^\circ$, a static condensation can be performed on eq.(5) by eliminating the free displacements $\underline{u}_f$. Some algebraic manipulations lead to

$$\underline{C}\ \underline{a} = \underline{u}^* \quad ; \quad \underline{D}\ \underline{a} = \underline{o} \tag{6a,b}$$

where

$$\underline{C} = [\ \underline{K}_{22} - \underline{K}_{23}\ \underline{K}_{33}^{-1}\ \underline{K}_{32}\ ]^{-1} \cdot [\ \underline{S}_2 - \underline{K}_{23}\ \underline{K}_{33}^{-1}\ \underline{S}_3\ ] \tag{6c}$$

$$\underline{D} = \underline{K}_{12}\ \underline{C} + \underline{K}_{13}\ \underline{K}_{33}^{-1} \cdot [\ \underline{S}_3 - \underline{K}_{32}\ \underline{C}\ ] - \underline{S}_1 \tag{6d}$$

Eq.(6a) relates the parameters $\underline{a}$ describing the unknown pressure distribution to the known displacements and eq.(6b) insures that no "fictitious" nodal reactions are generated at the nodes where the displacements $\underline{u}^\circ$ are defined.

The measured earth pressure components $\underline{p}^*$ can be expressed, through eqs.(1), as linear relations between some of the coefficients of vector $\underline{a}$, i.e.

$$\underline{L} \cdot \underline{a} = \underline{p}^* \tag{7}$$

In eq.(7), matrix $\underline{L}$ is composed of vectors $\underline{\ell}_n$ and $\underline{\ell}_t$ computed at the locations where the earth pressure components are measured. By assembling eqs.(6) and (7) in an unique system, one obtains

$$\begin{bmatrix} \underline{C} \\ \underline{L} \\ \underline{D} \end{bmatrix} \cdot \underline{a} = \begin{Bmatrix} \underline{u}^* \\ \underline{p}^* \\ \underline{o} \end{Bmatrix} \tag{8}$$

By applying the "least square" minimization procedure to eq.(8), the following final relation, leading to the "optimal" vector $\underline{a}$, is arrived at:

$$\begin{bmatrix} \underline{C} \\ \underline{L} \\ \underline{D} \end{bmatrix}^T \cdot \begin{bmatrix} \underline{C} \\ \underline{L} \\ \underline{D} \end{bmatrix} \cdot \underline{a} = \begin{bmatrix} \underline{C} \\ \underline{L} \\ \underline{D} \end{bmatrix}^T \cdot \begin{Bmatrix} \underline{u}^* \\ \underline{p}^* \\ \underline{o} \end{Bmatrix} \tag{9}$$

Note that other conditions, expressed by linear combinations of coefficients $\underline{a}$, could be considered in the problem formulation. For instance, one may impose continuity between the pressure distributions and their derivatives at the boundary between one set of elements and the next. Concentrated loads can be taken into account as well.

The solution of the identification problem is strongly influenced by three characteristics of the experimental data, i.e. their number, their "quality" and the errors affecting them. The "quality" of the experimental information concerns, e.g., the locations in which the measurements are performed, the directions of the displacements measured, etc. For instance, it is easy to see that the earth pressure distribution on the entire liner cannot be identified on the basis of displacement and pressure measurements concentrated close to the tunnel crown only.

It is well know that every experimental measurement is affected by errors that depend on the type of measured quantity, on the instrument adopted, on the field conditions, etc. An approximated way to take into account these errors consists in associating to each experimental information a "weight" depending upon the measurement accuracy. Other approaches, leading to constrained minimization problems, can be adopted in order to allow for the influence of input errors. A discussion on this point falls outside the limits of this study and will constitute matter for a separate paper.

ILLUSTRATIVE EXAMPLE

The approach described in the preceding section was applied to the back calculation of the earth pressure acting on the liner shown in fig.1 . For the purposes of this example the "experimental" data were generated numerically by means of a two dimensional finite element analysis assuming a geometry (cf. fig.1a) similar to that adopted by Sakurai and Yamamoto (1976). The displacement distribution along the liner, as obtained by this analysis, is reported in fig.2 . Note that in fig.2 the displacements are referred to the nodal points of the mesh adopted for the identification analysis (cf. fig.1b) and that they are divided by the radius R of the upper part of the liner.

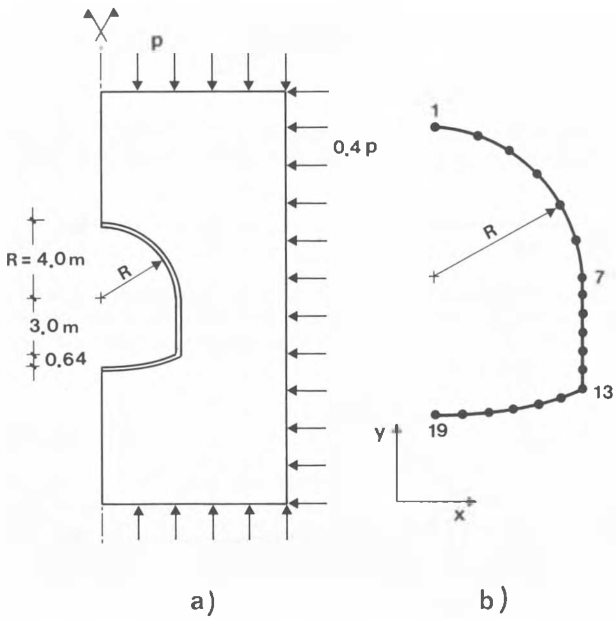

Fig. 1 Illustrative example geometry.

Two identification analyses, referred to in what follows as cases b) and c), were carried out. For both analyses the input data consist of the displacements of the nodes of fig.1b characterized by odd numbers; thus, only 20 displacement components were specified. In order to attempt a simulation of the experimental accuracy, the displacements were approximated to 1/10 mm.

For analysis b) the liner was divided into three zones: first zone from node 1 to node 7; second zone from node 7 to node 13; third zone from node 13 to node 19 (cf. fig.1b). Quadratic distributions of normal and tangential pressure components in the first and second zone, and linear distributions in the third zone were assumed. Continuity of both pressure components at the boundary between the first and second zone was imposed.

For analysis c) the liner was divided into two zones: first zone from node 1 to node 13 and second zone from node 13 to node 19. In the first zone quadratic pressure distributions were assumed, while in the second zone the pressure distributions were linear.

The results of analyses b) and c), compared with the "real" pressure distribution a), are shown in fig. 3 . The computed distributions of the pressure component normal to the liner approximate the real distribution with a reasonable accuracy, while a certain difference can be observed between real and computed distributions of tangential pressure. This is due to the fact that (because of the liner geometry) the liner deformation and, in turn, the displacements adopted as input data for the identification analyses strongly depend upon the pressure normal to the liner, while they are weakly influenced by the distribution of tangential pressure.

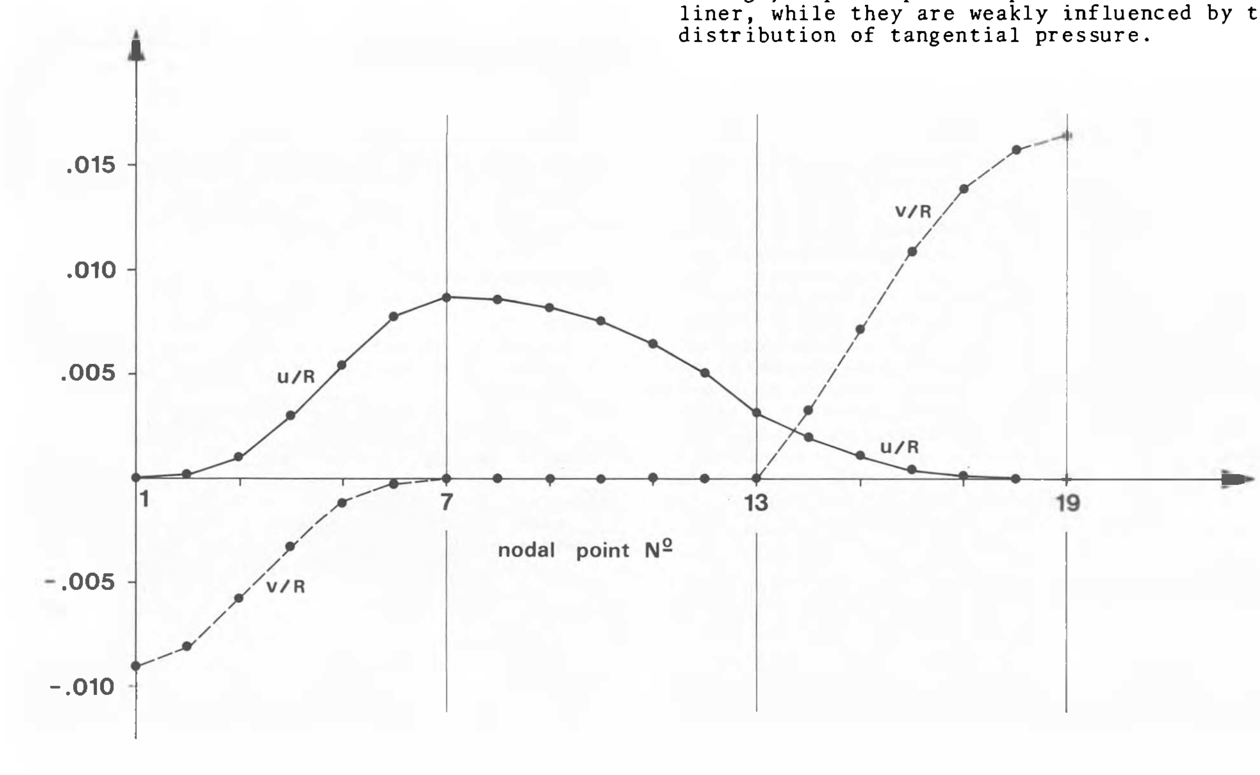

Fig. 2 Displacement distribution along the liner (u and v are the displacement components in the x and y directions, respectively).

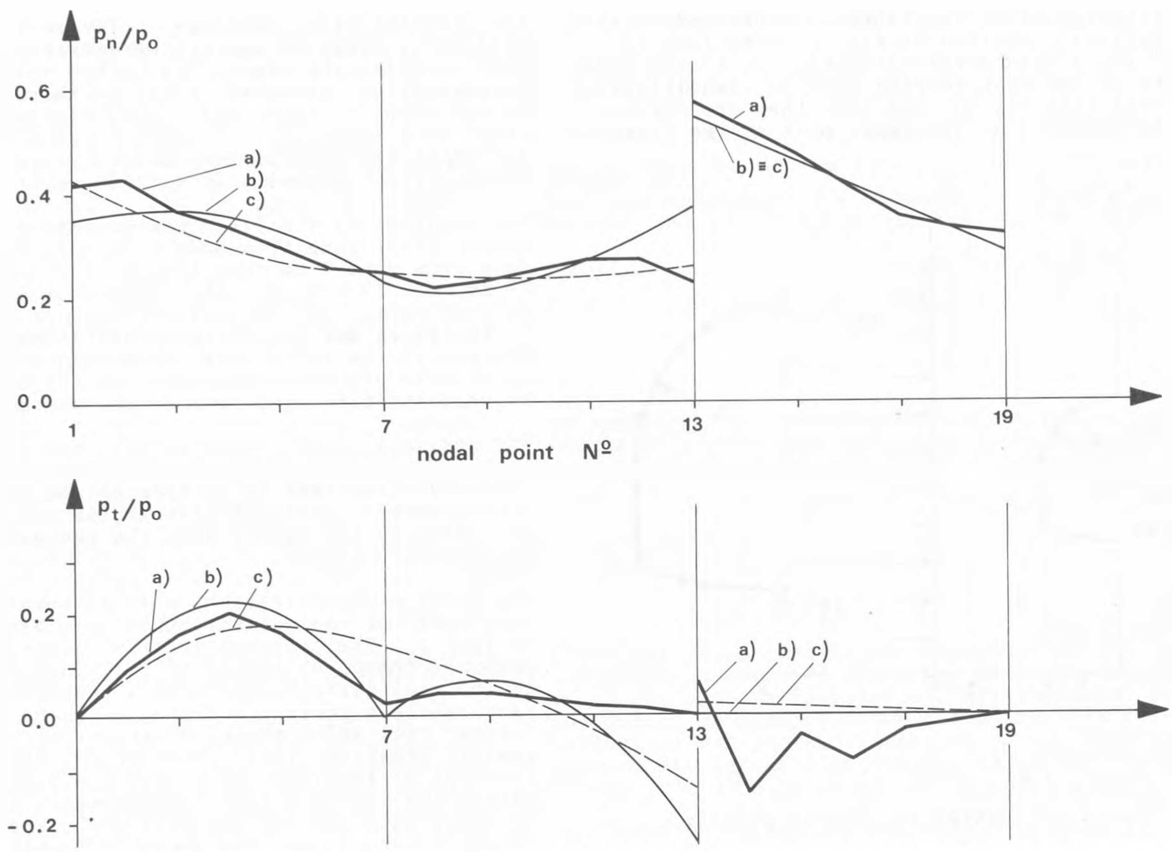

Fig. 3 Normal, p_n, and tangential, p_t, pressure distributions along the liner. a) "real" pressure distributions; b) and c) results of the identification analyses.

CONCLUSIONS

A numerical procedure has been outlined for the identification, or back calculation, of the earth pressure acting on tunnel liners, on the basis of in situ measurements performed on the full scale structure. Various types of measurements can be adopted as input data, e.g. displacements and rotations of structural points, values of the earth pressure at some locations, values of concentrated forces, etc. Such a procedure represents an useful tool for checking the agreement between the earth pressure "predicted" by well established theories and the actual pressure distribution. It could be also adopted, in the spirit of Terzaghi's "observational method", for detecting possible variations of the earth pressure during the construction works and, if necessary, for modifying the structural characteristics of the liner.

An application to a simple problem has been presented. The results of analyses indicate that the procedure could be applied to the solution of problems of practical interest. This suggests further efforts for research on this topic, especially related to assessing the influence of experimental errors on the identification results.

ACKNOWLEDGEMENTS

This study is part of a research project supported by the C.N.R. (National (Italian) Research Council).

REFERENCES

Gioda, G., Jurina, L. (1980). Numerical identification of soil-structure interaction pressure. To appear on Int.J.Numer.Anal.Methods Geomech.

Kovari, K., Amstad, C. and Fritz, P. (1977). Integrated measuring technique for rock pressure determination. Proc.Int.Symp.Field Measurements in Rock Mechanics, 289-316, Zurich.

Sakurai, S., Yamamoto, Y. (1976). A numerical analysis of the maximum earth pressure acting on tunnel linings. Proc. 2nd Int.Conf.Numer. Methods Geomech., 821-833, Blacksburg.