# Drawdown Capacity of Groundwater Wells

## Capacité de Rabattement des Puits Filtrants

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SYNOPSIS The drawdown of the groundwater surface near a well is restricted due to the needs of the groundwater flow towards the cylindrical face of the well. There is no explicit solution for the actual drawdown at the surface of a drained well, since the phreatic line does not correspond to that after Dupuit. In the paper, the results of numerical analyses (FEM) are presented in form of the quantitative relation between the actual drawdown in the mantle of a fully drained well and the relevant parameters such as radius of well, radius of influence, and height of aquifer. The results, obtained for the conditions of steady flow (fixed radius of influence) and a fully penetrating well, are given in graphical form and include the influence of anisotropy.

## INTRODUCTION

In connection with the use of groundwater wells in foundation engineering, the determination of the yield of a well and of the position and shape of the free water-surface (phreatic line) are the main problems. As is well known, Dupuit's solution for the phreatic line is not valid in the vicinity of a groundwater well, this in particular if the well is drained to a major portion of its height. There are a lot of practical cases like that one shown in Fig. 1, where the maximum possible drawdown of deeply drained wells is of great interest to the engineer.

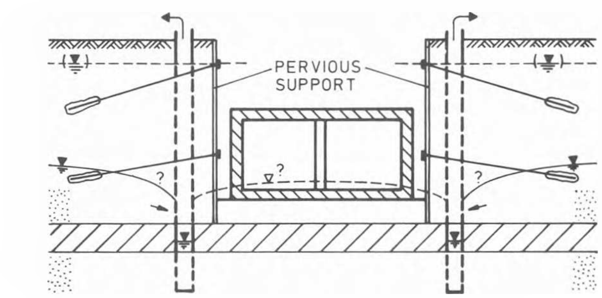


Fig. 1 Dewatering for Subway Construction Near a Shallow Layer of Impervious Soil

From the literature available so far, a clear answer cannot be given to the question of the minimum height of the seepage face (maximum drawdown) of a gravity well in which the water level is lowered to a niveau near or at its bottom end.

## PROBLEM AND FRAME

If water is taken from a deep well in a homogeneous soil, the groundwater table is lowered in a circular region, the radius of which (radius of influence, R) increases with time. A very useful and simple solution to estimate the radius of influence in function of the duration of water extraction from the subsoil, has been given by Weber (1928):

$$R = 3\sqrt{\frac{kHt}{n}} \qquad (1)$$

k = coefficient of permeability
H = height of aquifer
t = duration of water extraction
n = drainable porosity

In practice, it is common to assume that the radius of influence - instead of growing to infinity - is restricted so that, after a certain period of dewatering, the process of water migration towards a well can be regarded as quasi-stationary. From a number of observations in practice, Sichardt (see Kyrieleis and Sichardt, 1930) has derived a simple empirical equation for R which is still frequently used in Germany:

$$R = 3000\, s\sqrt{k} \quad \text{units: } R, s\,[\text{m}],\; k\,[\text{m/s}] \qquad (2)$$

s = drawdown of water in well (compare Fig. 2)

For steady-state conditions (water-level in well, h, and radius of influence, R, fixed), the well-known Dupuit-solution (eq. 3, 4 , Fig. 2) has proved to be a very powerful tool (see also: Heinrich, 1963).

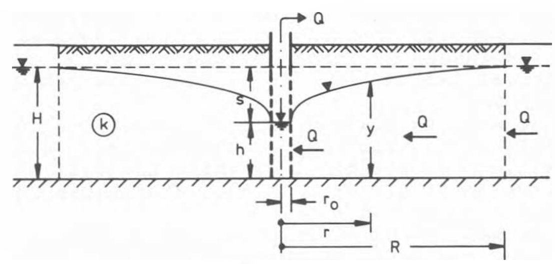


Fig. 2 Steady-State Flow Towards a Well According to Dupuit's Solution

$$\frac{Q}{k} = \pi \frac{H^2 - h^2}{\ln \frac{R}{r_o}} \qquad (3)$$

$$\frac{y}{H} = \sqrt{1 - \left(1 - \frac{h^2}{H^2}\right) \frac{\ln \frac{R}{r}}{\ln \frac{R}{r_o}}} \qquad (4)$$

(for the symbols, see Fig. 2)

It is to be seen from eq. (4) that the form of the phreatic line is not a function of k, but purely a function of the geometry of the system, as long as R is fixed.

In fact, the transient process of dewatering around a well has been analysed in more detail in the last few decades, this with particular regard to pumping tests. A survey of the vast literature in this field has been given by Kruseman and de Ridder (1973), for instance. Furthermore, new numerical methods have been developed which allow to make calculations for very complex conditions of transient flow problems. Nevertheless, simple analytical solutions such as those given above, are still of great value to the engineer. In cases where such explicit solutions are not possible, the application of numerical methods of calculation in form of parametric investigations can lead to valuable results of common interest.

One of the problems of this kind is the actual drawdown of the groundwater surface near a dewatering well in case of a low water-table in the well. If we regard a single well in the centre of its region of influence, three different situations of drawdown can be distinguished (compare Fig. 3):

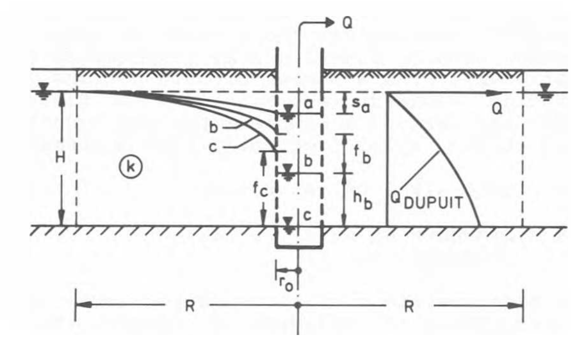


Fig. 3 Different Drawdown Conditions of a Single Well

a) The drawdown, s, in the well is only a small portion of the height, H, of the aquifer; in this case, the phreatic line joins the cylindrical face of the well at the water-table in the well ($y_{r=r_o} = h$).
b) The well is drained to a major portion of its height; in this case, a seepage-face develops, while the phreatic line joins the well at a level by f higher than the water-table in the well ($y_{r=r_o} = h + f$).
c) The well is completely drained (= empty); then, the water is entering the well over a certain portion of its height ($y_{r=r_o} = f$), which is a seepage-face.

As is well known from various investigations, Dupuit's solution for the phreatic line (eq.3) is a good approximation only in the first of the three cases. On the other hand, Dupuit's eq. (2) for the yield of the well, Q, is valid throughout $0 \leq h \leq H$, as has been shown by various authors (Chapman, 1957; Heinrich, 1963; see also Hunt, 1970).

In german literature, the presence of a seepage-face and, thus, of an upper limit of the drawdown around a well has often been related to a certain maximum hydraulic gradient of entrance along the cylindrical face of the well, such as proposed by Sichardt (1928) on the basis of empirical data:

$$I_{max} = \frac{1}{15\sqrt{k}} \quad \text{units: } k\ [m/s] \qquad (5)$$

In the case of a fully drained well, the assumption of a uniform hydraulic gradient along the mantle of a well would be far from reality. This can be seen from the detailed studies of Nahrgang (1954) on this subject, from which an example is given in Fig. 4:

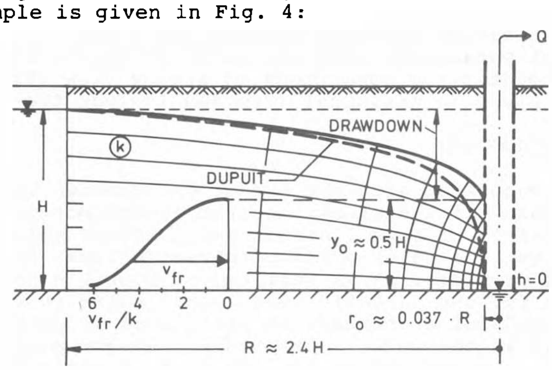


Fig. 4 Actual Flow-Net and Entrance-Velocity for a Fully Drained Well (after Nahrgang, 1954).

The increase of the hydraulic gradient of entrance with depth can easily be seen from the flow-net and from the profile of radial filter-velocities along the seepage-face, $y_o$, in Fig. 4, which also gives the phreatic line after Dupuit, for comparison. It becomes obvious from this figure that the developement of a seepage-face and the existance of an upper limit of drawdown around a well is purely a question of water-escape from the soil body under the aggravating conditions of axisymmetry (convergence of flow towards the well in plan). In order to discuss the parameters which define the maximum possible drawdown, let us consider Fig. 5 where steady-state conditions of flow are shown for fixed R- and H-values, but for well-radii ranging from $r_o = 0$ up to $r_o = R$.

We start regarding a fully drained or emptied well with small radius, $r_{o1}$, and we may have a maximum possible drawdown according to $y_{o1}$. If we enlarge the diameter of the well ($r_{o1} \rightarrow r_{o2}$), a cylinder of soil around the well of radius $r_{o1}$ is removed. This must result in an increase in flow, Q, and, thus, in a steeper phreatic line leading to a smaller height of the seepage-face, $y_{o2}$. Enlarging $r_o$ more and more, say up to $r_o \rightarrow R$, the three-dimensional problem finally deforms to the case of plane flow through a thin-walled hollow cylinder (see $r_{o3}$ in Fig. 5 for this), for which higher values of $y_o$ result again, since: $y_o \rightarrow H$ for $(R - r_o) \rightarrow 0$ or $r_o \rightarrow R$. Therefor, the function $y_o = f(r_o)$ has the form as shown in the left part of the section in

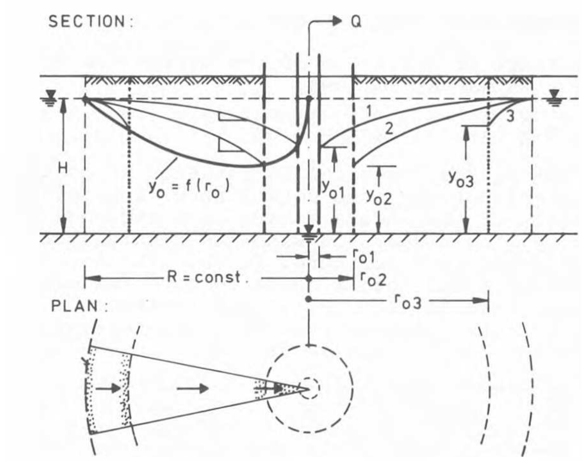


Fig. 5 Maximum Drawdown of a Fully Drained Well with Varying Radius

Fig. 5. This indicates that there is a steep relation between the maximum drawdown and the radius of a well, particularly for small $r_o/R$-ratios which prevail in the major portion of cases in the practice of dewatering. In addition to the relation shown in Fig. 5, the position and form of the phreatic line must be a function of the relative height of the system, i.e.: the H/R-ratio. The relative drawdown-effect will be the less, the higher the percolated cylinder.

In conclusion from this qualitative discussion, we find the following statements for the height of the seepage-face in a well:

$y_o = f(r_o/R, H/R)$,

$y_o$ = independent from k,

and that there are no special physical effects needed to define and to quantify the maximum drawdown-capacity of groundwater-wells.

MAXIMUM DRAWDOWN OF FULLY DRAINED WELL

Available Solutions

In order to allow a better guess for the actual drawdown around a well than is possible with Dupuit's handy but inexact solution (eq. 4), several authors have tried to take the presence of a seepage-face into account. In the following, a comparison of such solutions is given for the particular case of a completely drained well (h=0, compare Fig. 5).

A first relation to define the maximum drawdown of such a well can be obtained by combining the maximum gradient of entrance after Sichardt (eq. 5) with Dupuit's formula for the yield of a well (eq.3). This leads to the following equation which is very often used in german practice:

$$\frac{y_o}{H} = \sqrt{\left(\frac{1}{15\sqrt{k}}\,\frac{r_o}{R}\,\frac{R}{H}\,\ln\frac{R}{r_o}\right)^2 + 1} - \frac{1}{15\sqrt{k}}\,\frac{r_o}{R}\,\frac{R}{H}\,\ln\frac{R}{r_o}$$

units: k [m/s] (6)

(for the symbols, see fig. 5)

This relation is shown in a $y_o/H$-$r_o/R$-diagram in Fig. 6 for three values of k (solid lines), and for the arbitrary example R/H = 10. The log-scale of the abszissa is used in order to better be able to show the results for low $r_o/R$-values.

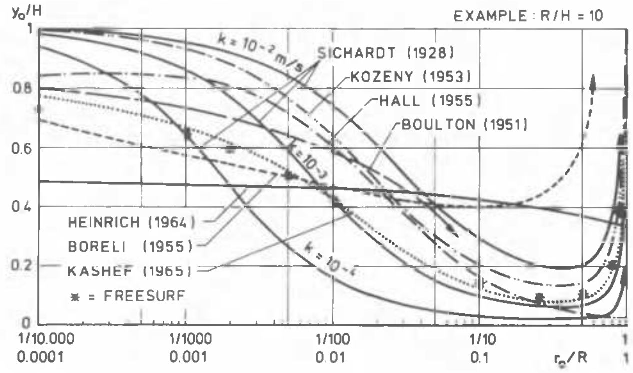


Fig. 6 Height of Seepage-Face in a Fully Drained Well, after Several Authors

A numerical analysis of the problem of the free water-surface near a gravity well by means of the relaxation method has been published by Boulton in 1951. From his investigation, the following expression for the conditions of a fully drained well can be obtained:

$$\frac{y_o}{H} = 1 - \frac{\xi_o}{2\ln\frac{R}{r_o}} \qquad (7)$$

in which: $\xi_o$ is a factor depending on ratio $r_o/H$ to some extent ($\xi_o \approx 3.75$ for common geometrical conditions)

In Fig.6, Boulton's relation is given with a broken line which shows that for higher $r_o/R$-values, $y_o$ results negative.

A relation similar to eq. (7) has been given earlier by Babbitt and Caldwell (1948) who obtained their results from some electric-analogy-tests.

Hall (1950, 1955) has also made numerical analyses with the relaxation method, and he has performed sector-model-tests with sand. His empirical solution in the form for h = 0 is:

$$\frac{y_o}{H} = \frac{1}{\left(1 + 0.02\ln\frac{R}{r_o}\right)\left(1 + 5\,\frac{r_o}{R}\,\frac{R}{H}\right)} \qquad (8)$$

This relation (which is independent from the ratio R/H) is shown in Fig. 6 through a —·— line. It gives a very small value of $y_o/H$ =0.02 (instead of 1) for $r_o/R \to 1$.

Kozeny (1953) has given a solution for the height of the seepage face based on the assumption that the maximum possible entrance velocity of the water is equal to k:

$$\frac{y_o}{H} = \frac{r_o}{R}\,\frac{R}{H}\,\ln\frac{R}{r_o}\left[\sqrt{1 + \left(\frac{H}{R}\,\frac{R}{r_o}\,\frac{1}{\ln\frac{R}{r_o}}\right)^2} - 1\right] \qquad (9)$$

This relation (independent from R/H again) is showen through —··— in Fig. 6.

Another empirical solution has been proposed by Boreli (1955). His solution is quoted by Mansur and Kaufman in "Foundation Engineering" (Leonards, Editor, 1962):

$$\frac{y_o}{H} = 1 - \frac{\ln\frac{R}{H} + 2.3}{\ln\frac{R}{r_o}}\left[0.13\ln\frac{R}{r_o} - 0.0123\left(\ln\frac{R}{r_o} - 2.3\right)^2\right] \quad (10)$$

For our example (R/H = 10), eq. (10) results in the dotted line (Fig. 6), which has a strange course in comparison to most of the other lines discussed up to now.

A further approximative analytical solution in explicit form is that of Heinrich (1964):

$$\frac{y_o}{H} = \frac{\ln\left[\frac{1}{2}\left(\frac{R}{r_o} + 1\right)\right]}{\ln\left[\frac{1}{2}\frac{R}{r_o}\left(\frac{R}{r_o} + 1\right)\right]} \quad (11)$$

This relationship which is independent from the parameter H/R, is given by a solid line again in Fig. 6; it has no similarity with all the other curves shown there. In the wide range of $r_o/R$-values given, $y_o/H$-values between 0.33 and 0.49 result only.

Regarding the wide scattering of the several approximative analytical solutions to the problem discussed here, it is to be expected that one or the other equation may be valid for certain conditions of geometry. But it seems questionable whether one of the solutions is able to describe the problem over a major portion of the possible range of variation of the parameters involved. Besides this, some of the relations contain the coefficient of permeability, k, as a parameter (this is not meaningful for steady-state conditions), and some do not include the parameter H/R, which must be of some influence on the result.

In more recent time, the problem of flow toward a gravity well has been treated with less simplifying assumtions and with better consideration of the actual boundary conditions. In particular, the papers of Kirkham (1964) and of Kashef (1965) are to be mentioned here. Due to the complexity of the problem, solutions are not obtained in explicit form; instead, the free water surface must be calculated by some iteration process. Whereas the exact solution of Kirkham makes use of a lot of mathematics and comes out with a relatively complicated algorithm for an iterative evaluation, Kashef's solution is eased through some simplifications, and the free surface can easily be obtained by stepwise calculation starting from the outer face of the percolated cylinder (i.e.: at r = R) and moving towards the well. In Fig. 6, Kashef's solution is shown through a fine-dotted line. It is of the same form as those after Sichardt and Kozeny; for both $r_o/R = 0$ and $r_o/R = 1$ it gives $y_o/H = 1$, as is necessary so, and the height of the seepage-face in the well is a function of H/R also.

<u>Results of Numerical Analyses by FEM</u>

In order to provide some more reliable data for the maximum possible drawdown of a fully penetrating gravity well than available up to now, systematic numerical analyses have been performed using a finite-element-program ("Freesurf 1") available at our institute (Brauns and Zangl, 1976). This program is a slightly changed and completed version of a program developed at the University of California/Berkeley. It allows to solve two-dimensional or axisymmetrical seepage-problems with free surface.

The range of variation of the parameters R/H and $r_o/R$ was as follows:

$$\frac{1}{10} \le \frac{R}{H} \le 100; \qquad \frac{1}{10000} \le \frac{r_o}{R} \lessapprox 1$$

As far as the numerical computation is concerned, we shall not get into detail here. Some of the results of the analysis have been included in Fig. 6, for comparison with the results available from the literature. It reveals that the solution after Kashef seems to give the most reliable results (see below also). The whole of the results obtained from the computer-analysis is graphically shown in Fig. 7.

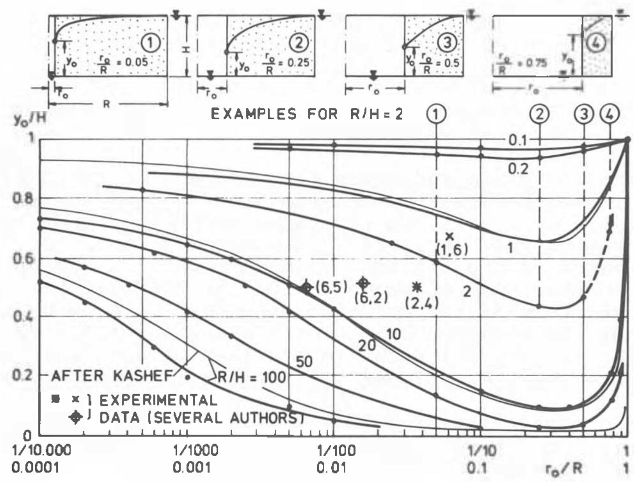


Fig. 7 Height of Seepage-Face in Fully Drained Well, Results of FEM-Analysis

As to be expected, there is a typical relation $y_o/H = f(r_o/R)$ for each value of the R/H-ratio. For both $r_o/R \to 0$ and $r_o/R \to 1$, all curves tend to $y_o/H = 1$. Depending on R/H, the relative height of the seepage-face in a well, $y_o/H$, may vary in the full range 0 through 1, except for very small values of $r_o/R$. There is a minimum $y_o/H$, corresponding to a maximum drawdown effect, in the range of $1/10 < r_o/R < 1/2$, but these values are much higher than those for dewatering wells in common practice which may be in the order of magnitude of $r_o/R < 0.01$. Even in this region of $r_o/R$-values, the height of the seepage-face varies within wide borders. Thus, lump values which sometimes are given in the literature (e.g.: $(y_o/H)_{min} = 0.5$, see Nahrgang, 1954, 1965; Klüber, 1975), are unfounded and indefensible.

In Fig. 7, three lines after Kashef are also given (see thin lines), for comparison with the own numerical results. It can be seen that both methods give nearly identical results. In addition, some experimental results from model-tests are included, such as taken from the literature (◆Babbitt and Caldwell, 1948; ✱Nahrgang, 1954; ✕Hall, 1955). The figures in brackets give the R/H-values corresponding to the test-conditions. As can be seen by interpolation, the experimental data fit pretty well with the analytical results. In this connection, it is to be mentioned that an accuracy problem, both in tests and analysis, arises from the fact that the junction of the phreatic line and the mantle of the well is a tangential point. Besides this, capillarity-effects - even if the evaluation accounts for them - can influence the observations with flow models (sand models) to a certain degree.

All in all, Fig. 7 may help to make a better guess for the maximum possible drawdown of a well in homogeneous and isotropic soil, than has been possible up to now. It is to be pointed out, in addition, that reliable results not only for $y_o$ but also for the form of the phreatic line can be obtained by application of the relatively handy analytical method of Kashef, for which not more than a programmable pocket-computer is necessary.

## Well yield

It has been mentioned already that Dupuit's solution for the yield of a well has been proved to be exact throughout $0 \leq h \leq H$, i.e.: independent from the development of a seepage-face. Accordingly, the values of the yield obtained in our numerical calculations do fit very well with the theoretical line following from Dupuit's eq. (2); see Fig. 8 for this.

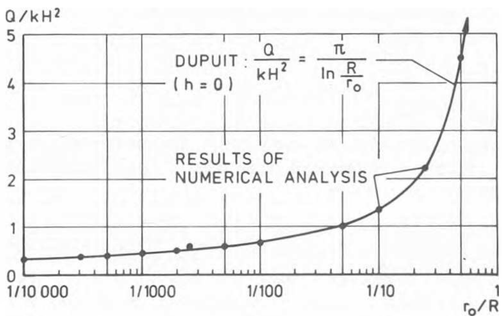


Fig. 8 Well Yield after Dupuit and from Numerical Analysis

## Form of Phreatic Line

Besides the height of the seepage-face or the maximum possible drawdown of a gravity well, the form of the entire free surface (phreatic line) very often is of great interest. In the literature quoted above, we find one or the other approximation, from which the various equations for $y_o/H$ given in the preceeding chapter are deduced.

In an numerical analysis such as discussed here, the entire free surface is part of the solution in each case. Also, if Kashef's method is applied, the form of the phreatic line is determined over its full length. For the particular example of H/R = 10, the free surface is plotted in Fig. 9 for four different values of $r_o/R$ = 0.05, 0.25, 0.50, 0.75.

The dashed line which is the locus of all possible points of maximum drawdown for $0 \leq r_o/R \leq 1$, is identical with the corresponding line in Fig. 7. As can be checked by simple calculations, Dupuit's equation for the phreatic line (eq. 4) gives a path of the free surface which is very close to that obtained from the numerical analysis, provided h (which is zero in all our cases here) is replaced by $y_o$.

## Effect of Anisotropy

In all the above discussion, we have assumed the soil to be isotropic with respect to permeabili-

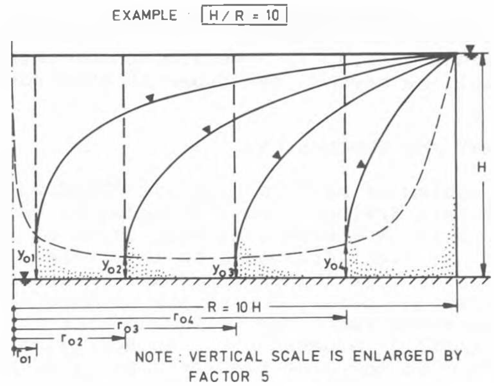


Fig. 9 Shape of the Phreatic Line (Examples) Obtained from Numerical Analysis and from Modified Dupuit's Formula (lines are nearly identical)

ty. Sedimentary soils, as is well known, very often show a more or less pronounced anisotropy in permeability ($k_h > k_v$). As has been shown in special tests (Brauns and Witt, 1981) and through analysis (Wittmann, 1980), this anisotropy is mainly due to low-scale stratification (not due to flatness and orientation of particles).

In fact, the form of the phreatic line around a well and, thus, the height of the seepage-face is a function of the ratio $k_h/k_v$. If $k_h/k_v > 1$, the free water-surface is higher than for isotropic conditions (at extremum: $k_h/k_v = \infty$, i.e.: $k_v = 0, \rightarrow y_o = y = H$); on the contrary, if $k_h/k_v < 1$, the phreatic line is lower than in the case of isotropy (see Heinrich, 1963).

With the FEM-program, anisotropic conditions can easily be investigated. Such computations have been performed in order to demonstrate the influence of the $k_h/k_v$-ratio. Some of the results are presented in Fig. 10, namely for two values of R/H: R/H = 10, 1.

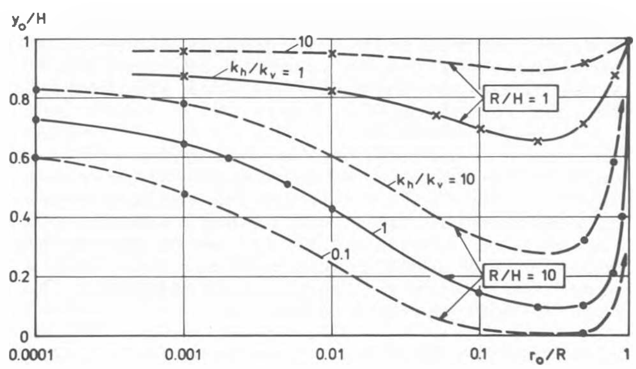


Fig. 10 Effect of k-Anisotropy on Height of Seepage-Face in Fully Drained Well

As can be seen from the diagram, an anisotropy-ratio of $k_h/k_v$ = 10 which may sometimes prevail in natural sediments due to "low-scale-stratification", is of a noteworthy influence in the height of the seepage-face in an well. Note: the lowest curve in Fig. 10 is not relevant for practice, but it shows the influence of $k_h/k_v$-values less than unity, in tendency.

Complete information on the effect of anisotropy would call for a full set of curves for each of the lines in Fig. 7. Presentation of this material would be beyond the frame of this contribution.

## SUMMARY AND CONCLUSIONS

The problem of maximum possible drawdown of a completely drained single groundwater well, fully penetrating a homogeneous soil layer of finite depth has been discussed in this paper. It has been shown that the presence and extent of a seepage-face are purely due to the needs of the seeing water to reach the well and that the numerous explicit equations for an approximative calculation do not give realistic results. New data, obtained from a parametric numerical investigation using a finite-element-program, are presented here (Fig. 7). They show that - depending on the geometrical boundary conditions - the height of the seepage-face may vary in wide borders and that lump values in terms of a certain ratio $y_o/H$ are untenable. The numerical results obtained fit very well with a relatively simple and handy solution after Kashef (1965), which is of the non-explicit type.

It has further been shown that a very good guess for the free water-surface (phreatic line) can be made with the help of the classical equation after Dupuit, provided the height of the seepage-face is used instead of the actual water-level in the well (Fig. 9). As far as the yield of a well is concerned, the FEM-analysis confirmed that Dupuit's solution holds independently from the presence of a seepage-face. In regard to the influence of an anisotropy in permeability, its effect proved to be remarcable in that the greater $k_h/k_v$, the higher the seepage-face (Fig. 10).

These investigations are going to be extented towards cases of arbitrary water levels in the wells. It is expected that by means of some exemplary calculations a sufficient accuracy of Kashef's method can be proofed here also. A much more complex problem is that of the maximum drawdown of wells in well-groups. Handy solutions of common applicability cannot be given here, since these are three-dimensional problems with further variables.

In order to clarify the relations for wells in multi-well-groups, the conditions of drawdown in the region of each individual well must be considered in a simplifying manner. Starter point for such an investigation may be Forchheimer's solution for a multi-well-system. The results presented herein may serve as first hints in such a configuration also.

## ACKNOWLEDGMENTS

The investigations, the results of which are presented in this paper, have been financially supported by the Kuratorium für Wasserwirtschaft, Bonn-Bad Godesberg. The author wants to express his thanks for this. In addition, the author wants to thank Mr. K.-J. Witt for his assistance in the evaluation of the literature and Mr. G. Straub for his diligence in performing the numerical computations.

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