Analytical Study of NATM

Etude Analytique du NATM

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SYNOPSIS An analytical method to estimate earth pressures and displacements of steel supports and shotcrete executed by the New Austrian Tunneling Method is newly proposed considering not only mechanical properties of linings and the ground but also some executive conditions. Because the analytical results can be verified by field measurements, this method is adequate for engineering applications. Therefore, such executive conditions as the interval of supports, the thickness of shotcrete, the advance velocity of a tunnel and the distance between tunnel face and location of lining construction may be effectively determined by the proposed approach.

INTRODUCTION

Recently, the New Austrian Tunneling Method(NATM) has been applied to such time dependent ground as soils, for which it should be noted not to cause surface displacements or damage of structures surrounding a tunnel. In the case of NATM, linings composed of steel supports and shotcrete are generally executed near the tunnel face. Thus, not only time dependent characteristics of the ground but also variational characteristics of tunnel displacement along tunnel axis should be taken into consideration in the analysis of earth pressure acting on linings. Furthermore, it is experimentally known that the earth pressure is significantly affected by various executive conditions. However, there is little information about effective tunnel construction considering executive conditions and mechanical properties of the ground and linings.

In this paper, such an analytical study of the NATM as applied to the viscoelastic ground is presented, taking the elastoplastic behavior of linings and executive conditions into account. Earth pressures and displacements of tunnel supports measured in the highly time dependent ground are compared with the analytical results. Subsequently, the effective execution of NATM is discussed, based on analytical results.

SURFACE DISPLACEMENT OF UNSUPPORTED TUNNEL

The solid curve in Fig.1, which is obtained by three dimensional elastic analysis of a circular tunnel having a diameter D under a uniform initial stress P (Ronken and Ghaboussi, 1975), shows variational characteristics of a radial displacement u on both real and virtual tunnel boundaries. From the above result, it may be possible to consider that the length of the span where the displacement u changes along the tunnel axis is approximately twice as long as the diameter D, and the nondimensional displacement curve may be approximated by the dotted curve

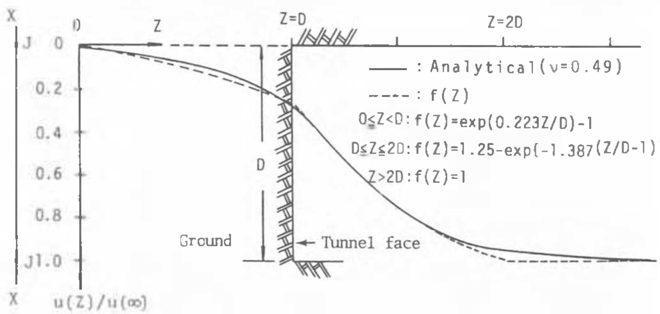

Fig.1 Variational characteristics of radial displacement u on both real and virtual tunnel boundaries in the elastic ground

expressed by the function f(Z), as shown in Fig.1. As the nondimensional displacement curve for the elastic ground is not much affected by Poisson's ratio of a given ground (Panet and Guellec, 1974), u can be expressed by the following equation,

$$u(Z)=\frac{DP}{4G}f(Z),\quad \text{(G: Shear modulus)}$$

If the tunnel is excavated with a constant velocity V in the viscoelastic ground, the displacement u of a point J on the any section X-X in Fig.1 is affected both by time dependent characteristics of the ground and by the advance velocity (Ito and Hisatake, 1979a), as follows.

$$u(t)=\frac{DP}{4}\int_0^t \phi(t-\tau)\frac{\partial}{\partial\tau}f(V\tau)\,d\tau \tag{1}$$

where t is the time after the origin of the coordinate Z (which moves with the face keeping the distance by D) has reached the section X-X, and $\phi(t)$ is a creep function concerning shear deformation. Therefore, $\phi(0)$ equals to 1/G.

ANALYSIS OF LINING

Time dependent earth pressure, axial stress and radial displacement of a lining composed of steel supports and shotcrete are analyzed, considering the elastoplastic behavior of the lining and time dependencies of both the modulus of elasticity and the strength of shotcrete. In this analysis, the lining is supposed to resist the earth pressure with its axial force of circumferential direction, since the thickness of the lining is very small in relation to the tunnel diameter in the case of NATM. And displacements of advance direction are neglected.

Analysis before yielding of support

It may be possible to consider that the radial displacements of a support and shotcrete(u_s and u_c respectively) which are executed at the same time are approximately equal. That is,

$$u_s(t_i) = u_c(t_i) \tag{2}$$

where t_i is the time after the lining has been executed. Those displacements which are brought about by earth pressures p_s and p_c acting on the support and the shotcrete, respectively, can be shown by the following equations,

$$u_s(t_{m+1}) = B_1 p_s(t_{m+1}),\ B_1 = a_0^2 L/(AE_e)$$
$$u_c(t_{m+1}) = \frac{a_0^2}{h}\sum_{j=1}^{m}\frac{p_c(t_{j+1})-p_c(t_j)}{E_c(t_j)} \tag{3}$$

where the time scale is divided into intervals by the time values t_j, j=1, 2, $\cdots$(m+1), with $t_1=0$ and $t_{m+1}=t_i$, and A and E_e are the cross section area and the modulus of elasticity of a support, respectively, L is the interval between supports, E_c and h are the time dependent modulus of elasticity and the thickness of shotcrete, respectively, and $a_0 = D/2$. And a total pressure p* acting on the lining is

$$p^*(t_{m+1}) = p_s(t_{m+1}) + p_c(t_{m+1}) \tag{4}$$

After solving Eqs.(2) and (3) about $p_c(t_{m+1})$, substitution of $p_c(t_{m+1})$ into Eq.(4) yields the following equation,

$$p_s(t_{m+1}) = Z_1(t_m)p^*(t_{m+1}) + Z_2(t_m)$$
$$Z_1(t_m) = \frac{1}{1+hLE_c(t_m)/(AE_e)} \tag{5}$$
$$Z_2(t_m) = Z_1(t_m)\left\{E_c(t_m)\sum_{j=1}^{m-1}\frac{p_c(t_{j+1})-p_c(t_j)}{E_c(t_j)} - p_c(t_m)\right\}$$

On the other hand, as the displacement of the tunnel is reduced by the lining, the boundary condition of displacement, after the lining is executed at the position apart from the tunnel face by distance L, becomes

$$u_s(t_{m+1}) = \Delta u(t_{m+1}) - \frac{a_0}{2}\int_{t_1}^{t_{m+1}}\phi(t_{m+1}-\tau)\frac{\partial}{\partial\tau}p^*(\tau)\,d\tau \tag{6}$$

where Δu is a displacement on the tunnel boundary which would be brought about after the time $t=t_0=(D+L)/V$ if the lining were not executed. And Δu and u_s can be calculated from Eqs.(1), (3) and (5), as follows,

$$\Delta u(t_{m+1}) = u(t_0+t_{m+1}) - u(t_0)$$
$$u_s(t_{m+1}) = B_1\{Z_1(t_m)p^*(t_{m+1}) + Z_2(t_m)\} \tag{7}$$

As the integral in Eq.(6) may be transformed to the finite approximation,

$$\int_{t_1}^{t_{m+1}}\phi(t_{m+1}-\tau)\frac{\partial}{\partial\tau}p^*(\tau)\,d\tau = \phi(0)p^*(t_{m+1}) - \phi(t_{m+1})p^*(0)$$
$$-\frac{1}{2}\sum_{j=1}^{m}\{p^*(t_{j+1})+p^*(t_j)\}\{\phi(t_{m+1}-t_{j+1})-\phi(t_{m+1}-t_j)\} \tag{8}$$

$p^*(t_{m+1})$ is determined successively in term of values already obtained, by substituting Eqs.(7) and (8) into Eq.(6),

$$p^*(t_{m+1}) = \frac{\Delta u(t_{m+1}) + M_1(t_m) + K_1(t_{m+1})}{M_2(t_m) + K_2(t_{m+1})} \tag{9}$$

$$M_1(t_m) = -B_1Z_2(t_m),\ M_2(t_m) = B_1Z_1(t_m)$$
$$K_1(t_{m+1}) = 0.25a_0\Big\{\{\phi(0)-\phi(t_{m+1}-t_m)\}p^*(t_m)$$
$$+\sum_{j=1}^{m-1}\{p^*(t_{j+1})+p^*(t_j)\}\{\phi(t_{m+1}-t_{j+1})-\phi(t_{m+1}-t_j)\}\Big\}$$
$$K_2(t_{m+1}) = 0.25a_0\{\phi(0)+\phi(t_{m+1}-t_m)\}$$

And p_s, p_c and u_s are easily determined by Eqs. (4), (5), (7) and (9).

Subsequently, the circumferential axial stresses σ_{ns} and σ_{nc} of the support and the shotcrete, respectively, are obtained by considering equilibrium conditions between earth pressures and axial stresses,

$$\sigma_{ns}(t_{m+1}) = a_0Lp_s(t_{m+1})/A,\ \sigma_{nc}(t_{m+1}) = a_0p_c(t_{m+1})/h \tag{10}$$

Analysis after yielding of support

The displacement of a support after its yielding can be shown by the following equation, considering Eqs.(3) and (10),

$$u_s(t_{m+1}) = a_0^2L\bar{p}_s/(AE_e) + a_0^2L\{p_s(t_{m+1})-\bar{p}_s\}/(AE_p) = B_2p_s(t_{m+1}) + B_3$$
$$B_2 = a_0^2L/(AE_p),\ B_3 = a_0\sigma_{ys}(E_p-E_e)/(E_eE_p)$$

where σ_{ys}, $\bar{p}_s$ and E_p are respectively the yield stress of a support, the earth pressure on a support at the yielding and the modulus of elasticity of a support after its yielding. In this case, earth pressures and displacements are easily calculated in the same manner as mentioned above, as follows,

$$p^*(t_{m+1}) = \frac{\Delta u(t_{m+1}) + M_4(t_m) + K_1(t_{m+1})}{M_3(t_m) + K_2(t_{m+1})}$$
$$p_s(t_{m+1}) = [M_3(t_m)p^*(t_{m+1}) - \{M_4(t_m)+B_3\}]/B_2$$
$$p_c(t_{m+1}) = p^*(t_{m+1}) - p_s(t_{m+1})$$
$$u_s(t_{m+1}) = B_2p_s(t_{m+1}) + B_3$$

$$M_3(t_m)=B_2/\{1+hB_2E_c(t_m)/a_0^2\}$$

$$M_4(t_m)=-B_3+M_3(t_m)\Big\{E_c(t_m)[hB_3/a_0^2 - \sum_{j=1}^{m-1}\frac{p_c(t_{j+1})+p_c(t_j)}{E_c(t_j)}]+p_c(t_m)\Big\}$$

and the axial stresses are calculated by the above equations and Eqs.(10).

COMPRESSIVE STRENGTH AND MODULUS OF ELASTICITY OF SHOTCRETE

Compressive strength of shotcrete σ_{yc} at t= 28 days after it is executed in a tunnel is approximately (JAPAN Soc. of Civil Engrs., 1974)

$$\sigma_{yc}(t=28\ \text{days})=15\sim20\ \text{MN/m}^2$$

Therefore, if $\sigma_{yc}(t=28\ \text{days})=18\ \text{MN/m}^2$ is adopted, the time dependent compressive strength of shotcrete $\sigma_{yc}(t)$ can be expressed by the following equation using the time dependent characteristic of the strength of rapid hardening concrete , as shown in Fig.2 (JAPAN Soc. of Civil Engrs., 1964),

$$\sigma_{yc}(t)=18\{C_1+C_2\ln(1+t)\}\ \text{MN/m}^2,\ (t\text{:Days}) \quad (11)$$

where C_1 and C_2 are shown in Fig.2.

Moreover, modulus of elasticity E_c of shotcrete also varies with time. And E_c is calculated by the following equation (Ban, 1952),

$$E_c(t)=\{1-0.0384\sigma_{nc}(t)/\sqrt{\sigma_{yc}(t)}\}^2\sqrt{\sigma_{yc}(t)}/\beta\ \text{kgf/cm}^2 \quad (12)$$

$$\beta=7.44\times10^{-5}\times0.402^{x},\quad x\text{:Water cement ratio}$$

After the stress of shotcrete reached the compressive strength shown by Eq.(11), E_c is decided as one percent of the result of Eq.(12).

ANALYTICAL RESULTS AND FIELD MEASUREMENTS

To verify the method mentioned above, field data of earth pressures and displacements of tunnel supports executed by NATM in the time dependent

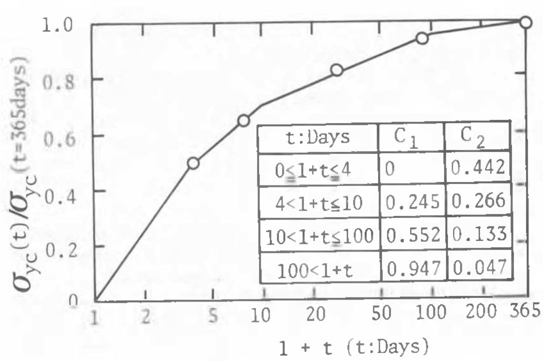

Fig.2 Time dependent characteristic of the strength of rapid hardening concrete, and constants C_1 and C_2 in Eq.(11)

ground (Adachi et al., 1969) are compared with analytical results. In addition, considerations about effective execution of NATM are given. Parameters of linings and executive conditions are shown in TABLE I.

TABLE I
Parameters of linings and executive conditions

A cm²	105.94	E_e MN/m²	206000	E_p MN/m²	3040
$\varepsilon_y=\sigma_{ys}/E_e$	0.0013	$\bar{L}$ m	12.5	L m	0.7
V m/day	1.5	D m	7.6	P MN/m²	7.88
1/a MN/m²	80	1/b MN/m²	80	h m	0.25
x	0.4				

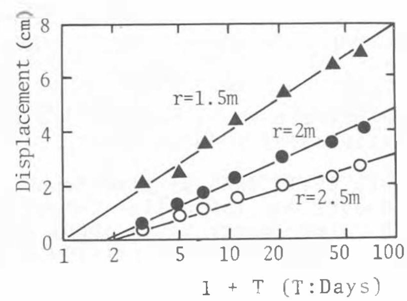

Fig.3 Radial displacements of the ground surrounding a circular tunnel

Creep function of the ground

Fig.3 shows the radial displacement Δu_0 of the ground surrounding a circular tunnel (D=3 m) without linings, which was experimentally excavated at a shallower site before a main tunnel (D=7.6 m) was excavated. Δu_0 does not include the displacement which was already brought about before the time t_1 when measurement started, and it can be shown by the following equation,

$$\Delta u_0(T)=c+m\cdot\ln(1+T),\ (T\text{:Days}) \quad (13)$$

where c and m are constants, and T is the time after the tunnel has been excavated. On the other hand, the analytical displacement u can be expressed by the following equation, assuming the plane strain condition,

$$u(T)=a_0^2P\phi(T)/(2r) \quad (14)$$

where r is the distance of radial direction from tunnel center. From Eqs.(13) and (14), it is easily understood that $\phi(T)$ may be expressed by a logarithmic function $\phi(T)=a+b\cdot\ln(1+T)$, and by substituting this function into Eq.(14), Δu_0 can be shown as

$$\Delta u_0(T)=u(T)-u(t_1)=\frac{a_0^2Pb}{2r}\{\ln(1+T)-\ln(1+t_1)\} \quad (15)$$

As the Measured value m in Eq.(13) corresponds to $a_0^2Pb/(2r)$ in Eq.(15), retardation shear modulus 1/b can be determined as 80 MN/m². Subsequently, the value of shear modulus 1/a may be estimated as 80 MN/m² by using Fig.4, which shows the relationship between 1/a and 1/b,

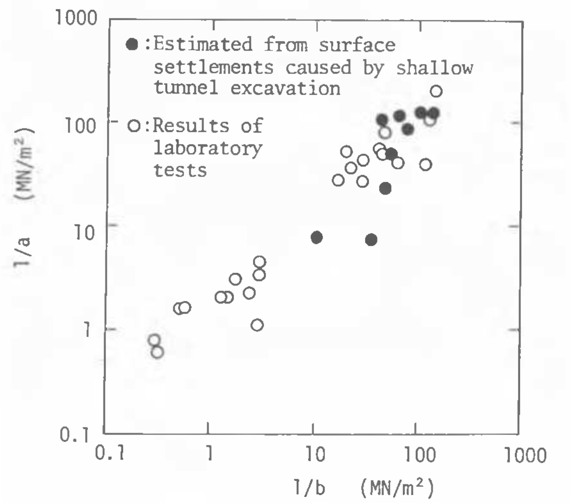

Fig.4 Relation between shear modulus 1/a and retardation shear modulus 1/b

obtained by both field data of time dependent surface settlements due to shallow tunnel excavation and laboratory creep tests about several soils and soft rocks (Ito and Hisatake, 1979b, 1981).

Analytical and measured results

Fig.5 shows the analytical and measured earth pressures acting on supports. From the analytical result, it can be understood that the earth pressure increases with time until it reaches the value $\bar{p}_s = A\sigma_{ys}/(a_0 L)$ at which the support yields. Then the increase of the earth pressure stops temporarily and resumes after the shotcrete yields. Such analytical behavior is in good agreement with that of field measurements.

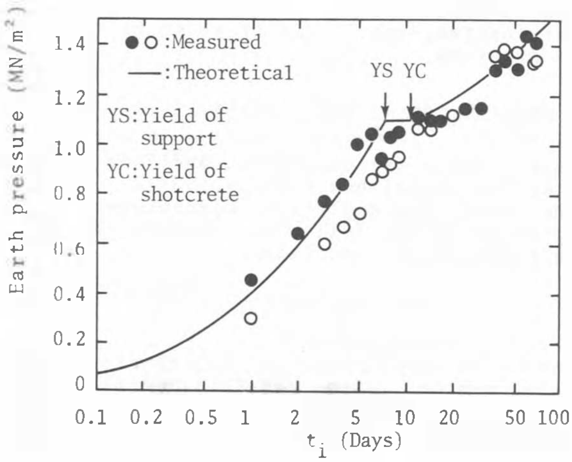

Fig.5 Analytical and measured earth pressures acting on supports

The curves in Fig.6 show the analytical tunnel displacements u after the lining is executed at the time $t=t_0$ in both the cases of the NATM and the conventional method in which the effect of supports preventing tunnel displacements is neglected. The measured settlements at crown are also plotted in Fig.6. The analytical displacement in the case of NATM is clearly less than that of the conventional method. The

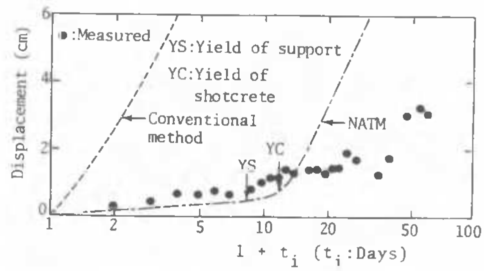

Fig.6 Analytical displacements (the NATM and the conventional method) and measured ones

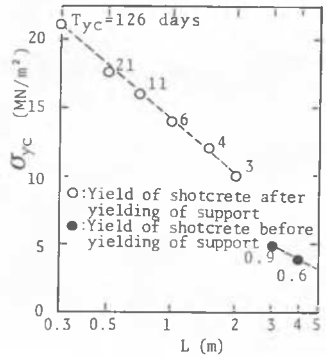

Fig.7 Effect of the interval of supports L on σ_{yc} and T_{yc}

measured displacements correspond more closely to the analytical result of NATM than that of conventional method. Fig.7 obtained analytically shows the effect of the interval of supports L on the compressive strength of shotcrete σ_{yc} and on the time span T_{yc} from the time of execution of shotcrete to that of its yielding. If L is larger than 2m, the strength of shotcrete becomes very low.

CONCLUSION

In this study, an analytical method to calculate earth pressures, displacements and axial stresses of the support and shotcrete executed by NATM is newly proposed. Consequently, various executive conditions can be effectively determined by this method.

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