Centrifuge Tests of the Uplift Capacity of Anchors

Essais Centrifugaux sur la Capacité de Sous-Pression des Tirants

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SYNOPSIS The use of model testing for solving design problems in Soil Mechanics is discussed with special reference to the scaling law relationship. As an example the uplift capacity of anchor slabs in sand is considered. The potential of the centrifugal testing technique is demonstrated. On basis of centrifugal model tests formulas are devised for the uplift capacity of circular and square anchor slabs in sand pulled in a vertical and in a slanting direction.

INTRODUCTION

A number of theories to determine the uplift capacity of anchor slabs in sand is reported by various authors in the Soil Mechanics literature. In order to justify their theories many of the authors (for example Balla (1961), Matsuo (1968), Meyerhof and Adams (1968), and Vesić (1971)) employ the results of model tests at a reduced length scale. These tests are normally performed with model anchor slabs having a width between 20 and 100 mm. Very few attempts, however, have been made to investigate the scaling law relationship for model tests with anchor slabs in sand. Research performed recently with various types of structures utilising the centrifugal testing technique seems to indicate that scaling errors might occur in conventional model tests at a reduced length scale. Part one of the paper attempts to investigate the scaling law relationship for tests with uplift capacity of anchor slabs in sand. Part two of the paper presents the results of an investigation performed by means of the centrifugal testing technique of the uplift capacity of anchor slabs in sand.

PART ONE: SCALING LAW RELATIONSHIP

The basic problem is illustrated in Figure 1a: A square anchor slab having a width B burried at a depth H in dry sand having a horizontal surface and being pulled in a vertical direction. Figure 2a shows the slab placed in a slanting position defined by the angle Θ between the anchor pull and the vertical.

Figure 2b illustrates how a circular anchor slab can be represented by a square slab (with an equivalent width B_e) having the same area as the circular slab.

The present investigation is limited to anchor slabs at a shallow depth, i.e. $H/B_e \leq 3.5$.

Dimensional analysis

Now, consider the problem: How should a model test be designed in order to determine experimentally the uplift capacity Q of a given prototype anchor?

As illustrated in Figure 1 and 2 the following ten independent quantities might influence Q:

B_e (m)	the effective width of the slab
H (m)	the depth of the slab
Θ (-)	the angle of the anchor pull to the vertical
γ (N/m³)	the unit weight of the sand
e (-)	the void ratio of the sand
ϕ_μ (-)	the angle of interparticle friction of the sand grains

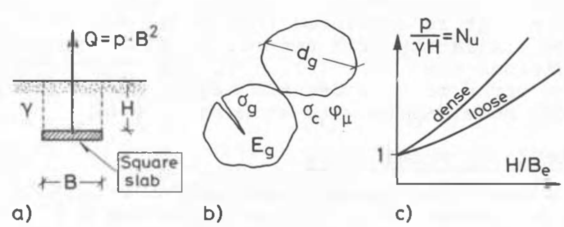

Figure 1: a) Definitions for an anchor slab in sand, b) sand grain properties, and c) dimensionless diagram for uplift capacity.

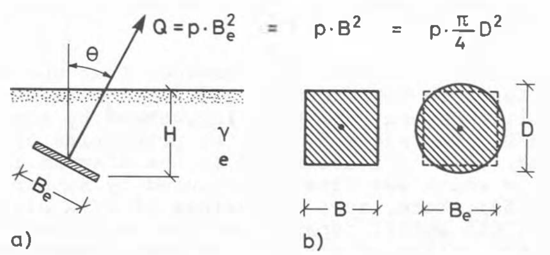

Figure 2: a) Anchor slab in a slanting position and b) definitions for circular/square slabs.

	Prototype Scale: 1:1 Gravity: g		Conventional Model Scale: 1:n Gravity: g		Centrifugal Model Scale: 1:n Gravity: ng	
1	$H/B_e = \frac{1}{B_e/H}$		H/B_e	similar	H/B_e	similar
2	Θ		Θ	similar	Θ	similar
3	e		e	similar	e	similar
4	ϕ_μ		ϕ_μ	similar	ϕ_μ	similar
5	$\frac{\sigma_c}{\gamma \cdot H}$		$\frac{\sigma_c}{\gamma \cdot H/n}$	not similar	$\frac{\sigma_c}{\gamma n \cdot H/n}$	similar
6	$\frac{\sigma_g}{\gamma \cdot H}$		$\frac{\sigma_g}{\gamma \cdot H/n}$	not similar	$\frac{\sigma_g}{\gamma n \cdot H/n}$	similar
7	$\frac{E_g}{\gamma \cdot H}$		$\frac{E_g}{\gamma \cdot H/n}$	not similar	$\frac{E_g}{\gamma n \cdot H/n}$	similar
8	$\frac{d_g}{H}$		$\frac{d_g}{H/n}$	not similar	$\frac{d_g}{H/n}$	not similar

Table 1: Similarity requirements for a conventional model and a centrifugal model.

σ_c (N/m^2) the interparticle cohesion between the sand grains

σ_g (N/m^2) the crushing strength of the grain material

E_g (N/m^2) the coefficient of elasticity of the grain material

d_g (m) the average grain size.

In the dimensional analysis the quantities γ and H are chosen to represent the basic units, length and force. The uplift capacity p per unit area of the slab can then be expressed in dimensionless form as an (unknown) function of eight independent dimensionless products in the following way:

$$\frac{p}{\gamma H} = F(H/B_e, \Theta, e, \phi_\mu, \frac{\sigma_c}{\gamma \cdot H}, \frac{\sigma_g}{\gamma H}, \frac{E_g}{\gamma \cdot H}, \frac{d_g}{H})$$

In order to obtain complete similarity the model test must be designed in such a way that the eight independent dimensionless products attain the same values for the model and for the prototype. The model and the prototype are then said to be completely similar.

Usually it is not feasible to impose complete similarity in a model test. Consequently, some of the independent dimensionless products which are believed to have secondary influence are allowed to deviate from their correct values. An important part of the work of the experimenter - indeed the most important part - is to justify his/her departures from complete similarity.

For the anchor problem it is obvious that the aim of the tests is to determine how the dimensionless uplift capacity $p/\gamma H$ is influenced by the ratio H/B_e, the angle Θ, and the properties of the sand. This is illustrated in the diagram in Figure 2c which was first introduced by Sutherland (1965). Note, that for values of H/B_e close to zero, the uplift capacity of the anchor slab equals the weight of the soil directly above the slab; consequently, the curves in the diagram must pass through point (0.1).

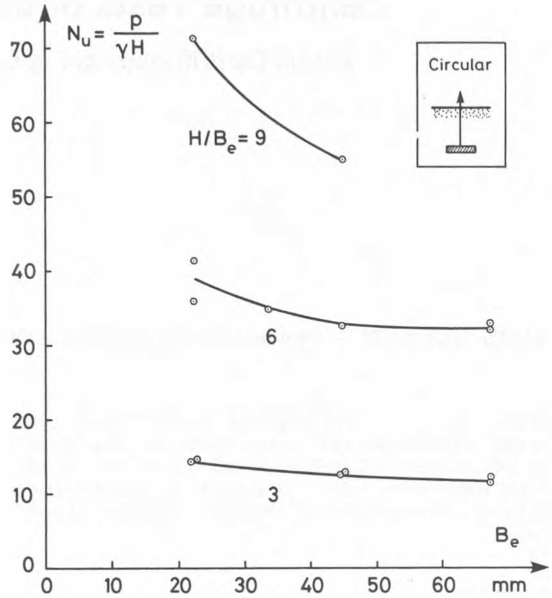

Figure 3: Results of conventional model tests with anchors performed by Baker and Kondner (1966).

Conventional model tests

In a conventional model test an anchor slab is built to the length scale 1:n. The slab is then tested in the same sand as that in the prototype. The eight similarity requirements for the prototype and the model can now be written in the way presented in Table 1.

The Table demonstrates that the model departs from complete similarity with the prototype in that four of the similarity requirements are not fulfilled. In order to investigate how this departure influences the results of model tests it is necessary to evaluate the results of such tests.

Baker and Kondner (1966) performed uplift capacity tests with slabs having diameters of 1, 1½, 2, and 3 inches at various depths. Some of their test results are presented in Figure 3 where the dimensionless uplift capacity factor $N_u = p/\gamma H$ is presented as a function of B_e for various H/B_e-ratios. Figure 3 clearly demonstrates that the test results are subjected to a scale error: The smaller width of the slab, the higher dimensionless uplift capacity factor. This scale error courses serious trouble in the transformation of the model test results to actual prototype size anchor slabs.

On the basis of the test results presented in Figure 3 it is concluded that departure from complete similarity with respect to the similarity conditions Nos. (5), (6), (7), and (8) causes a scale error of considerable magnitude in conventional model tests with anchors in sand.

Centrifugal model tests

The concept of a centrifugal model test is to expose a model built to the length scale 1:n with the same sand as that in the prototype to an artificial gravity field of the magnitude n·g, where g is natural gravity.

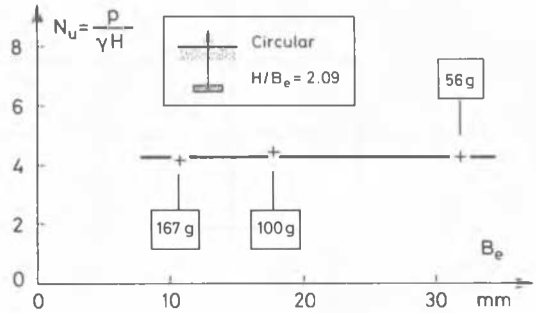

Figure 4: Results of Test Series 1; centrifugal tests with three different model anchors corresponding to a prototype, H = 3.70 m and D = 2.00 m.

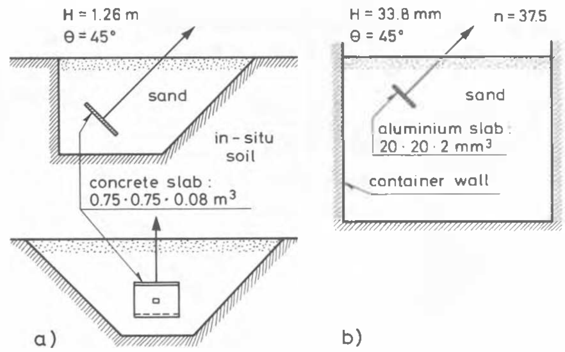

Figure 5: Geometrical parameters for a) DSB field test and b) equivalent centrifugal model test.

For this type of test all similarity requirements but No. (8) are fulfilled as demonstrated in Table 1. In other words: Seven similarity requirements are fulfilled in the centrifugal model test, where only four were fulfilled in the conventional model test. The centrifugal model test only deviates from the prototype in that the similarity requirement concerning the grain size is not fulfilled due to the fact that the prototype sand is used in the model. In order to investigate whether or not this departure from complete similarity results in a scale error in the centrifugal model test it is necessary to perform actual testing.

Centrifuge

Research on the uplift capacity of anchor slabs in dry sand have been performed in a centrifuge at the Danish Engineering Academy.

The centrifuge has an effective radius of approximately 0.72 m. The containers are of the swinging-bucket type; they have an inside diameter of 140 mm and a sand depth of 110 mm.

The uplift movement of the anchor slab was produced by a screw-spindle driven by an electric motor through a gear. The uplift force on the slab was measured by electrical strain gauges mounted on a proving ring; the movement of the anchor was measured by a linear displacement transducer. The force and movement and the centrifuge running speed were registered electrically during the test, which was strain controlled with a constant movement rate.

Test series 1

This series consist of three tests performed in the centrifuge and designed to model the same prototype. The tests were conducted with circular slabs of different diameters and pulled in the vertical direction.

The tests were made with the so-called G-12 sand which has been used extensively for model testing at the Danish Geotechnical Institute. The sand has an average grain size of 0.25 mm. It has a uniformity ratio U = 1.67 and its grains are rather rounded. The sand is air-dried and layered through a system of meshes which supplied a uniform deposit with a relative density I_D = 95%. The slabs were constructed of aluminium.

Figure 4 presents the results of the three tests. They were performed with anchors having different diameters and subjected to different accelerations; however, the product $n \cdot B_e$ is identical for the three tests; so is the ratio H/B_e = 2.09. Consequently, all three tests model a slab having a diameter of 2.00 m and buried at a depth of 3.70 m. (Actually, the ratio $H:B_e$ was not constant for the three tests; for experimental reasons it varied between 1.98 and 2.16. However, corrections were made to take this variation into account and thus enable a direct comparison of the test results).

Figure 4 demonstrates that no scale error is observed in the three anchor slab tests performed in the centrifuge. Note, that the three tests represent ratios of B_e/d_g ranging from 44 to 128.

Test series 2

This series consist of a field test and a centrifugal test modeling the field test as illustrated in Figure 5.

The field test was conducted in collaboration with DSB - the Danish State Railways. A square anchor slab was pulled by means of a hydraulic jack at an angle $\Theta = 45^o$ to the vertical. The fill is a well-graded sand with an average grain size of 0.8 mm; 25% of the grains (by weight) has a size between 2 and 8 mm. It has a uniformity ratio U = 6.0; the grains are fairly angular. The water content was w = 4.7%; the sand was compacted by a vibrator to relative density I_D = 1.0.

The same sand was used for the centrifugal test, except that all grains having a size larger than 2 mm were removed by sifting.

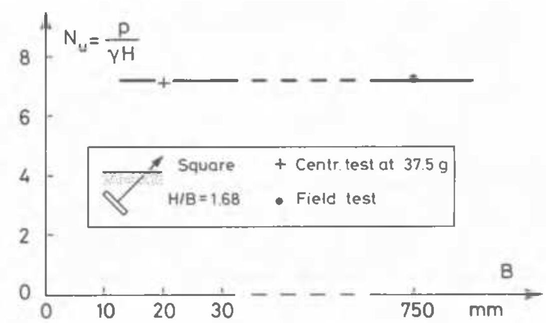

Figure 6: Results from Test Series 2; comparison between field test and centrifugal model test.

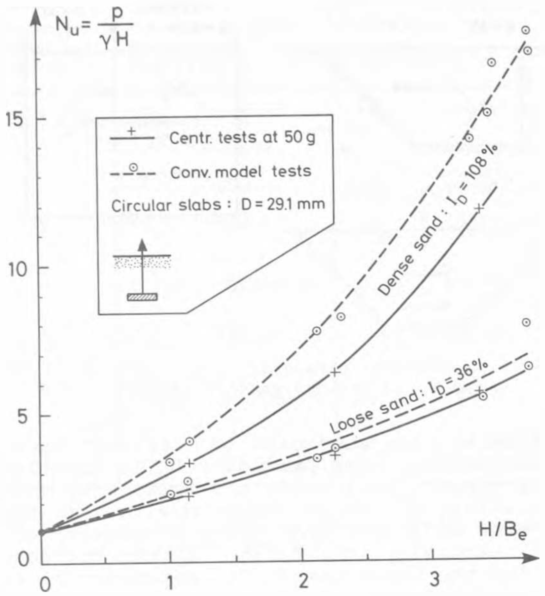

Figure 7: Results of Test Series 3 and 4; comparison between conventional and centrifugal model tests.

Figure 6, which presents the test results, demonstrates that no scale error is observed in the centrifuge test. Note, that the centrifugal model test represents a ratio of B_e/d_g equal to 25.

Test series 3 and 4

The two series consist of tests with circular anchor slabs pulled in the vertical direction; the tests were conducted on loose and dense sand within the range $0 \leq H/B_e \leq 3.5$. The tests in Series 3 were performed as conventional model tests and the tests in Series 4 as centrifugal model tests.

The sand used in these tests is a uniform diluvial sand - the so-called Dansk Normalsand No. 1. The grains are rounded and clean with diameters ranging from 0.3 to 0.6 mm; they consist almost exclusively of quartz. The sand is air-dried and layered through a system of meshes which supplied uniform deposits with almost constant void ratios. By using two different systems of meshes two different densities could be obtained corresponding to I_D = 108% and I_D = 36%, respectively.

All tests in series 3 and 4 were made with a model slab having a diameter D = 29.1 mm. The centrifugal model test in Series 4 were performed at an acceleration 50·g.

Figure 7 presents the results of 16 tests performed in Series 3 and 6 tests performed in Series 4. The lines in Figure 7 are drawn with the aim of representing the test results. The Figure demonstrates that for conventional model tests with the uplift capacity of anchor slabs in sand a scale error exists. The scale error might lead to an overestimation of the uplift capacity, especially for dense sand and for $H/B_e > 2$.

Conclusion

The following conclusion can be drawn concerning tests of the uplift capacity of anchors in sand:

1. Conventional model tests in a reduced scale are subjected to a scale error which might lead to an overestimation of the uplift capacity.

2. In centrifuge tests no scale error is observed for models where the model-width/average-grain-size ratios B_e/d_g were equal to or larger than 44 and 25 for Series 1 and 2 respectively.

The conclusion is in agreement with the conclusion drawn by the author (Ovesen (1979)) concerning centrifugal testing of circular footings on a sand surface where no scale error was observed for model-diameter/average-grain-size ratios larger than 30.

PART TWO: THE UPLIFT CAPACITY OF ANCHOR SLABS IN SAND

In order to investigate the uplift capacity of anchor slabs in sand two additional series of tests were performed. Series 5 examine the difference in uplift capacity between circular and square anchor slabs and Series 6 investigate the uplift capacity of an anchor slab placed in a slanting position.

A formula for the uplift capacity of anchor slabs in sand has be devised on basis of the test results.

Test series 5

A series of 6 centrifugal model tests were performed in order to examine the difference between the uplift capacity of a circular and a square slab. Table 2 gives the test programme: Tests were performed with a square slab having B = 20 mm, and with two circular slabs - one having D = 20 mm and another having D = 22.6 mm (i.e. the same area as the square slab). All slabs were pulled in the vertical direction.

Shape of slab		□ (square)	○ (large circle)	○ (small circle)
Size of slab (mm) H (mm) H/B_e		B = 20 60.2 3.01	D = 22.6 60.0 3.00	D = 20 60.0 3.39
I_D = 105%	$N_u = \frac{p}{\gamma H}$ =	10.1	10.0	12.4
I_D = 61%		7.3	7.3	8.1

Table 2: Results of tests in series 5: Comparison of vertical uplift capacity for square and circular anchor slabs.

The tests were performed with Dansk Normalsand No. 1, the same sand as the one used in Series 3 and 4. Three tests were performed with dense sand and three tests with medium-dense sand.

Table 2 presents the test results in terms of the uplift capacity factor $N_u = p/\gamma H$. The results show that the same uplift capacity is found from a circular and a square slab burried at the same depth, provided that the areas of the slabs are equal - or in others words: The same value of the uplift capacity factor N_u can be used for a circular and a square slab having the same H/B_e-ratio, provided the H/B_e-ratio for the circular slab is defined according to Figure 2.

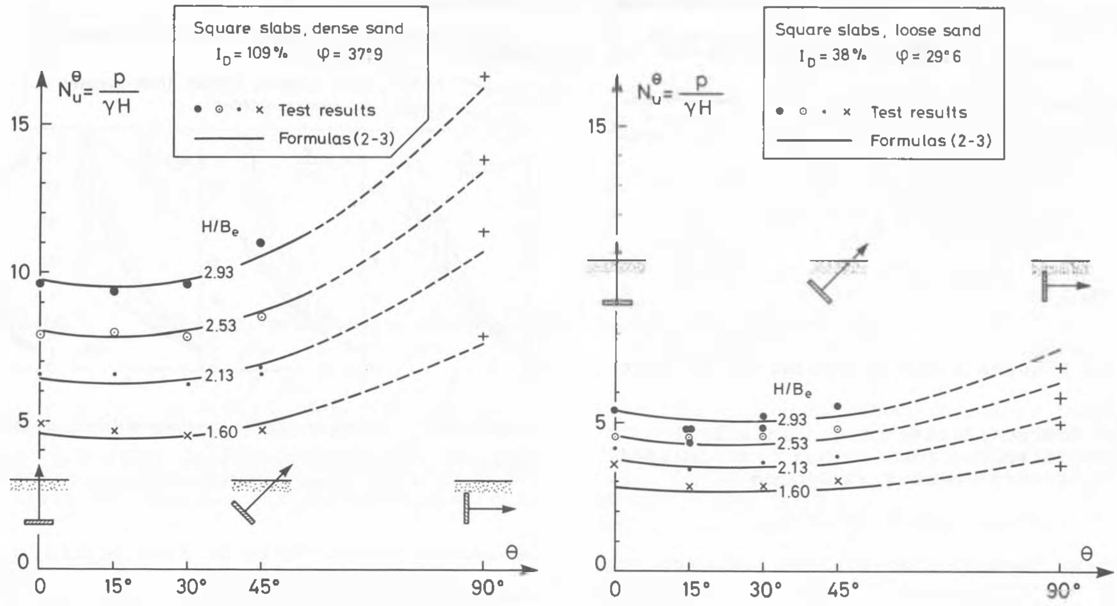

Figure 8: Result from Test Series 6; square anchor slabs in a slanting position.

Test series 6

A series of 35 centrifugal model tests were performed with anchor slabs placed in a slanting position. Tests were performed at four ratios H/B_e = 2.93, 2.52, 2.13, and 1.60 and at four angles Θ = 0°, 15°, 30°, and 45°. Square slabs having B = 20 mm were used. The tests were conducted at an acceleration 37.5·g.

The tests were performed with Dansk Normalsand No. 1, which was also used in Series 3, 4, and 5. Tests were performed at a dense and at a loose bedding.

Figure 8 presents the test results: The uplift capacity factor N_u^θ is given as function of the angle Θ for various ratios H/B_e for dense and loose sand respectively. From Figure 8 appears that N_u^θ decreases slightly when the angle Θ increases from 0° and up to about 20°; for larger values of Θ, N_u^θ increases, when Θ increases.

The author earlier performed an investigation of the resistance of vertical anchor slabs to a horizontal pull and proposed a semi-empirical method for the resistance of anchor slabs pulled in the horizontal direction (Ovesen and Strømann (1972)). This method has been used to estimate the horizontal resistance of anchors placed in a vertical position as indicated in Figure 8. However, it should be stressed that the values of N_u^θ for Θ = 90° in Figure 8 is estimated on basis of theoretical considerations and conventional model tests with a sand different from the one used in Series 6.

Summary of test results

Figure 9 summarizes the results of all centrifuge model tests performed in Series 4, 5, and 6 concerning the vertical uplift capacity.

All tests were performed on Dansk Normalsand No. 1. Due to difficulties in interpretation of triaxial test results with respect to the stress level, it has been chosen to relate the angle of internal friction ϕ to the properties of the sand in accordance with the Danish Code of Practice for Foundation Engineering 1977 according to which:" an estimate of the characteristic triaxial angle of friction may be obtained using the empirical formula:

$$\phi_{tr} \sim 30^\circ - \frac{3}{U} + (14 - \frac{4}{U}) I_D \qquad (1)$$

This expression is applicable to sand with fairly angular grains. A deduction should be made as shown below for silt content and more rounded grains:

10 per cent silt	- 2°
20 per cent silt	- 5°
rounded grains	- 3°

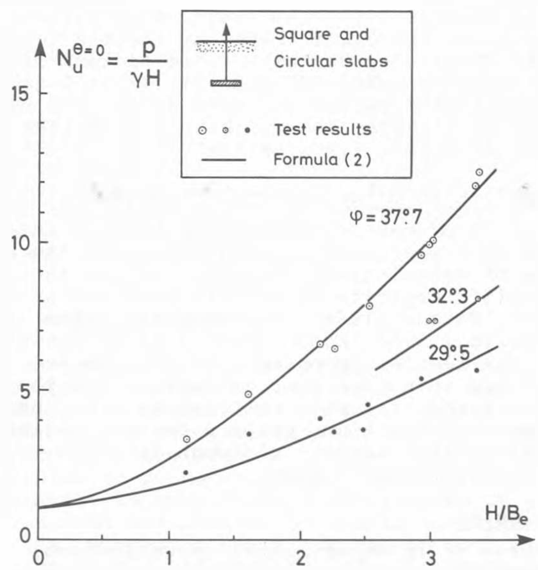

Figure 9: Results from Test Series 4, 5, and 6; vertical uplift capacity.

H	B_e	Θ	γ kN/m³	φ	p/γH test	p/γH formula (2)-(3)	Performed by
1.26	0.75	45°	19.8	42.2°	7.13	6.82	DSB-series 2
1.10	1.60	0	15.0	36°	1.78	1.89	Matsuo (1968) (I-5)
1.10	2.00	0	16.0	35°	1.59	1.59	Matsuo (1968) (III-2)
1.10	1.60	0	16.0	35°	1.88	1.83	Matsuo (1968) (I-2)
1.10	1.20	0	16.0	35°	2.20	2.27	Matsuo (1968) (IV-2)

Table 3: Comparison between results of field tests and formula (2) and (3).

The values of the angle of internal friction indicated in Figures 8 and 9 correspond to this estimate.

By means of "curve fitting" and statistical analysis the following formula has been devised to present the test results in Figure 9:

$$N_u^{\Theta=0} = \frac{p}{\gamma H} = 1+(4.32\cdot\tan\phi-1.58)(H/B_e)^{3/2} \qquad (2)$$

Note, that the formula fulfills the boundary condition $p/\gamma H = 1$ for $H/B_e = 0$. The curves drawn in Figure 9 correspond to Formula (2).

For anchor slabs in a slanting position the following formula in combination with formula (2) has been devised to represent the test results given in Figure 8:

$$N_u^{\Theta} = N_u^{\Theta=0}\cdot\left[1-0.33\left(\frac{\Theta}{90^\circ}\right)+1.27\cdot\tan\phi\cdot\left(\frac{\Theta}{90^\circ}\right)^2\right] \qquad (3)$$

Note that formula (3) fulfills the boundary condition $N_u^{\Theta}=N_u^{\Theta=0}$ for $\Theta = 0^\circ$. The curves drawn in Figure 8 correspondend to formulas (3) and (2).

Comparison with field tests

In order to investigate the validity of formulas (2) and (3) they have been checked against the field test conducted in Series 2 and against four field tests with anchor slabs in cohesionless soil reported by Matsuo (1968). Table 3 gives the geometrical parameters, properties of the sand etc. for the tests. Also given in Table 3 are the uplift capacity factors found in the tests together with those factors calculated from Formulas (2) and (3). Table 3 demonstrates that Formulas (2) and (3) yield a high degree of accuracy in predicting the uplift capacity of anchor slabs in the field.

Comparison to other authors' theories

Figure 10 presents a comparison between the results of the present investigation and the results of calculations according to the theories for uplift capacity of anchors proposed by Balla (1961), Matsuo (1968), Meyerhof and Adams (1968), and Vesič (1971). From Figure 9 it is evident that the teories porposed by Balla, Matsuo, and Vesič are in rather poor agreement with the results of the present investigation. The theory by Meyerhof and Adams gives a better prediction of the uplift capacity although it seems to be on the conservative side.

Conclusion

On basis of an experimental investigation, which employed the centrifugal testing technique, a set of formulas, (2) and (3), has been devised for the uplift capacity of circular

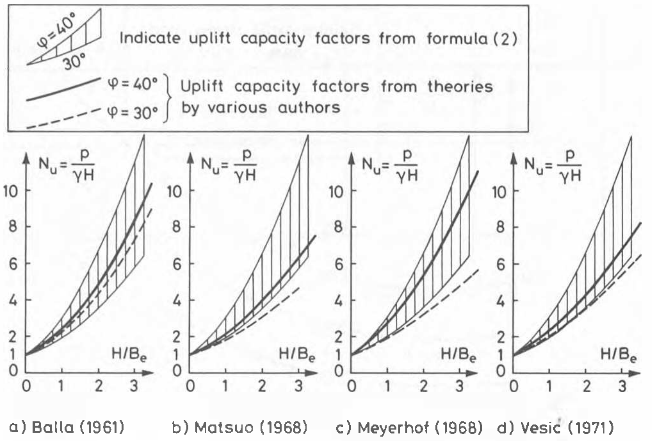

Figure 10: Results of centrifugal model tests compared to theories by various authors.

and square anchor slabs in sand pulled in a vertical and in a slanting direction. The set of formulas represents the test results with an accuracy which only in a few cases exceeds 5%; it shows also good agreement with the results of field tests.

The set of formulas can be used in practice to predict the uplift capacity of circular and square anchor slabs within the following limits:

$$0 \leq H/B_e \leq 3.5 \qquad 29^\circ \leq \phi \leq 42^\circ \qquad 0 \leq \Theta \leq 45^\circ$$

The angle of internal friction ϕ entering into the set of formulas is defined according to the Danish Code of Practice for Foundation Engineering 1977 as indicated by formula (1).

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