Seepage Erosion Analyses of Structures

Analyses d'Erosion par Ecoulement

D. VAN ZYL Senior Geotechnical Engineer, Steffen, Robertson & Kirsten, Inc., Johannesburg, South Africa
M.E. HARR Professor in Geotechnical Engineering, Purdue University, Indiana, USA

SYNOPSIS This paper presents an overview of seepage erosion analysis of geotechnical structures on granular soil deposits subjected to confined flow. The different approaches to the analysis of seepage erosion under confined flow conditions are discussed, namely: the global gradient approach, heave, and the exit gradient approach. Results from laboratory tests show that the failure gradient of a soil is higher than the critical gradient, and that both these gradients are random variables.

INTRODUCTION

The failure of geotechnical structures due to the flow of water under high seepage gradients is usually described as 'piping', 'heave', or 'internal erosion' failures. The term 'suffosion' (or its derivatives) is used in a number of languages (e.g. German, Spanish and Russian) to describe the adverse effects of seepage. The term 'seepage erosion' is much more general in describing the adverse effects of seepage and includes the failure mechanisms above. Most of the time, especially with respect to natural soil deposits, seepage erosion is a far better descriptor for this kind of failure because of the uncertainties involved in the exact failure mechanism.

This paper reviews the methods for analyzing seepage erosion of granular soils subjected to confined flow.

CHARACTERIZATION OF VARIOUS MODES OF SEEPAGE EROSION

Seepage erosion failures can be classified into three modes:

(i) heave,

(ii) piping, and

(iii) internal erosion.

Each of these modes is controlled by a different set of failure mechanisms, and each is analyzed in a different fashion. Table I presents a summary of the mechanisms and techniques of analysis for each failure mode as well as some examples for each. References where methods of analysis and discussion of failure modes are presented are also given in Table I.

ANALYSIS OF SEEPAGE EROSION

Global Gradient Approach

The principle of the global gradient approach to the design of weirs on permeable foundations was first stated by Buckley (1905)*. It was based upon his laboratory experiments in India during the last decade of the 19th century. Buckley concluded:

> 'It has been maintained that...the true measure of security of a weir in a permeable bed is the distance through the soil which a current of water would have to travel before it could rise up below the weir, and it is of little consequence whether masonry is laid horizontally on the weir bed or sunk vertically below it, so long as currents passing through the soil below the structure are exposed to the friction of the same legnth of passage.'

Studies based on the concept of the length of the path traveled by seeping water led to the development of the so-called creep ratios or creep coefficients. Two well-known design methods on this basis are those by Bligh (1927) (and earlier editions of this text) and Lane (1935). The method proposed by Lane (1935) is still widely used in the design of dams and weirs, despite criticism about its empirical nature. Table II gives the definition of the creep coefficients in both cases as well as the suggested limiting magnitudes. In concept, creep coefficients obtained from the equations in Table II are to be higher than those tabulated for the soil conditions under consideration. The limiting values suggested were obtained by analyzing a large number of structures founded on various soil conditions (Lane, 1935, analyzed 200 structures).

* Referred to by Khosla, et.al. (1936) as "Irrigation Works in India", no publisher is given.

TABLE I

Characterization of Seepage Erosion

MODE	MECHANISM	METHOD OF ANALYSIS	EXAMPLES	REFERENCES
Heave	·Simultaneous heaving of substantial soil volume ·Shear resistance of soil volume approximately zero	·Comparison of seepage force per unit volume with effective unit weight of selected critical volume of soil	·Heaving of excavations with internal groundwater lowering ·Blow-up of excavations under artesian conditions ·Heaving immediately adjacent to downstream cut-offs of weirs and spillways	·Sentko (1961) ·Terzaghi and Peck (1967) ·van Zyl (1979)
Piping	·Initiated locally at discontinuities in the soil, e.g., concentration of fines, particles with low specific gravity or fissures and cracks ·Removal of particles begin at soil surface ·A space is created at exit which progressively enlarges and works backwards to form irregular channels of 'pipes' ·Soil adjacent to pipes is stable	·Almost impossible due to control by discontinuities, however, use of 'global' gradient approach is an attempt	·Sand boils behind levees ·Piping underneath cutoff weirs	·Bligh (1927) ·Terzaghi (1929) ·Lane (1935) ·Khosla, et.al. (1936)
Internal Erosion (Matrix Erosion)	·Begins locally by fine particles being moved from the soil matrix into a coarser layer ·Cavities form, leading to collapse and failure	·Attempts have been made to combine effect of particle size distribution and gradient, generally very difficult to analyze. Prevent by using filters designed to proper filter criteria	·Transfer of particles between zones of earth and rockfill dams ·Dispersive soils	·Lubochkov(1968) ·Ronzhin (1969) ·Breth (1972) ·Sherard, et.al. (1972)

The global seepage gradient is defined as the head loss per unit length of flow path. The inverse of the creep coefficient is therefore the global seepage gradient. These values are also shown in Table II ($1/C$ and $1/C_w$).

Chugaev (1958) presented a more general approach to the analysis of seepage erosion of concrete dams on pervious foundations. Resistance coefficients are used to calculate the global gradient. Analysis of 174 structures resulted in the limiting values shown in Table III.

Although the definitions of global gradient are different, there is remarkable similarity between the values of $1/C_w$ in Table II and the allowable values of global gradient given in Table III.

The limiting gradients or creep-coefficient in the global gradient approach were determined from an analysis of a large number of structures. The sample of structures investigated in each case may, however, be an insignificant part of the statistical population, and does not necessarily include all the possible failure modes or adverse subsoil conditions. In some cases where failure took place, it might have been caused by seepage along geological discontinuities (Terzaghi, 1929). Concentrated seepage, instead of 'average seepage' therefore may be the cause (Davidenkoff, 1970). The prediction of the safety of a structure against seepage erosion caused by concentrated seepage is not possible with any of the global gradient techniques unless the source of the concentrated seepage is detected during site investigation or prevented during construction. Blind application of design methods based on global gradients can therefore lead to a false sense of security. However, as a first approach, these methods give a very good indication of potential problems.

An advantage of the global gradient approach is that the values of the suggested limiting gradients, or creep coefficients, are dependent on the subsoil conditions of the project site. It is therefore impressed upon the designer that, e.g., silt and fine sand erode more easily than gravel or clay.

TABLE II

Creep Coefficients and Global Gradient

	Bligh (1927)			Lane (1935)		
Expression	Creep Coefficient: $C = \frac{L}{h}$ where L = length of seepage path, measured along base of weir; h = total head loss			Weighted Creep Ratio: $C_w = \frac{\frac{H}{3} + V}{h}$ where H = distance along horizontal contacts ($<45^0$); V = distance along vertical contacts ($>45^0$); h = total head loss		
Limiting Values Suggested	Soil Type	C	1/C	Soil Type	C_w	$1/C_w$
	light silt and mud (60% passing 100 mesh sieve)	18	0.06	very fine sand or silt	8.5	0.12
				fine sand	7.0	0.14
	fine micaceous sand (80% passing 75 mesh)	15	0.07	medium sand	6.0	0.17
	coarse-grained sands	12	0.08	coarse sand	5.0	0.2
	mixture of boulders, gravel and sand	5-9	0.1-0.2	fine gravel	4.0	0.25
				medium gravel	3.5	0.29
				coarse gravel including cobbles	3.0	0.33
				boulders with some cobbles and gravel	2.5	0.4
				hard clay	1.8	0.56

Seepage Erosion Through Heave

From the results of laboratory tests, Terzaghi (1943), suggested that failure by heave downstream of a cutoff of embedment depth, s, takes place over a segment of width s/2 by depth, s, directly downstream from the cutoff. This was confirmed through laboratory tests by Sentko (1969). The summation of the forces acting on this segment is employed to estimate the factor of safety against failure by heave.

TABLE III

Allowable Global Gradient for Concrete Dams on Pervious Foundations (Chugaev, 1958)

Soil Type	Allowable Global Gradient
Fine-grained sand	0.12
Middle-grained sand	0.15
Sandy loam	0.20
Coarse-grained sand, gravel	0.25
Compact Clay	0.40

Terzaghi (1943) assumed that the seepage force acts vertically upwards. However, the seepage force acts in the direction of flow, i.e., tangential to a flow line. The direction of the flow changes from horizontal at the tip of an embedment to vertical at the exit point downstream. By applying this principle, van Zyl (1979) showed that the minimum factor of safety of seepage erosion by heave is obtained for a segment s/2 x s/2 (embedment depth s). It was furthermore shown that for a segment with these dimensions, it can be assumed that the seepage force acts vertically.

Exit Gradient Approach to Seepage Erosion

In 1922, Terzaghi published his classic work on the failure of dams by seepage erosion. This very important contribution was brought to the attention of the English-speaking community by Terzaghi only in 1929. The theoretical development is based on the summation of the vertical seepage forces exerted by the upward flow of water and the vertical downward weight of the submerged soil particles, so that the critical gradient, i_c, is given by:

$$i_c = \frac{G - 1}{1 + e} = (1 - n)(G - 1) \tag{1}$$

where G = specific gravity of soil particles
e = void ratio of soil
n = porosity of soil

For typical soils, assuming G = 2.65 and $0.28 < n < 0.52$, the value of i_c ranges from 0.8 to 1.2, with a mean of approximately 1.0 (Terzaghi, 1922).

The gradient where the first instability occurs was called the flotation gradient by Terzaghi (1925), the 'bursting gradient' by Haugh* and the 'critical gradient' by Khosla*. The last term has found more general use in literature.

Laboratory tests by Terzaghi (1922, 1925 and 1929), Sentko (1961), Nakajima (1968), and others, confirm the general validity of equation (1).

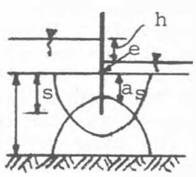
However, when friction forces are present, the critical gradient is usually higher than that obtained from equation (1). This effect will be further elaborated on below.

Various approaches have been proposed for estimating the exit gradient of a specific cross-section. Khosla, et.al.(1936) suggested theoretical methods for determining the exit gradients for confined flow under various boundary conditions. These, as well as the method of fragments, are reviewed by Harr (1962, 1977). Methods for estimating the exit gradient from flow nets are presented by Casagrande (1937) and Davidenkoff (1970).

A summary of the analyses methods above is presented in Table IV.

TABLE IV

Exit Gradient Approach to Seepage Analysis



1. Theoretical (Khosla, et.al. 1936; Harr 1962)

$$i_e = \frac{h\pi}{4\ KT m}$$

K = complete elliptic integral of the 1st kind with modulus $m = \sin \frac{\pi s}{2T}$

2. Casagrande (1937)

$$i = \frac{\Delta h}{a_s}$$

$\Delta h = \frac{h}{n_e}$ - number of equipotential drops

3. Davidenkoff (1970)

$$i = \frac{h_s}{s}$$

where h_s = head at bottom of cutoff

* Referenced by Khosla, et.al. (1936)

In general, the factor of safety with respect to seepage erosion for the exit gradient approach is defined by:

$$FS_i = \frac{i_c}{i_e} \qquad (2)$$

where i_c = critical gradient as defined in equation (1)

i_e = exit gradient for specific boundary conditions

It is customary in geotechnical engineering to specify a minimum allowable factor of safety when a deterministic (design) approach is followed. The recommended factor of safety is typically a result of experience. Table V gives a summary of different factors of safety against seepage erosion recommended by different authors. The large factors of safety with respect to seepage erosion reflect the degree of uncertainty in the knowledge of foundation conditions and the overall seepage erosion process. If the foundation material is homogeneous and the seepage boundary conditions can be defined with a high degree of certainty, lower values of the factor of safety can be used. This is reflected in the value of 1.5 recommended by Marsland (1953).

LABORATORY STUDIES OF SEEPAGE EROSION

Laboratory experiments to model seepage erosion have been conducted by many researchers since Terzaghi reported his study in 1922. Two types of experimental setups have been reported: (a) cylindrical (Terzaghi, 1922; Nakajima (1968); Kezdi, 1976; and Kälin, 1977), and (b) special models to simulate specific boundary conditions, e.g., sheetpile walls (Terzaghi, 1943; Marsland, 1953; Ehss, 1960; and Sentko, 1961).

Results of tests in cylindrical containers show the following:

- A complete rearrangement of particles takes place upon reaching about the critical gradient, i_c. This rearrangement of particles usually leads to an increase in porosity and therefore also an increase in permeability.
- If non-uniform materials are used, the fines are washed out first and are then deposited on the surface, prior to their complete rearrangement (Terzaghi, 1922; Mockmore and Dougherty, 1935*; Kezdi, 1976).
- Terzaghi (1922) observed the arching of the sand to the sides of a cylindrical container. It was concluded that arching of the sand sample was fully developed when the layer thickness of the sand was 1/3 of the diameter of the container. Arching causes the development of friction between the soil and the container sides. These artificial effects introduced during testing are very important in the interpretation of the test results.

* Discussion to Harza (1935).

TABLE V

Recommended Factors of Safety Against Seepage Erosion (Exit Gradient Approach)

Author	Soil Conditions	Factor of Safety
Casagrande (1935) (discussion of Lane's paper)	Homogeneous soil with anisotropy k_{max}/k_{min} $\geq$ 2 to 3	3
	Poor site exploration or irregular and stratified deposits	$\geq$10
Khosla, et.al. (1936)	Gravel	4 to 5
	Coarse Sand	5 to 6
	Fine Sand	6 to 7
Zaki and Leliavsky (1948)	Exit gradient from flownet. Allow for scouring downstream from weir.	15
Marsland (1953)	Variation of site conditions well established	1.5
Harr (1962)		$\geq$ 4

In order to reduce the arching effect observed in cylindrical containers, laboratory tests were performed in a funnel by Terzaghi (1925) and in a truncated cone-shaped apparatus made from an Erlenmeyer flask by van Zyl (1979). The latter study included tests on plastic balls as well as various granular soils (including Ottawa sand, copper tailings, and fly ash). Two important conclusions, confirming previous results, were reached by van Zyl (1979), namely:

(i) Failure due to seepage erosion takes place from the bottom of the sample. A piezometer at the bottom of the sample serves to illustrate this. The seepage head is increased gradually. After reaching a maximum value, the piezometer level decreases without any movement of sand particles on the sample surface. Shortly after the decrease in head has begun, the sample surface evidences heave. The time between the onset of decrease in head and heave depends on the sample characteristics; it is slightly longer for coarse sand than for fine sand, and it is longer for a dense sample than for a loose sample. Similar results were reported by Sentko (1961). The decrease in head prior to movement of particles on the sample surface is the result of expansion (heave) of the bottom of the sample. Holding the seepage velocity constant, the permeability increases and the head therefore decreases.

(ii) The failure gradient, i_f, is larger than the calculated critical gradient, i_c. Similar results were reported by Terzaghi (1925). When deriving the expression for the critical gradient, it is assumed that seepage forces are resisted only by the submerged unit weight of the soil and that there are absolutely no friction forces present between the soil particles. The results from this study indicate that $i_f = (1.06 \text{ to } 1.2)\ i_c$. Tests performed on sand in a funnel by Terzaghi (1925) produced $i_f = (1.14 \text{ to } 1.16) i_c$.

Considering equation (1), it is clear that the critical gradient is a random variable because the porosity of a soil is a spatial random variable, while the specific gravity of the soil may also be a random variable, depending on the mineral composition of the soil. From this observation and (ii) preceding, the failure gradient (i_c) of a soil can be written in general as:

$$i_f = n_s\ i_c \tag{3}$$

where n_s = a correction factor to account for material characteristics

and i_f, n_s and i_c are random variables.

From first order Taylor series approximation, the mean value of the failure gradient can be given by (van Zyl, 1979)

$$\overline{i_f} = \overline{n_s}\ \overline{i_c} \tag{4}$$

and the coefficient of variation *(V_{i_f})of i_f by:

$$V_{i_f} = \sqrt{V_{n_s}^{\ 2} + V_{i_c}^{\ 2}} \tag{5}$$

where V_{n_s} and V_{i_c} are the coefficients of variation of n_s and i_c.

Typical values of $\overline{n_s}$ are given in Table VI. It was furthermore shown that a good approximation of V_{i_f} is (van Zyl, 1979)

$$V_{i_f} \simeq 1.5\ V_{i_c} \tag{6}$$

$$V_{i_c} = \sqrt{\frac{V_n^{\ 2}\ \bar{n}^2}{(1 - \bar{n})^2} + \frac{V_G\ \bar{G}^2}{(\bar{G} - 1)^2}} \tag{7}$$

where n = porosity ($\bar{n}$ = mean, V_n = coefficien'. of variation)

G = specific gravity ($\bar{G}$ = mean, V_G = coefficient of variation)

---

*The ratio of the standard deviation to the mean, expressed as a percentage.

PRACTICAL CONSIDERATIONS:

The results of the laboratory tests by van Zyl (1979) and the findings of other researchers (e.g. Terzaghi, 1922, 1925 and Sentko, 1961), suggest that the following matters be given special consideration when concern centers on the seepage erosion of granular soils:

(i) The head at which seepage erosion failure occurs depends on the soil type. The amount of fines in the soil, the specific gravity of the constituent minerals and the porosity of the soil play the most important roles.

(ii) Discontinuities can lead to concentrated seepage ('piping') and increased probability of failure.

(iii) The mode of seepage erosion failure depends on the soil type, the rate of head increase and the flow condition (e.g. saturated or unsaturated flow). The soil type controls whether heave is followed by a quick condition as in clean sand, or whether heave leads to crack formation, concentrated flow and piping. The latter mode appears to be more prevalent in granular soils with a large percentage of fines.

TABLE VI

Typical Values of $\overline{n_s}$ Estimated From Laboratory Testing Results (van Zyl, 1979)

Material Description	$\overline{n_s}$
Dense Coarse Rounded Sand*	1.2
Loose Coarse Rounded Sand*	1.1
Dense Fine Rounded Sand*	1.15
Loose Fine Rounded Sand*	1.1
Dense Fine Angular Sand*	1.16
Loose Fine Angular Sand*	1.1

* <5% passing the no. 200 sieve

The rate of head increase determines whether seepage erosion is manifested by a quick condition on the soil surface, or whether heave of the soil surface will occur. A quick condition prior to heave can be produced when the head is raised very slowly.

A rapid increase in head results in a heaving of the surface, leading eventually to a quick condition. Failures of this kind are expected in excavations where the dewatering system fails or downstream from a water retention structure being filled rapidly.

Field observations by Domjan (1961), Nakajima (1968) and Kezdi (1976) indicated that unsaturated soil fails at lower gradients than the critical gradient of the soil. The first filling of a reservoir may induce this type of failure.

(iv) The heave of an excavation floor is the result of disturbance throughout the total depth of cutoff embedment (failure initiates at bottom of sample). If there is leakage along the depth of the cutoff, heave may originate at the level where excessive leakage occurs. Careful investigations of the relative density of the soil with depth should therefore be made immediately upon the appearance of a heave failure.

(v) The heaving of filters may result in considerable mixing of the lower filter layers. The mixing may limit the ability of the filter to prevent movement of the fine particles from the soil and may result in the piping of the foundation soil.

CONCLUSIONS

(i) Seepage erosion is a more general term to describe the adverse effects of seepage.

(ii) Seepage erosion can be classified into three modes: heave, piping, and internal erosion. Each mode is characterized by different failure mechanisms and each mode is analyzed differently.

(iii) The global gradient approach for seepage erosion analysis is empirical and should only be used when no more accurate information is available on the seepage boundary conditions.

(iv) Heaving failures should be considered by summations of forces over a segment s/2 x s/2 directly downstream from a cutoff of embedment depth, s.

(v) Arching of granular soil develops in cylindrical containers when a sample thicker than about 1/3 the container diameter is subjected to a seepage force vertically upwards. This artificial effect can be reduced by using a funnel-like testing apparatus.

(vi) Failure due to seepage erosion takes place from the bottom of a sample.

(vii) The failure gradient is larger than the critical gradient. Both these gradients are random variables.

ACKNOWLEDGEMENTS

The authors would like to thank their colleague, Gerald A. Leonards, for his contribution towards the development of Table I. This research was sponsored by Purdue Research Foundation and the South African Council for Scientific and Industrial Research.

REFERENCES

Bligh, W.G. (1927) The Practical Design of Irrigation Works, Van Nostrand Co., New York, 425 pp.

Breth, W.G. (1972) Der derzeitige Stand des Staudammbaues, Wasserwirtschaft, Vol. 62, No. 1/2, pp. 20-33.

Casagrande, A. (1937) Seepage Through Dams, Journal of the New England Water Works Association, June, Republished in 'Contributions to Soil Mechanics 1925-1940' BSCE, pp. 295-336.

Chugaev, R.R. (1958) Design and Calculation of the Underground Profile of Dams on Pervious Foundation, Proc. of the 6th ICOLD, New York, Question No. 21, Paper R. 134, Vol. 2, pp. 1403-1433.

Davidenkoff, R. (1970) Unterläufkigkeit von Stauwerken, Werner-Verlag, Düsseldorf, 103 pp.

Domjan, J. (1969) Piping of Soils Near Dams, Proc. of the 5th Int. Conf. on SM & FE, Vol. 2, pp. 581-583.

Ehss, H. (1960) Grundlagen zum hydraulischen Grundbruch bei Stauhaltungen im durchlässigen Erdreich, Der Bauingenieur, Vol. 35, No. 11, pp. 423-431.

Harr, M.E. (1962) Groundwater and Seepage, McGraw-Hill Book Co., 315 pp.

Harr, M.E. (1977) Mechanics of Particulate Media - A Probabilistic Approach, McGraw-Hill, 543 pp.

Harza, L.F. (1935)Uplift and Seepage Under Dams on Sand, Transactions of ASCE, Vol. 100, Paper No. 1920, pp. 1352-1385, Discussions, pp. 1386-1406.

Kälin, M. (1977) Hydraulic Piping - Theoretical and Experimental Findings, Canadian Geotech nical Journal, Vol. 14, No. 1, February, pp. 107-124.

Kezdi, A. (1976) Process of Hydraulic Soil Failure, Acta Technica Acad. Sci. Hungary, Vol. 82, No. 3-4, pp. 245-254.

Khosla, A.N., N.K. Bose and E. McKenzie Taylor (1936) Design of Weirs on Permeable Foundations, Central Board of Irrigation, Publication No. 12, India, 200 pp.

Lane, E.W., (1935) Security from Under-Seepage: Masonry Dams on Earth Foundation, Trans. ASCE pp. 1235-1272, Discussions, pp.1273-1351.

Lubochkov, E.A. (1968) Calculation of the Piping Properties of Cohesionless Soils with Use of a Nonpiping Analog, Hydrotechnical Construction (Translation from the Russian of Gidrotekhnischeskoe Stroitel'stvo), No. 3, pp. 233-237.

Marsland, A. (1953) Model Experiments to Study the Influence of Seepage on the Stability of a Sheeted Excavation in Sand, Geotechnique, Vol. 3, No. 2, pp. 233-240.

Nakajima, Y. (1968) Study of the Failure of Piping in Polder Dikes for Land Reclamation, Soils and Foundations, Vol. VIII, No. 4, pp. 30-47.

Ronzhin, I.S. (1969) On Water Seepage under Concrete Hydraulic Structures and Some of Its Specific Features, Proc. of 13th Congress of the Int. Ass. for Hydraulic Res., Japan, Vol. 4, pp. 143-150.

Sentko, M. (1961) Der Zeitliche Ablauf des Schwimmsand aufbruches und der Einfluss der geometrischen Anordumg der Baugrubenumschliess ungen auf das kritische Gefälle, Ph.D. Thesis, Veroffentlichungen Institut für Bodenmechanik, Technische Hochschule, Karlsruhe,No.7, 130 pp.

Sherard, J.L., R.S. Decker and N.L. Ryker (1972) Piping in Earth Dams of Dispersive Clay, Proc. of the Specialty Conf. on Performance of Earth and Earth-Supported Structures, ASCE, Purdue University, June 11-14, Vol. 1, Part 1, pp. 589-626.

Terzaghi, K. (1922) Der Grundbruch an Stauwerken und seine Verhütung, Die Wasserkraft, Vol. 17, pp. 445-449, Reprinted in 'From Theory to Practice in Soil Mechanics', Wiley, 1960, pp. 114-118.

Terzaghi, K. (1925) Erdbaumechanik auf Bodenphysikalisher Grundlage, Franz Deuticke, 369 pp.

Terzaghi, K. (1929) Effect of Minor Geologic Details on the Safety of Dams, American Institute of Mining and Metallurgical Engineers, Technical Publication 215, pp. 31-44, Reprinted in 'From Theory to Practice in Soil Mechanics' Wiley, 1960, pp. 119-132.

Terzaghi, K. (1943) Theoretical Soil Mechanics, John Wiley & Sons, Inc., New York, 510 pp.

Terzaghi, K. and R.B. Peck (1967) Soil Mechanics In Engineering Practice, 2nd Edition, John Wiley & Sons, 729 pp.

van Zyl, Dirk (1979) Seepage Erosion of Geotechnical Structures Subjected to Confined Flow-A Probabilistic Design Approach, Ph.D. Thesis, Purdue University, Aug., 244 pp.

Zaki, H. and S. Leliavsky (1948) Tail Erosion as a Factor Affecting the Safety Coefficient Against Piping, Proc. of the 3rd ICOLD, Vol. II, Questions No. 10, R.4, 10 pp.