Soil Interaction in Buried Structures

Interaction du Sol avec les Structures Souterraines

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SYNOPSIS This paper deals with the arching-effect induced by the inflection of a deformable slab in an overlying layer of soil of finite thickness. The analysis is developed by assuming a linearly elastic constitutive law both for the soil and for the structure. This model can adequately describe the soil-structure interaction involved in the deformation of the upper slabs of artificial tunnels having a rectangular cross-section. The results of the analysis are represented in normalized form suitable for practical applications and they show the distribution of the contact pressures on the slab and the corresponding deformations of the ground level.

INTRODUCTION

The use of artificial tunnels in crossing hillocks or for the construction of underground railways and of ducts in urban areas is increasingly becoming widespread. One of the most outstanding geotechnical problems for these constructions is the evaluation of the contact stresses on the upper slab of the tunnels, keeping account of the soil-structure interaction.

The deformability of the soil and of the upper slab brings about a distribution of the contact stresses which may not correspond to the weight of the overlying soil, due to a phenomenon commonly called *arching-effect*.

The mechanism of the arching was explained by Terzaghi in his Theoretical Soil Mechanics and by other authors among whom Bjerrum et al.(1972) who stated that (see fig.1):

> *"... arching involves two parts: a reduction of the earth pressure on a yielding portion of a structure, and an increase of the earth pressure on the adjoining portions."*

The arching-effect in a layer having a finite thickness produced by a prescribed deformation of a flexible strip in the lower limiting plane is studied in this paper and the relevant results are used in the analysis of the interaction between the upper slab of an artificial tunnel and the overlying soil.

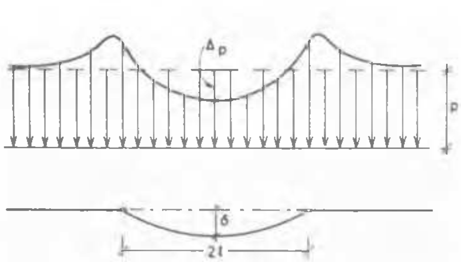

Fig. 1 - Simple arching problem (after Bjerrum et al., 1972)

REVIEW OF PREVIOUS WORKS

The first studies on the arching-effect in buried structures date back to the end of the last century. During the following decades the analysis of the mechanical behaviour of the soil and of the structure was developed on the basis of a number of different assumptions. The main constributions to this topic have been collected and illustrated in a previous work (Belli and Burghignoli, 1980). A short outline is given here of the main results provided by the theories that are currently used, with particular reference to those solutions that can be applied to practical problems.

A traditional solution was suggested by Terzaghi (1943) by assuming the soil to have a plastic behaviour. The upper slab of the tunnel is considered to be a rigid horizontal strip capable of moving vertically. The value of the contact stresses is obtained by equating the forces corresponding to a sliding mechanism where two slip surfaces are present at the boundaries of the strip.

Terzaghi's analysis is currently used in practice but the perfectly plastic constitutive law for the soil makes it impossible to link the flexibility of the slab to the contact pressures. Therefore it is not possible to analyse the interaction between the soil and the structure and to assess the states of stress and strain prior to failure.

The limitations of such hypotheses are overcome if the soils were considered to have an elastic behaviour as in the case of buried structures (i.e. artificial tunnels), in which the deformations involved are usually relatively small. Indeed, many studies on the arching-effect are based on this assumption. An elegant approach to the problem was developed by Finn (1963) under the hypothesis that the soil is linearly elastic; Finn obtained the expressions for induced stresses in a weightless half-space by the vertical displacement and the rotation of a rigid strip in the boundary plane (Fig. 2).

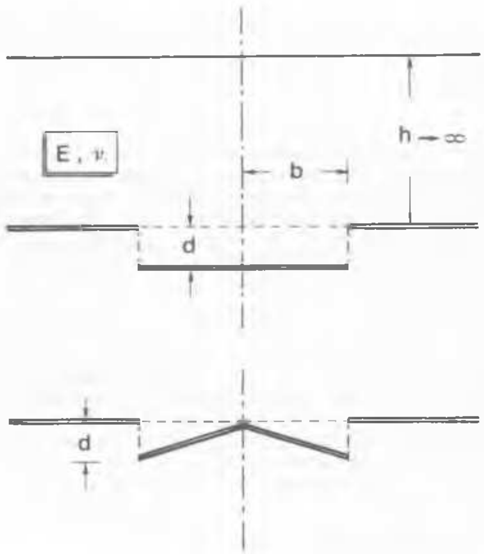

Fig. 2 - Strip displacement and rotation in Finn's arching analysis.

The following expressions are obtained for the variation of the contact pressures on a smooth strip:

$$\Delta\sigma_v = \frac{db}{\beta\pi}\,\frac{1}{x^2-b^2} \qquad \text{(vertical displacement)}$$

$$\Delta\sigma_v = \frac{db}{\beta\pi}\,\frac{1}{x^2-b^2} - \frac{d}{2\beta\pi b}\ln\frac{x^2}{x^2-b^2} \qquad \text{(rotation)}$$

where $\beta = (1-\nu^2)/E$ represents the elastic properties of the soil.

Finn's work was further developed by others. Chelapati (1964) studied the case of a finite layer over a rigid strip. On the same assumptions Burghignoli and Ricciardi (1978) and Belli and Burghignoli (1980) dealt with the case of a flexible strip. For layers of unlimited thickness the latter authors obtained the following expression for the contact pressure variation due to whatever deformation of the strip (adhesion being absent):

$$\Delta\sigma_v = -\frac{1}{2\beta\pi}\int_0^b\left[\frac{1}{(\eta-x)^2}+\frac{1}{(\eta+x)^2}\right]v(\eta)\,d\eta \qquad (1)$$

where b is the half-width of the strip, β is the constant of elasticity as already defined, and $v(\eta)$ is a function describing the shape of the deformed strip. If the following replacements are made in Eq. 1, $v(\eta) = -d$ or $v(\eta) = -\frac{d}{b}\eta$, the expressions obtained by solving the integral are the same as those obtained by Finn for the translation or rotation of a rigid strip.

In the following, attention is focused on the analysis of the contact pressures on the slab and on the deformation of the upper limiting plane of the overlying finite layer.

DEVELOPMENT OF THE PROPOSED ANALYSIS

For this problem a linear and isotropic model of an elastic continuum is expected to provide a sufficiently approximate description of the mechanical behaviour of the soil. Under these conditions the interaction between the slab and the overlying soil is studied by seeking the analytical function expressing the stress variation in the soil caused by any prescribed deformation of the slab, and by coupling this function with the one expressing the deformation of the slab brought about by any prescribed distribution of the contact stresses.

As to the first part of the problem, reference is made to the layout in Fig.3 where the slab is described by a deformable strip of a rigid plane that is the lower boundary of an elastic weightless layer having a finite and constant thickness.

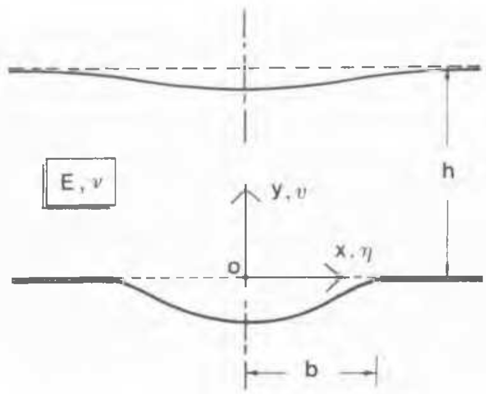

Fig. 3 - Scheme of strip deformation in the present analysis

For the assumption of a state of plane deformation the stress variations in the elastic medium can be placed under the following form (tensile stresses are taken as positive):

$$\Delta\sigma_x = \frac{\partial^2\phi}{\partial y^2} \qquad (2a)$$

$$\Delta\sigma_y = \frac{\partial^2\phi}{\partial x^2} \qquad (2b)$$

$$\Delta\tau_{xy} = \frac{\partial^2\phi}{\partial x \partial y} \qquad (2c)$$

The function ϕ that appears in Eq. 1 and that must satisfy the bi-harmonicity condition

$$\nabla^2\phi = 0 \qquad (\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})$$

can be expressed by the integral (Finn, 1963):

$$\phi = \int_0^\infty \frac{1}{\alpha^2} \left[(A_1 + A_2 y\alpha) \exp(y\alpha) + (A_3 + A_4 y\alpha) \exp(-y\alpha) \right] \cos x\alpha \, d\alpha \qquad (3)$$

The constants A_1, A_2, A_3 and A_4 are numerically defined by imposing the boundary conditions. For the case under study they are:

$$\Delta\sigma_y = 0 \qquad \text{for} \quad y = h \qquad (4a)$$

$$\Delta\tau_{xy} = 0 \qquad \text{for} \quad y = h \qquad (4b)$$

$$\Delta\tau_{xy} = 0 \quad \text{or} \quad \varepsilon_x = 0 \quad \text{for} \quad y = 0 \qquad (4c)$$

$$v(x) = \int_0^h \varepsilon_y \, dy \qquad (4d)$$

The conditions (4c) are to be used as an alternative to define the shear stresses or the strains on the soil-slab interface and in practice they specify whether the slab is perfectly smooth or rough. The former of these two conditions was adopted for the work described in this paper. The function $v(x)$ represents the deformation of the strip, and the unit strains ε_x and ε_y are defined as follows:

$$\varepsilon_x = \beta\sigma_x - \rho\sigma_y$$

$$\varepsilon_y = \beta\sigma_y - \rho\sigma_x$$

where $\beta = \frac{1-\nu^2}{E}$ and $\rho = \frac{\nu(1+\nu)}{E}$.

The displacement function v(x) can be expressed by a Fourier cosine integral

$$v(x) = \int_0^b \bar{v}(\alpha) \cos x\alpha \, d\alpha \qquad (5)$$

where

$$\bar{v}(\alpha) = \frac{2}{\pi} \int_0^b v(\eta) \cos \alpha\eta \, d\eta \qquad (6)$$

Taking into account Eq.s 5 and 6, the boundary conditions in Eq.s 3 lead to a system of linear equations, A_i (i=1, ..., 4) being the unknowns.

By solving the system, the following expressions can be obtained:

$$A_i = \frac{F_i(h\alpha)}{Den(h\alpha)} \qquad (i = 1, \ldots, 4) \qquad (7)$$

$F_i(h\alpha)$ and $Den(h\alpha)$ being some functions of exponentials.

The stresses induced by the deformation of the strip in the elastic medium are therefore:

$$\Delta\sigma_x = \frac{1}{\beta}\int_0^\infty \frac{\alpha\bar{v}(\alpha)}{Den}\left\{\left[F_1 + (2 + y\alpha) F_2\right]\exp(y\alpha) + \left[F_3 - (2 - y\alpha) F_4\right]\exp(-y\alpha)\right\}\cos x\alpha \, d\alpha \qquad (8a)$$

$$\Delta\sigma_y = -\frac{1}{\beta}\int_0^\infty \frac{\alpha\bar{v}(\alpha)}{Den}\left\{\left[F_1 + y\alpha F_2\right]\exp(y\alpha) + \left[F_3 + y\alpha F_4\right]\exp(-y\alpha)\right\}\cos x\alpha \, d\alpha \qquad (8b)$$

$$\tau_{xy} = \frac{1}{\beta}\int_0^\infty \frac{\alpha\bar{v}(\alpha)}{Den}\left\{\left[F_1 + (1 + y\alpha) F_2\right]\exp(y\alpha) - \left[F_3 - (1 - y\alpha) F_4\right]\exp(-y\alpha)\right\}\sin x\alpha \, d\alpha \qquad (8c)$$

The integrals in Eq.s 8 cannot be solved in a closed form. Indeed, two types of difficulties arise: firstly, rational functions of exponentials are present; secondly, the function $\bar{v}(\alpha)$ is generally unknown since it depends on the deformation of the strip. The first difficulty can be overcome by approximating the function $\frac{1}{Den}$ through the following expression:

$$\frac{1}{Den} \cong 0.5 + \frac{C_1}{h\alpha} + C_2 h\alpha \exp(-C_3 h\alpha) \qquad (9)$$

where C_1, C_2 and C_3 are numerical constants obtained by means of a fitting procedure of the actual function.

The second difficulty is overcome by introducing a linear relationship, as the function $\bar{v}(\alpha)$, into the integral in Eq.s 8, therefore approximating the deformed strip by a finite number of line segments. This is remarkably attractive in view of the use of such relationships for the analysis of the interaction with the structure. It is possible to go on, therefore, by applying the principle of superimposition, which can be used in virtue of the linearly elastic constitutive law. Then Eq.s 8 can be integrated for each strip segment and the induced stresses added (Fig.4). A solution is therefore obtained for whatever shape of the deformed strip.

Expressions of the following type are obtained:

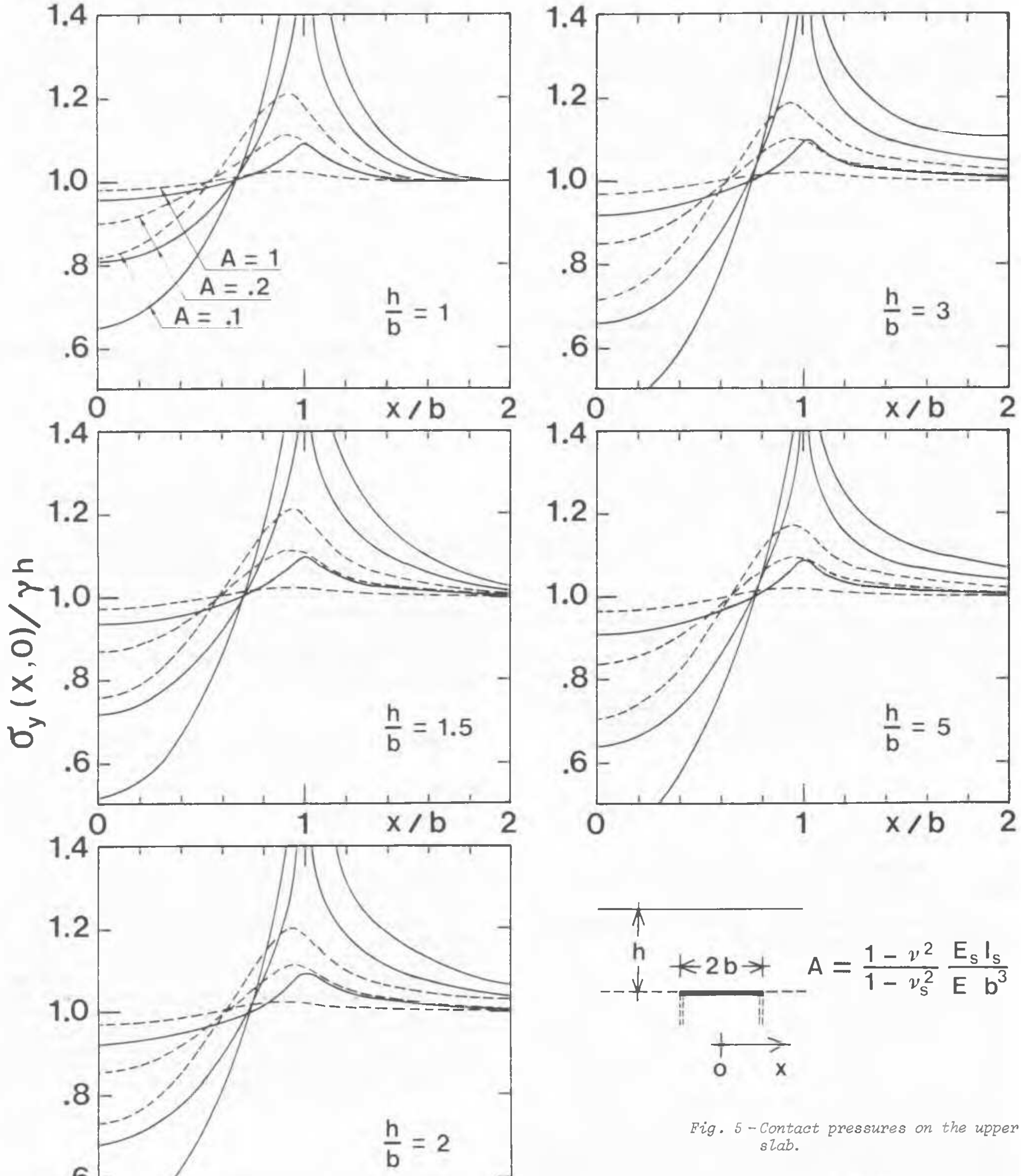

Fig. 5 - Contact pressures on the upper slab.

——— α = .4
------ α = 1

$$\Delta\sigma_y(X,Y) = -\frac{1}{\beta}\sum_{i=1}^{n}\varphi\left[H,Y,X;\ v_{i-1},v_i\right] \qquad (10)$$

and likewise for $\Delta\sigma_x$ and $\Delta\tau_{xy}$, where $H=\frac{h}{b}$, $X=\frac{x}{b}$, $Y=\frac{y}{b}$, n is the number of the linear segments by which it is possible to describe the deformation of the strip, and v_i the value of the displacement of the i-th node.

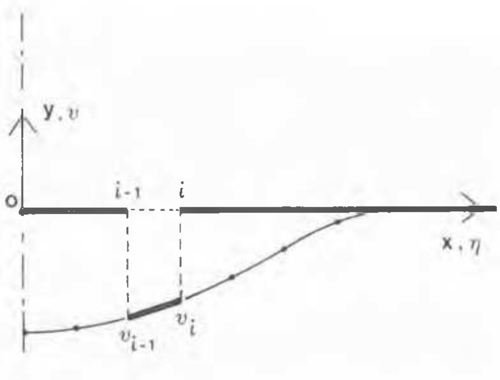

Fig. 4 - Displacement of a single element of the strip.

Finally, the problem of the interaction between the soil and the slab has been solved by associating Eq.10, where $Y = 0$, with the equation of the elastic line

$$-\sigma_y(X,0) = \frac{E_s I_s}{(1-\nu_s^2)\,b^4}\,\frac{d^4 v}{dX^4} \qquad (11)$$

where $-\sigma_y(X,0) = -\gamma h + \Delta\sigma_y(X,0)$, expressed in terms of finite differences with the same grid. An iterative solving procedure has been easily applied.

RESULTS AND FINAL REMARKS

The results of the analysis are summarized in the diagrams in Fig.s 5, 6 and 7. Fig. 5 shows the contact stresses on the slab calculated for some values of the ratio $\frac{h}{b}$ between the thickness of the overlying soil and the semi-width of the slab and for different values of the constant

$$A = \frac{1-\nu^2}{1-\nu_s^2}\,\frac{E_s I_s}{E\,b^3}$$

that defines the relative stiffnesses of the soil and the structure. A double set of curves is given in each diagram, obtained for two different values of the parameter α, which is the ratio of the bending moment at the boundary of the slab to the fixed end moment. The diagrams show a decrease in contact pressures, at the centre of the slab, and an increase in the areas adjacent to and outside the structure. This trend significantly increases if the bending moments at the slab ends and the structure stiffness decrease. As already noticed by other authors (i.e. Terzaghi, 1943 and Allgood, 1964), when the height of the overlying layer is from 1.5 to 2 times the breadth of the slab, the ratio between the contact stresses and those simply corresponding to the weight of the soil, remains practically constant.

The curves in fig.6 represent the vertical displacement of the upper limiting plane with respect to the maximum value at the axis of the slab. The curves practically depend only on the values of $\frac{h}{b}$ to which they refer and it must be pointed out that their shape does not depend on the restraining conditions of the slab nor on the relative stiffnesses of the soil and the structure.

It can furthermore be noticed that the curves are sensitively similar to the error function proposed by Peck (1969) to describe the deformation of the ground level following the construction of tunnels having a circular cross-section. The two dashed lines in the diagram show the deflection and maximum curvature points of the error function, and intersect the displacement curves close to the deflection and maximum curvature points.

In order to practically utilize the results illustrated in Fig. 6 it is necessary to know the value of the maximum vertical displacement. To this aim the diagrams in Fig. 7 can be used for determining the value of $\left[E_s I_s/\gamma h b^5\right] v(0,h)$, and therefore of $v(0,h)$ too, versus the different geometrical and stiffness characteristics.

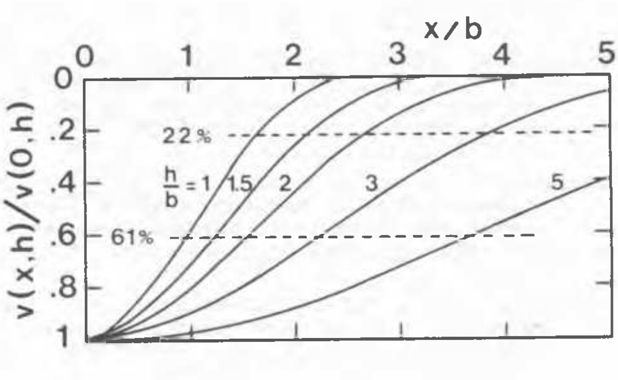

Fig. 6 - Deformation of the upper limiting plane of the layer overlying the slab.

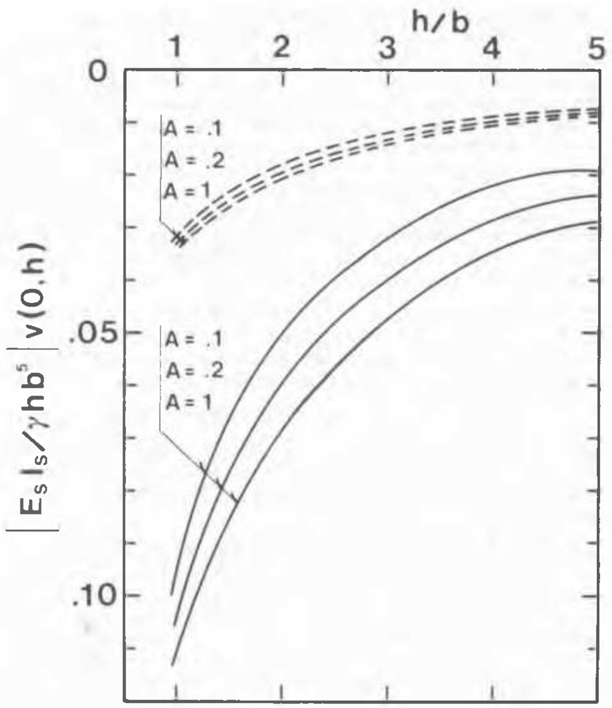

Fig. 7 - Values of the maximum vertical displacement of the upper limiting plane.

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