On the Validity of Winkler's Principle

Sur la Validité de Principe de Winkler

Th. DIETRICH Dr.-Ing., Bundesanstalt für Materialprüfung (BAM), Berlin

SYNOPSIS Winkler's principle, claiming the interaction problem to be one-dimensional, is capable of various interpretations. Beside the early interpretation as a bed of independent springs, more realistic interpretations have been offered by several authors. Thus Winkler's principle is made an attractive means for the solution of practical problems as well as for parameter studies. In this paper Winkler's principle is shown to apply to a wide class of statically loaded structures on half-space due to the beamlike or rodlike shape of the structure and due to certain features of the stress-strain-behavior of the material of the half-space, quite common with soils.

INTPODUCTION

Winkler (1867) assumed the reaction of the subsoil upon some point of a foundation structure to depend solely on the settlement of this point and to be independent of the settlements of neighboring points. This assumption will be called Winkler's principle in the sequel. Additionally Winkler assumed the reaction of the subsoil to be proportional to settlement. The two assumptions toqether may be considerd to represent an arrangement of independent elastic springs. (A bed of such springs is sometimes called a Winkler-material). As Gibson (1967) showed, a half-space whose modulus of elasticity increases proportional to the vertical distance from the surface also justifies the two assumptions.

Holzlöhner (1969) showed, that the two assumptions apply to a foundation-slab of arbitrary shape and loaded arbitrarily, which is supported by an elastic layer resting on a rigid base, provided the depth of the elastic layer is small compared to the dimensions of the slab and to other length-like parameters of the problem.

Whereas the first and second interpretation of the two assumptions specify the constitution of the subsoil, Holzlöhner's interpretation restricts the geometry of the class of systems considered. Due to this restriction, Winkler's principle (the first assumption) is valid. The second assumption may or may not apply depending on the constitution of the soft layer and on the amount of compression its particles suffer.

Holzlöhner then, by introducing geometrical restrictions, reduces the generally three-dimensional problem of soil-structure-interaction to a one-dimensional problem. This reminds of similar approaches in continuum mechanics. A. e. the reduction of the three-dimensional problem of the determination of the strains and stresses of a structural member to a one-dimensional problem by invoking the assumptions of de Saint-Venant's theory of bending rods.

Zimmermann (1888) argued already, that the geometrical peculiarities of rails supported by sleepers justify Winkler's principle. His argument is deficient however.

In the sequel, it will be shown, that a suitable enlargement of the body of assumptions of de Saint-Venant's theory reduces the problem of interaction between rodlike foundation structures and a material half-space of very general constitution to a set of two-dimensional problems, the elements of which are linked by the rodlike structure, and finally to a one-dimensional problem. Consequently Winkler's principle applies.

MOST CONTPIBUTING DOMAIN

Material Half-Spaces

A material half-space is a deformable body which in its initial configuration completely occupies the space extending from a certain plane to infinity. A half-space-problem with prescribed time dependant tractions on some part of its boundaries is considered properly stated, if any point of the material half-space under the action of the prescribed loading process will undergo but a finite displacement relative to the infinitely remote boundary. Whether this requirement can be met depends, once the constitution of the material half-space is given, generally on the kind of the applied loading process. It is a. e. not met by a homogeneous elastic half-space, which on an infinitely long strip of its surface is evenly loaded (cf. Selvadurai, 1979).

The Domain of the Material Half-Space Which Contributes Most to the Displacement of the Foundation Structure.

In the following the material half-space is supposed to obey the principle of continuity by virtue of which every particle will (except for some particles along discontinuities) retain its neighbors for ever. The boundary at infinite distance from the surface is supposed to be rigid and is taken for frame of reference. Furthermore the material is assumed to lack any property the physical dimension of which involves the dimension of time (which implies that it is nonviscuous). Consider now a material surface A, containing the locus S of loading and keeping everywhere a finite distance off the top surface of the material half-space (Fig. 1).

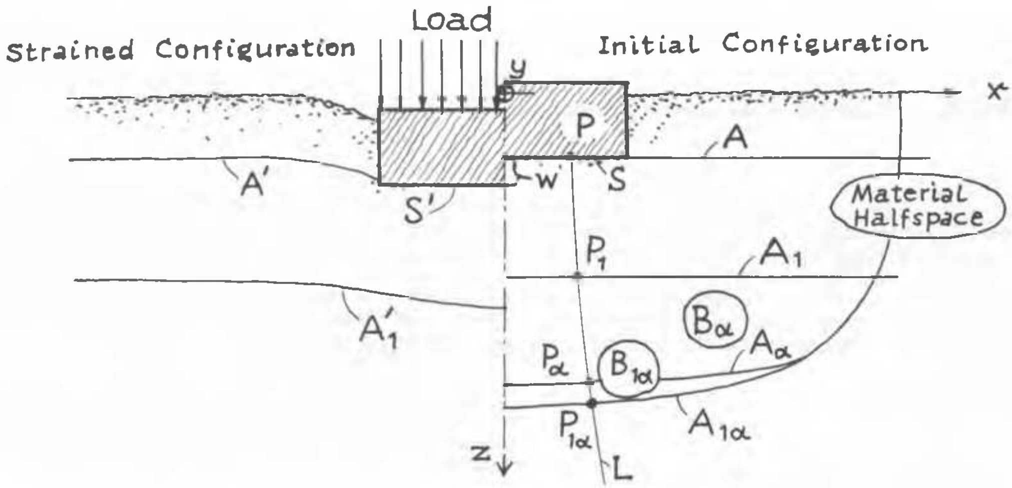

Fig. 1 The Domain B_α Wherein the Portion αw of the Deflection w Originates.

Assume A to transform into A' during the loading process whereby every material particle of A is displaced into a point of A'. The displacements may be obtained by integrating the strains of the particles below the material surface A. Where large strains are to be expected the loading process must be dissected into finite steps and the integration must be carried out for each step. Obviously the strains at great distance from S must vanish if the displacements are to remain finite. When this necessary condition is fulfilled, then, below some surface A_1 sufficiently distant from S, the integration may be carried out at once. For the rest of this investigation it suffices to consider the displacements of the particles of the material surface A_1 as will become apparent. To keep the displacements finite however it does not suffice that the strains vanish at all as the depth increases (this has been demonstrated by the example of the infinite strip load on a homogeneous elastic half-space). The strains must vanish fast enough moreover. This sufficient condition is fulfilled by the phenomenon of load scattering as long as S is a finite part of A. If S is infinite the finiteness of displacements cannot be ensured unless the material half-space possesses special properties as a. e. a modulus of elasticity which increases with depth.

Imagine now the material half-space to consist of material lines L connecting the surface A to the remote boundary. Every material line gives rise to a relation of order among its particles. Consider two particles P_1, P_2 of the material line L i. e. $P_1 \in L$, $P_2 \in L$. Let $P_1 \prec P_2$ if P_1 is situated between P_2 and P, where $P = A \cap L$. Let $w(P_1)$ denote the displacement of the point P_1 (where $P_1 = A_1 \cap L$) parallel to the local element of L and directed away from the surface. $w(P_1)$ is obtained by integrating the strains ε_λ parallel to the respective element of L along L from P_1 to the boundary infinitely below. If these decline fast enough so that the integral converges, one will obtain already an arbitrarily large portion of $w(P_1)$ if one carries out the integration - starting from point P_1 - along a finite part of L. Let $\alpha w(P_1)$ be that portion, then there exists a point $P_{1\alpha}$ at finite distance from P_1 and $P_1 \prec P_{1\alpha}$, so that $\alpha w(P_1)$ originates between P_1 and $P_{1\alpha}$, formally:

$$\bigwedge_{P_1 \in A_1} \bigwedge_{\alpha<1} \bigvee_{P_{1\alpha} \in A_{1\alpha}} w(P_1) - w(P_{1\alpha}) = \alpha w(P_1) \qquad (1)$$

where $w(P_{1\alpha})$ is the settlement of point $P_{1\alpha}$. All the points $P_{1\alpha}$ constitute a surface $A_{1\alpha}$. This and the surface A_1 bound a domain $B_{1\alpha}$ wherein α-times the settlement of the surface A_1 originates. Corresponding to the points $P_{1\alpha}$ there exist points P_α, where $P \prec P_\alpha$, which together with the surface A, bound a domain B_α wherein α-times the settlement of the surface A originates.

If the strains ε_λ along L are compressive throughout, then

$$\bigwedge_{P \in L} \bigwedge_{P_1 \in L} \bigvee_{P_\alpha \in L} \bigvee_{P_{1\alpha} \in L} P \prec P_1 \Rightarrow P_\alpha \prec P_{1\alpha} \qquad (2)$$

For brevity let $w_{1\alpha} = w(P_{1\alpha})$ etc. By definition

$$w_\alpha = w - \alpha w = (1-\alpha)\, w \qquad (3)$$

$$w_{1\alpha} = w_1 - \alpha w_1 = (1-\alpha)\, w_1 \qquad (4)$$

By proposition

$$P \prec P_1 \Rightarrow w > w_1 \qquad (5)$$

by (3) $\wedge$ (4) $\wedge$ (5)

$$w_\alpha > w_{1\alpha} \qquad (6)$$

and by proposition

$$w_\alpha > w_{1\alpha} \Rightarrow P_\alpha \prec P_{1\alpha} \text{ q.e.d.} \qquad (7)$$

Hence it suffices to investigate the settlement of the surface A , below which the strains are small enough so that the integration may be carried out at once, in order to make sure whether the settlement of the surface A (or of the locus of loading S respectively) is finite.

Without regard of the peculiar value of α the result of the above investigation may be summed up by saying that - provided certain conditions of convergence are fulfilled - most of the settlement of the surface A is contributed by a finite domain of the material half-space. Shortly, this domain will be referred to as the most contributing domain. For practical purposes one may choose, say, $\alpha = 90$ %.

The shape of B_α is affected by the choice of the material lines L by which it was defined. However this ambiguity does not invalidate the usefulnes of the notion of the domain B_α .

Once the domain B_α and the surface A_α have been established, let w denote the vertical displacement in the sequel.

BOUNDARY LAYER OF QUASIPLANE MOTION

Conditions of Plane Deformation of a Material Half-Space Under an Evenly Loaded Rigid Beam

It is customary to assume the (plastic) deformation of the subsoil under an evenly loaded foundation beam to be plane if the ratio of the length l of the beam to its breadth b exceeds a certain value (see a. e. DIN 4017, Tab. 2). Conditions will be discussed now that justify this assumption.

length l increases. Therefore a beam whose length is finite but surpasses a certain measure will induce a nearly plane deformation of the material half-space (see Fig. 2).

Boundary Layer of Plane Motion of a Cylindrical Foundation-Structure

Imagine the evenly loaded rigid foundation-beam as it starts penetrating into the material half-space. The particles of the half-space adjacent to the penetrating faces of the structure will, except for points or lines of discontinuity, partake in the movement of the rigid faces: they will have the same velocities as the adjacent points of the faces. In its immediate vicinity an evenly loaded beam therefore will always induce plane deformation of the material half-space. By use of an appropriate quantity, measuring the deviation of three-dimensional motion from plane motion, one may define therefore a boundary layer B_π of plane motion induced by a rigid cylindrical foundation-structure, translating normally to its generatrices. If it happens that the most contributing domain (introduced in the preceding chapter) does not exceed the boundary layer of plane motion (except for negligible portions) then the foundation structure causes plane motion of the material halfspace.

To continue the investigation the material of the half-space is specified as psammic material. This is a continuous representation of an assembly of an unlimited number of unbreakable, rigid, heavy grains, the interparticle forces of which are governed by Coulomb friction (Dietrich 1977). Except for its mass density psammic material possesses dimensionless properties only.

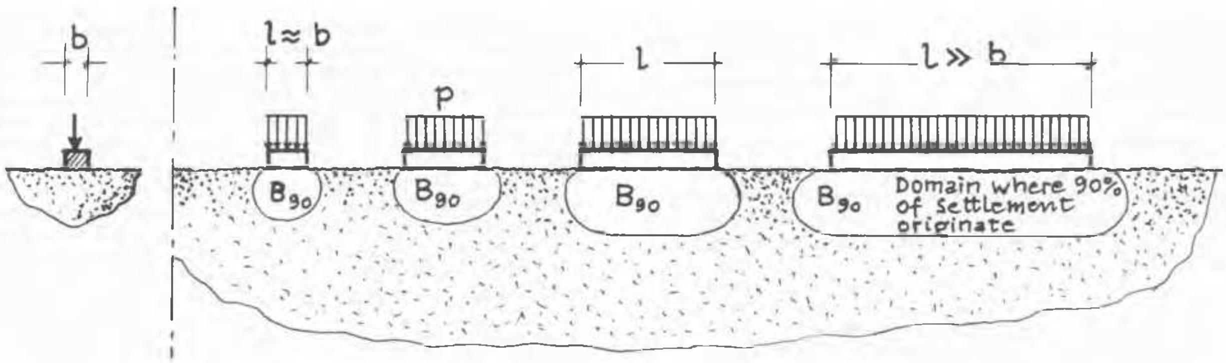

Fig. 2 Development of a State of Plane Deformation Under a Slender Foundation-Beam on a Psammic Half-Space

If the constitution of the material half-space is such that evenly distributed loading of its entire surface produces a finite settlement only (and therefore a finite depth z_α of the bottom of the domain B_α) as long as the intensity of the load remains finite, then the settlement due to any other not evenly distributed loading of finite intensity will also be finite and the depths z_α of these problems will also be finite. For reasons of symmetry the deformation under an evenly loaded beam of infinite length will be plane then and the domain B_α will be bounded by a cylinder. Due to the principle of continuity invoked, plane behavior and the cylindrical boundary of B_α will develop gradually as the

The settlement w(t) of the rigid beam of length l and breadth b may now be written as

$$w(t) = f(\gamma, b, l, p(\tau))_{\tau=0}^{t} \tag{8}$$

where γ denotes the initial density of the psammic half-space (PSH), t the present instant, o the time of beginning of loading and τ the time coordinate. p(t) denotes the evenly, distributed load at time t , while $p(\tau)_{\tau=0}^{t}$ denotes the loading (i. e. the process of load from O until t . By dimensional analysis (8) is transformed as follows

$$\frac{w(t)}{b} = \varphi\left(\frac{p(t)}{\gamma b}, \frac{l}{b}, \frac{p(\tau)}{p(t)}\right)_{\tau=0}^{t} \tag{9}$$

for brevity this is rewritten as

$$\frac{w}{b} = \varphi(\frac{p}{\gamma b}, \frac{l}{b}, \mathscr{P}) \quad (10)$$

where w and p denote the present values of these variables and $\mathscr{P}$ denotes the dimensionless process of load. As has been shown elsewhere (Dietrich 1979) a PSH settles by a finite amount only if the intensity p/γ of the arbitrarily distributed load remains finite. Therefore the limit $\lim \varphi_{l\to\infty}$, obtainable from Eqs (9) and (10) exists. According to what has been said above, there exists a value l_ε then also, such that a beam whose length $l > l_\varepsilon$ causes a motion of the PSH the deviation of which from plane motion - appropriately defined - will remain below a prescribed bound. But Eq (10), respectively the limit $\varphi_{l\to\infty}$, may be interpreted differently yet. Instead of increasing l, one may diminish b and compensate this variation by a corresponding variation of p in the first argument. Therefore a short beam will cause a nearly plane motion of the PSH if it is narrow enough. Whether plane motion prevails will depend then - for fixed values of $p/(\gamma b)$ and $\mathscr{P}$ - not on the absolute size of the foundation structure but on its shape only, i. e. on the ratio l/b. Observe, that this result required the existence of a peculiar property of the material half-space, having the physical dimension $KL^{-\nu}$, where $\nu > 2$ (in case of PSH $\nu = 3$). Let z_π denote the depth of the bottom of the domain B_π, the boundary layer of quasiplane motion, pertaining to the beam considered. Obviously

$$l/b \to \infty \Rightarrow z_\pi/b \to \infty \quad (11)$$

i.e.: as the length l increases the regime of plane motion will take over the entire half-space. In case of a PSH one may write

$$\frac{z_\alpha}{b} = \psi(\frac{p}{\gamma b}, \frac{l}{b}, \mathscr{P})$$

Since $\lim \varphi_{l\to\infty}$ of Eq (10) exists with a PSH, $\lim \psi_{l\to\infty}$ of Eq (12) exists also, i.e.

$$l/b \to \infty \Rightarrow z_\alpha/b = \text{const} \quad (13)$$

Comparing (11) and (13) one notices that any domain B_α - regardless of the value of α - will stay within the boundary layer of plane motion B_π, if only the beam will be slender enough.

QUASIPLANE MOTION OF MATERIAL HALF-SPACE

Quasiplane Motion with Rigid Foundation-Structures

Consider a rigid foundation beam on a newly sedimented PSH being unevenly loaded as depicted in Fig. 3. The load varies according to

$$p(x,t) := (1 + \frac{2x}{l})\, p_1 t \quad (14)$$

The resultant load $P(t) = \int_{-1/2}^{+1/2} p(x,t)dx$ acts at the distance $l/6$ from the centerpoint of the beam. Its intensity increases proportional to time t starting at $t = 0$. The beam will translate as the evenly loaded beam but also rotate. Consider now a set of rigid beams of equal breadths b but different lengths l' and arrange them in order of increasing l', from $l' = l$ until $l' = \infty$. Let their loads be denoted by p' and let them vary according to

$$p'(x,t) := (1 + \frac{2x}{l'})p_1 t \quad (15)$$

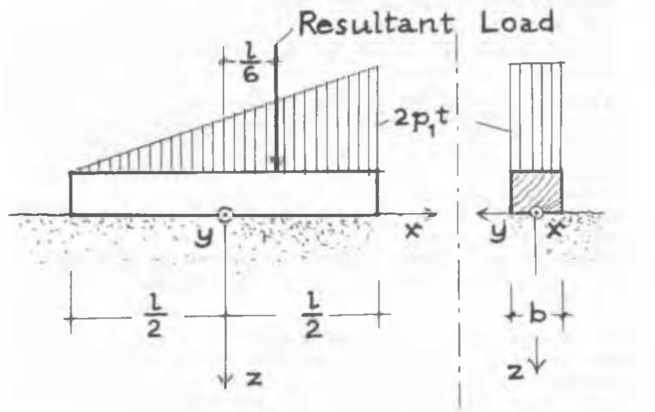

Fig.3 Unevenly Loaded Rigid Foundation-Beam on Material Half-Space

All beams of the series carry the same average load, namely $p_1 t$ units per unit of area of the base of each beam. Therefore all beams will exhibit about equal settlements $w(\xi l',t)$ at equal abscissas $\xi = x/l'$ and equal times t. Each resultant load possesses the same excentricity of 1/6 of the beams length. All beams of the series will settle unevenly therefore. The slope $\mu(t) := \partial w(x,t)/\partial t$, constant along each beam, will decrease, however, as l' increases. Therefore, as l' increases, the motions of the psammic particles at cross-section $x = \xi l'$ will differ ever less from the motions of the particles at cross-sections $x + dx$ and $x - dx$. Moreover, the motion of the PSH at $x = \xi l'$ will differ ever less from the plane motion of the PSH under an evenly loaded beam of equal dimensions. At an other cross-section $\bar{x} = \bar{\xi} l'$ a plane state of deformation will develop also but different from the one at $x = \xi l'$. This type of motion of the PSH under a slender, rigid, excentrically loaded beam will be called quasiplane.

Quasiplane Motion with Flexible Foundation Structures

The locally plane states of deformation of the quasiplane regime under a slender, rigid excentrically loaded beam are related in an especially simple way due to the rectilinear line of deflection of the rigid beam. But quasiplane motion may occur also under a flexible foundation - beam exhibiting a curvilinear line of deflection. The motion will be exactly quasiplane, if the distortions in sections normal to the beam's axis will vanish compared to the distortions in sections parallel to the beam's axis and if the influence of both tips of the beam will vanish. This is secured by the following conditions

$$\frac{\partial w(x,t)}{\partial x} \to 0 \quad (16)$$

$$\frac{\partial w(x,t)}{\partial x} \Big/ \frac{w(x,t)}{b} \rightarrow 0 \qquad (17)$$

$$b\frac{\partial^2 w(x,t)}{\partial x^2} \rightarrow 0 \qquad (18)$$

$$b/l \rightarrow 0 \qquad (19)$$

which must be fullfilled at any point x of the beam's axis and at any time t. Condition (16) is well known as a requirement for the application of de Saint-Venant's theory of beams. The denominator of (17) characterizes the locally plane state of distortion . The expression (18) represents the ratio of the beam's breadth to the radius of curvature of the elastic line of the beam. A foundation-structure complying to the four conditions (16) to (19) will be called "bending-rod-like structure in material halfspace". This notion is useful because quasiplane motion is at hand already when the expressions (16) to (19) assume values current in engineering practice but still far from zero. The usefulness of de Saint-Venant's beam-theory is due to similar reasons.

The four conditions (16) to (19) cannot be fullfilled simultaneously in every kind of material half-space. Consequently quasiplane motion requires a certain specification of material properties of the material half-space as has been pointed out for the elastic half-space and for the PSH. Such properties are quite common with soils.

WINKLER'S PRINCIPLE AND QUASIPLANE MOTION OF MATERIAL HALF-SPACE

The behavior of a foundation-beam and of the supporting half-space may be determined not only by the loading process but also by the process of the elastic line. The latter determines the reactions of the material half-space, the bending moments of the beam and finally the corresponding load (more precisely the processes of bending moments and loads). In quasiplane motion a particle of the material half-space exchanges energy but with particles within its local plane of deformation. Therefore its state of stress is determined exclusively by the process of the displacement w of the beam's cross-section coincident with that plane. This is true especially of those particles adjacent to the beam and transmitting the reaction q of the material half-space unto the penetrating beam. In case of PSH:

$$q(x,t) = f(\gamma, b, w(x,\tau))_{\tau=0}^{t} \qquad (20)$$

Eq (20) expresses Winkler's principle. Indeed, if the foundation-structure may be considered as bending-rod-like structure in a material half-space the stiffness of which increases with depth (as a. e. of a PSH) then Winkler's principle applies, i. e.: The reaction of the half-space unto some point of the structure depends solely on the displacement of this point and is independant of the displacements of neighboring points.

BENDING-ROD-LIKE STRUCTURES IN MATERIAL HALF-SPACE

Unlimited Distortion of Half-Space

Under the heading of bending-rod-like structures in material half-space come many of the most common foundation-structures as foundation-beams, strip-foundations, single piles vertical and inclined, piles in groups, grids and others.

The conditions (16) to (19) place no limitation on the distortions within the planes of deformation making up the quasiplane deformation of a material half-space. Hence Eqs (16) to (19) may be employed along with a theory of plasticity. They justify a. e. the analysis of interaction between a laterally loaded rigid pile and a Coulomb-half-space presented by Brinch-Hansen (1961) and similar approaces by other authors. The practical value of such an analysis seems limited however because the stiffness of ordinary construction-materials is so low and the strength of soils is so high as to forbid the simultaneous fulfillment of the failure-condition of the material half-space and the conditions (16) to (19). It may be possible to meet those conditions simultaneously in a material of very low bulk density. In this respect the fact is of importance that only the buoyant weight enters the analysis of quasistatic problems.

Bending-Rod-Like Structures in Material Half-Space with Small Deformations

To the conditions (16), (17), (18) and (19) a further condition will be added now, requiring the distortions of the material half-space to be small. As has been stated before, the intensity of the quasiplane state of distortion of the material half-space around a bending-rod-like structure is characterized by the denominator of expression (17). Hence the additional condition reads:

$$\frac{w(x,t)}{b} \rightarrow 0 \qquad (21)$$

Condition (16) may be recovered as consequence of Eqs (17) and (21). (No other condition than (16) may be eliminated between (16), (17), (18), (19) and (21).) Hence, there are again not more than four independant conditions, namely Eqs (17), (18), (19) and (21). They require not only small distortions but small deformations all over the half-space. Therefore these conditions represent the type of structure mentioned in the heading of this chapter.

On account of (21) and with regard to the hardening law of psammic material (Dietrich 1977a) Winkler's principle may be transformed from (21) into the following incremental form (Dietrich 1979):

$$\Delta q = \gamma b\left(\frac{\Delta w}{b}\right)^{\varkappa} k(x,t) \qquad (22)$$

where $\varkappa$ denotes the minimal exponent of hardening of psammic material $(\varkappa<1)$. In case of an elastic half-space Eq (22) is replaced by a differential equation.

Self-Similar Bending-Rod-Like Structures in Psammic Half-Space

In case of self-similar structures in PSH the methods of an extended dimensional analysis together with considerations of topology and measure theory (Dietrich 1979) yield finite expressions for quantities of interest in the form of power laws. Consider a. e. an elastic straight beam of infinite length resting on a PSH under the action of a cyclically wandering, distributed load as depicted in Fig. 4.

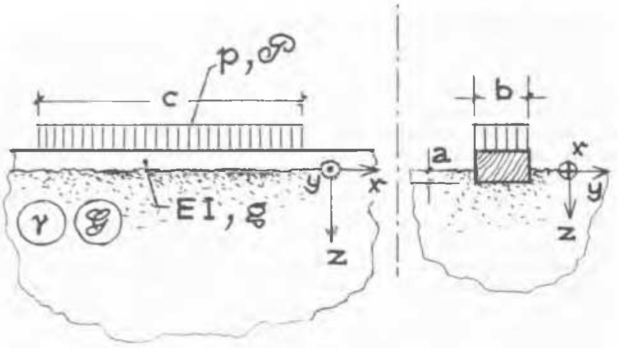

Fig. 4 Modelling of a Railroad-Train by a Wandering Distributed Load on a Flexible Beam Resting on a Psammic Half-Space

This system was developed for the modelling of settlements due to railroad-traffic (Dietrich 1979). If the length c of the trains is made infinitely short (i. e. the distributed load p is replaced by a concentrated load P) or if the trains are made so long that no particle in the most contributing domain can notice the front end and rear end of a train simultaneously, and if the distance between consecutive trains is large, the system will become self-similar. In case of the short train one obtains

$$w = b\left(\frac{P}{\gamma b l_E}\right)^{\frac{4}{1 + 3\varkappa}} \varphi(n) \qquad (23)$$

where $b, \gamma, \varkappa$ have been explained already. $l_E = \sqrt[4]{s/\gamma}$ denotes a kind of elastic length. $s = EI/b$ denotes the flexural rigidity per unit of breadth of beam. n counts the trains that have passed over the beam. w denotes the settlement at a fixed instance of the load cycle. A. e. the settlement of a certain cross-section of the beam when a concentrated load passes that section or a. e. when two consecutive loads have equal distances from that section.

Corresponding formulas exhibiting a power of the load's intensity with an exponent rational in $\varkappa$ may be obtained for other bending-rod-like structures in PSH with small deformations. Elsewhere (Dietrich 1977 b) experimental data have been published verifying the power-law-form for a self-similar type of laterally loaded flexible pile.

CONCLUSIONS

In this paper Winkler's principle, claiming the interaction problem to be one-dimensional, is shown to apply to a wide class of statically loaded structures on half space due to the beamlike or rodlike shape of the structure and due to certain features of the stress-strain-behavior of the material of the half-space, quite common with soils. Foundation members complying to the requirements of elementary beam theory and being of narrow width (compared to length or to radii of bending curvature respectively) cause the adjacent soil to move in quasiplane fashion. If the deformations of the soil particles, responsible for the deflections of the foundation structure, mainly occur within this boundary layer of quasiplane motion, then the three-dimensional problem of soil-structure-interaction degenerates into a set of two-dimensional problems, the elements of which are linked by the elastic foundation structure. Consequently, Winkler's principle applies.

The reduction of three-dimensional problems to two dimensions or one dimension respectively is of immediate profit to analytical, numerical and experimental investigators.

The investigations upon which this paper is based have been sponsored by the Federal Minister of Research and Technology (BMFT) of the Federal Republic of Germany under contract No.TV 78172 C

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