Stability of the Soil Base under Rigid Structures

Stabilité des Sols sous les Structures Rigides

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SUMMARY The first part of the paper (M.I.Gorbunov-Posadov) gives a method for stability analysis of a centrally loaded rigid plate on sand base that involves integrating Karman differential equation defining limit stress behaviour of the soil medium. The second part (D.D.Sapegin and A.L.Goldin) deals with interaction of a rigid plate on elasto-plastic base. The problem involved the usage of V.N.Nikolayevsky method and application of finite elements method. The third part (V.V.Belenkaya and V.M.Perley) considers the problem of bored wall stability that is solved assuming circular cylinder shape of the sheared mass which is taken to be a rigid solid.

1. M.I.Gorbunov-Posadov's paper (1965) at the VI-th Conference on Soil Mechanics and Foundation Engineering gave the stability analysis technique for the sand base under a rigid rough centrally-loaded plate the said technique complying with test data on the shape, structure and size of the forced-out mass as well as with the magnitude of critical load. In De Beer's General Report (1965) at the same Conference this approach was critically scrutinized. The solution assumes formation, right under the plate, of an elastic zone of the densified core that is shaped as a Curvilinear triangle with curved-in sides, the elastic part of the core being surrounded by said triangle's major part which is in the limit state. The core, after having been formed forces the soil apart thus acting as a retaining wall. There are some questionable points in the method, e.g. soil displacement trajectories are identified with slip-lines. These shortfalls, however, are made up for by coincidence with experimental data for the plane problem obtained in a test-box by the technique involving photo-registration of sand grains movements.

The method, however, involves complicated computations. The problem has been only solved for the case of internal friction angle $\varphi = 40^{\circ}$. Further use of the method in practical applications necessitaes solutions for other values of the friction angle.

Another technique has been also suggested by the author (M.I.Gorbunov-Posadov, 1962) which, however not so well coincident with the shape of theforced-out prysm obtained from test data, still better corresponds to the experiment than most other methods while involving a strict and rather simple computation technique.

The used scheme stems from the assumption that under the rough bottom of the plate a core is formed that has an isosceles traingle shape with the angle at the base equal to the angle of internal friction as was proposed by Terzaghi (1943). It is also assumed that the angle of friction of sand against the faces of the core δ is also equal to φ

Along the sides of the triangle the following relationship is fulfilled:

$$\sigma = \gamma r s(\theta) \quad (1)$$

with $\sigma = (\sigma_1 - \sigma_2)/(2\sin\varphi) = (\sigma_1 + \sigma_2)/2$ (2)

r and θ as polar coordinates (Fig.1). The pole is located at the summit of the triangle (Fig.2)

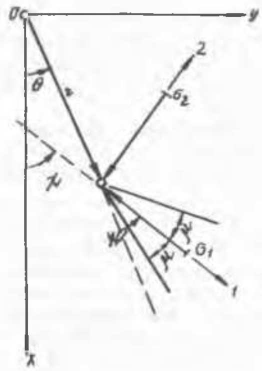

Fig.1. Notations to the solution of Karman-Sokolovsky

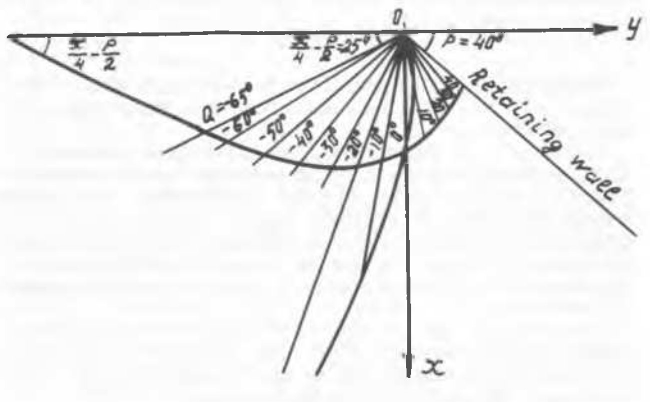

Fig.2. Slip-lines of both families when solving Karman problem ($\varphi = \delta = 40^{\circ}$)

The limit stress state theory yields the following

$$\left.\begin{matrix}\sigma_r \\ \sigma_\theta\end{matrix}\right\} = \sigma(1 \pm \sin\varphi\cos 2\psi); \quad \tau_{r\theta} = \sigma \sin\varphi \sin 2\psi \quad (3)$$

where ψ is the inclination of the major principle normal stress to the radius-vector passing through the point under consideration (Fig.1).

Stresses in the plastic zone can be determined and slip-lines found by simultaneous solution of two ordinary differential equations:

$$\frac{d\psi}{d\theta}+1=\frac{\cos\theta-\sin\varphi\cos(2\psi+\theta)-s\cos^2\varphi}{2s\sin\varphi(\cos 2\psi-\sin\varphi)} \quad (4)$$

$$\frac{ds}{d\theta}=\frac{-\sin(2\psi+\theta)+s\sin 2\psi}{\cos 2\psi-\sin\varphi}$$

The slip-lines' inclination to radius r drawn to the point under consideration is $\psi\pm\mu$ and to axis x is $\chi\pm\mu$ with $\mu=\pi/4-\varphi/2$. The equation for the slip-line is as follows:

$$r=C\exp\left[\int^{\theta}\operatorname{ctg}(\psi\pm\mu)d\theta\right] \quad (5)$$

We fix a point on the core's boundary which is assumed to be the face of the retaining wall for this point $\theta=\theta_0=\pi/2-\varphi$(Fig.2). The first family of slip-lines departs from the point vertically downward therefore $\chi_0-\mu=0$, wherefrom

$$\chi_0=\mu;\ \psi_0=\chi_0-\theta_0=\mu-\theta_0=-\frac{\pi}{4}+\frac{\varphi}{2}=-\mu \quad (6)$$

The value $\psi=\psi_0$ is to be incorporated in formulae (4,6) and then,assuming various values $S=S_0$ at the same point,we integrate equations (4) with the help of finite differences technique. The final magnitude of S_0 is obtained by the trial and error method so that, having approached to the zone of the maximal Renkin stress state with this value of S_0, to obtain at its boundary the present values of

$$\psi_1=-(\pi/4-\varphi/2);\ S_1=\cos\theta_1/(1-\sin\varphi) \quad (7)$$

However,for the value of ψ_0 from (6) the derivatives defined by equations (4) on the wall become infinite when $\delta=\varphi$. This is why according to Sokolovsky (1952) the arguments should be ψ rather than θ and obtained from the solution of differential equations (4) should satisfy boundary conditions

$$\theta=\theta_0;\qquad S=S_0 \quad (8)$$

Theoretical results for $\varphi=40^\circ$ are shown on Fig.2 and all slip-lines from the same family are similar.

Contrary to Karman,our results have shown that the slip-line of the second family starts right from the pole itself. Therefore the core's boundary ("the retaining wall") does not envelope the slip-lines, as Karman believes,but is rather the ultimate straight line approached by the lines from the second family when $C\to\infty$.

In accordance to equation (1) and to values $0=50^\circ$ $S=250$, $r=y/\cos 40^\circ=1.3055y$ where y is the horizontal distance from the plate's edge (Fig.2). $\sigma=1.3055\cdot 250\gamma y\approx 326\gamma y$

It can be shown that the sum of vertical reactions applied to the core

$$V=2\int_0^a\sigma(y)dy=326\gamma a^2$$

where a is a half width of the plate. The weight proper of the densified core with $\varphi=40^\circ$ is $G=\operatorname{tg}\varphi a^2\gamma=0.98\gamma a^2$. Critical load $p_{\kappa p}=\bar{p}\gamma a$; $\bar{p}=325,2/2=162.6$. (For 34° $\bar{p}$=44,6; 37° $\bar{p}$=80,9)

K.Terzaghi solution (1943) (by interpolation) for 40° $\bar{p}$=100. Solution based on the circular slip-surfaces technique (Gorbunov-Posadov, 1951) $\bar{p}$=68. A Caquot solution (1953) $\bar{p}$=114. Experimental data $\varphi=39^\circ$ (Kananyan,1954) $\bar{p}$=160. The improved solution (Gorbunov-Posadov,1965) gives $\bar{p}$=191.

II. Analysis of bearing capacity of subsoil under rigid structures is based on the solution of the elasto-plastic problem. Presently this problem is being solved by numerical methods, finite elements included. Both associated and non-associated soil plastic flow laws are being used. This paper tackles the problem of interaction between a rigid plate and the elasto-plastic base,defined by B.N.Nikolayevsky model (1975).

$$d\varepsilon_{ij}=d\varepsilon^e_{ij}+d\varepsilon^p_{ij};\ d\varepsilon^e_{ij}=\frac{1}{9K}d\sigma\delta_{ij}+\frac{1}{2G}\left(d\sigma_{ij}-\frac{d\sigma}{3}\delta_{ij}\right);\ d\varepsilon^p_{ij}=\left[\sigma_{ij}+\frac{2}{3}\alpha\Lambda Y\delta_{ij}-\left(1+\frac{2}{3}\Lambda\alpha\right)\frac{\sigma}{3}\delta_{ij}\right]d\lambda \quad (9)$$

$d\lambda\geq 0$ if the flow condition is verified

$$\Phi(\sigma_{ij})=T+\frac{\alpha}{3}\sigma-Y=0 \quad (10)$$

and $d\lambda=0$ if $\Phi(\sigma_{ij})<0$

Conventional denotations are used here,α and Y as analogues of the friction angle and cohesion,Λ as dilatancy rate. These values are the functions of hardening and according to Nikolayevsky (1975) are chosen in the following form

$$\alpha=\alpha_0+\sin(\gamma-\alpha_0)\frac{e_p}{e_{\kappa p}};\ Y=Y_0=\mathrm{Const} \quad (11)$$

$$\Lambda=\cos\gamma-\sqrt{1+2\cos\gamma(\sin\gamma-\alpha)-\alpha\sin\gamma}$$

where α,γ,Y_0 and $e_{\kappa p}$ are soil parameters, $e_p=\varepsilon^p_{11}+\varepsilon^p_{22}+\varepsilon^p_{33}$. The functions of hardening (11) can be obtained by processing experimental data, e.g. stabilometer tests described by Arnold and Mitchell (1973). Conditions (9) and (10) along with equilibrium equations, geometrical relationships and boundary conditions form a confined system of equations. Finite elements method is used that defines the problem in question as system of non-lines equations that is

$$[K]\{u\}=\{F\} \quad (12)$$

with $[K]=[K(\sigma_{ij},d\varepsilon_{ij},e_p)]$ as the matrix of the system, $\{u\}$ as nodal displacements vector,$\{F\}$ as external nodal forces vector.To solve (12) successive loading technique has been used and a system of the following linear equations at every load step has been solved

$$[K](\delta u^{(n)})]\{\delta u^{(n+1)}\}=\{F_{n+1}\} \quad (13)$$

where $\{\delta u^{(n+1)}\}=\{u^{(n+1)}-u^{(n)}\}$, $\{u^{(n)}\}$ are nodal displacements after n-th loading.

Elastic solutions technique yields an iterative process for each load step

$$[K_0]\{\delta u^{(n+1)}_{\ell+1}\}=\{F^{(n+1)}\}+[K_0]\{\delta u^{(n+1)}_{\ell}\}-[K(\delta^{(n)}_{ij},e^{(n)}_p,\delta u^{(n+1)}_{\ell})]\{\delta u^{(n+1)}_{\ell}\} \quad (14)$$

The initial approximation is obtained from equations $[K_0]\{u^{(0)}\}=\{\mathcal{F}_1\}$ where $[K_0]$ is the matrix for the system of equations relative to elastic solid.

The series of elastic problems is computer processed with the help of finite elements technique. Supplement loads are computed using the following formula

$$[K]\{\delta u\} = \int [B]^T (\delta\sigma)\, dV \qquad (15)$$

where $\{\delta\sigma\}$ is the stress variation that results from displacements $\{\delta u\}$. $\{\delta\sigma\}$ is determined through the use of relationships (9) and (10):

$$\{\delta\sigma\}=[\mathcal{D}^e]\left(\{\delta\varepsilon\}-\{\psi\}d\lambda\right)=\{\delta\sigma^e\}+\{\delta\sigma^p\} \qquad (16)$$

with $\{\delta e^e\}=[\mathcal{D}^e]\{\delta\varepsilon\}$, $\{\delta\sigma^p\}=-[\mathcal{D}^e]\{\psi\}d\lambda$

Here $[\mathcal{D}^e]$ is elastic matrix of the element

for the plane problem
$$\{\psi\}=\begin{Bmatrix}\sigma_{11}+\frac{Q-\sigma}{3}\\ \sigma_{22}+\frac{Q-\sigma}{3}\\ \sigma_{12}/2\end{Bmatrix}$$

$$Q=2\Lambda\alpha\left(\nu-\frac{\sigma}{3}\right);\quad \{\delta\sigma^p\}=\begin{Bmatrix}\sigma_{11}+\frac{Q-\sigma}{3}+\nu\psi_0\\ \sigma_{22}+\frac{Q-\sigma}{3}+\nu\psi_0\\ \sigma_{12}/2\end{Bmatrix} \qquad (17)$$

$\psi_0=\dfrac{\sigma+2Q}{3(1-\nu)}$, for the plane stress-state,

$\psi_0=Q/(1-2\nu)$ for the plane strain $\nu=\dfrac{3K-2G}{6K+2G}$

$\delta\lambda$ is determined assuming that deformations of each element within one load step develop proportionally: $\{\delta\varepsilon(a)\}=a\{\delta\varepsilon\}$ $0<a\le 1$

If $0\le a\le a_0$ elastic deformations only occur and a_0 is obtained from equation $\Phi(\sigma_{ij}+a_0\delta\sigma_{ij}^e)=0$ and $\delta\lambda=0$. If $a_0<a\le 1$ $d\lambda>0$ and its magnitude is determined from the following equations:

$$\Phi\left(\sigma_{ij}+a\delta\sigma_{ij}^e+\delta\sigma_{ij}^p(a),\ e_p+\delta e_p(a)\right)=0 \qquad (18)$$

$$de_p=Q(e_p)\,d\lambda$$

with initial conditions

$$\delta\sigma_{ij}^p=\delta e_p=0 \quad \text{with } a=a_0 \qquad (19)$$

Condition (18) yields the following relationship:

$$Q(e_p)\frac{\partial\Phi}{\partial\sigma_{ij}}\delta e_{ij}^e\,da+2T\left(\frac{\lambda(e_p)\sigma(\sin\gamma-\alpha)}{3e}-K\alpha(e_p)\lambda(e_p)-G\right)de_p=0 \qquad (20)$$

Equations (18) and (20) with condition (19) enable to determine $\delta\lambda$ and δe_p for each finite element if a=1. Supplement loads are obtained from formulae (16),(17).

Research worker from VNIIG V.S.Prokopovich has compiled a computer program. The case of high sand base supporting a symmetrically loaded rigid plate having the length $2L$=10m has been treated. The area of the base was 81x35,5m. The base consisted of low density river sand. Sand deformation graphs have been borrowed from the paper of Arnold (1973). The parameters of elasto-plastic model γ =0 corresponding to the graphs are K=60MPa, G=21,5MPa, ψ=0,0006, α_0=0.6, e_{kp}=0.008. Fig.3 shows data displaying how plastic zones in subsoil develop for various loads. On the figure one sees how plastic deformations are initiated under the plate's edges. When mean normal contact pressure becomes 140 the plastic zones join each other and an elastic zone forms itself right under the plate. Fig.4 shows how the

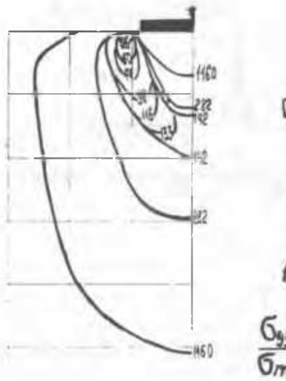

Fig.3. The zones of plastic deformations versus mean pressure (KPa) from the plate

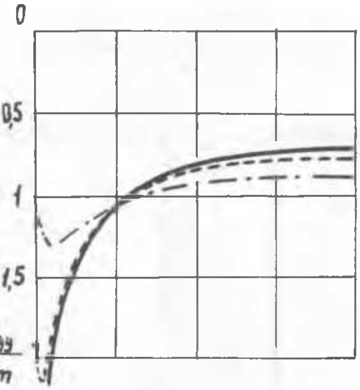

Fig.4. Evolution of contact pressure versus loads applied to the plate.

diagrams of normal contact pressure change under the plate due to the load increase and to the development of plastic zones. Smaller loads cause the contact pressure diagram close to that of linear elasticity. Larger plastic zones cause greater normal contact stresses under the center of the plate.

III. When the "Bored wall" technique is used for constructing underground premises in urban or industrial environment brings there arises complicated problem of safety of buildings and structures located close to the trench. The congested construction site forbids, as a rule, to adopt a distance from the side of the trench to the existing unit to be large enough to provide for the strength of the foundation while excavating the trench. This effect may be lessened by shortening the length of the section excavated with clay slurry used.

This necessitates the technique to compute the length of the section that would provide for the assessment of the state of subsoil under existing structures regarding stability and deformations. Analytical solution is given below for the problem of stability of a footing located nearby the trench.

The method is based on the plane problem solution with the use of circular slip-line technique (M.I.Gorbunov-Posadov and V.V.Krechmer,1951). This solution is free from technique that contradicts statical approach and drastically simplifies stability analysis for the case of vertically loaded plate supported by cohesionless soilbase.

We have extended the basic points of the method over the stability problem of soil mass confined in a quarter-plane and consisting of cohesive soils with a loaded free surface.

As one deals with short sections of the trench and therefore stress and strain behaviour state of the subsoil is far from being planar one has to take into account the third dimension. This is done by approximately including shear resistance on lateral faces of the sheared soil mass. The sheared soil mass is a solid confined between the trench wall, subsoil surface and circular slip-line.

and two lateral walls of the trench.

The lateral crossections of the sheared mass and the applied loads are shown on Fig.5.

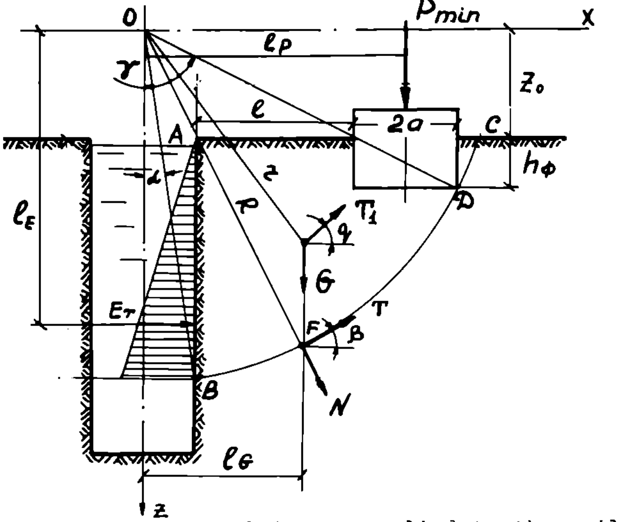

Fig.5. Diagram of forces applied to the soil mass.

P^{min} minimal load applied to the footing; G soil mass weight; N and T normal and tangential components of the reaction from the resting soil to the moving one; E^T tne pressure from thixotropic solution; T friction over lateral faces of the mass; $R, z, \ell_P, \ell_E, \ell_G$ distances from forces acting on the sheared soil mass to the center of rotation.

The length of the section and of the sheared mass (along the axis normal to the crossection) equals to L.

Stress behaviour of the sheared mass ABC in limit equilibrium acted upon by the above forces is described by the following system of equations :

$$E_T + T\cos\beta - N\sin\beta + 2T_1\cos\varphi = 0; \quad (21)$$

$$P_{min} + G - N\cos\beta - T\sin\beta - 2T_1\sin\varphi = 0;$$

$$TR + E_T\ell_E - G\ell_G - P_{min}\ell_P + 2T_1 z = 0;$$

$$T = N\,tg\,\varphi + C_1$$

with φ as soil friction angle, C_1 as resultant cohesion along the slip-surface; other notations are clear from Fig.5.

Unknown are radius R and coordinates of the center (axis) of the slip-line as well as minimal load P_{min} applied to the footing defined by shearing moment over resultant resistance ratio equal to unity. The change of the distributed reaction for its resultant with component T and N applied to one point of the slip-line (point F) was proposed by M.I.Gorbunov-Posadov (1951), who showed that such an assumption corresponds to the minimal stability of the soil mass.

Solving the system of equations (21) we obtain :

$$P_{min} = \frac{E_T\,tg\,\varphi + C_1\sin\beta + 2T_1\cos\varphi\,tg\,\varphi}{\cos\beta\,(tg\,\beta - tg\,\varphi)} \times \left(\frac{\cos\beta}{tg\,\varphi} + \sin\beta - G - C_1\frac{\cos\beta}{tg\,\varphi} + 2T_1\cos\varphi\right) \quad (22)$$

The values in (22) are the functions of pre set parameters and of angles α and γ that outline the size and location of the sheared mass. The angles should provide for minimal P, i.e. $\partial P/\partial\alpha = 0$ and $\partial P/\partial\gamma = 0$. A grid has been used to find minimal value of the load , said grid having nodes defined by variable couples of parameters α_o and γ_i with virtual angles adopted between 0° and 90°.

Safety of the footing neighbouring a trench is ensured if the following condition is complied with :

$$K^z = P_{min}/P_{comp} \geq 1{,}2 \quad (23)$$

where K^z is the safety factor for the subsoil, P_{comp} is the actual load on the footing; P_{min} is the load outlined by expression (22).

Closeness of theoretical and experimental results enables to conclude that the proposed method for computing critical load is sufficient to realistically display soil mass stress behaviour nearby trenches of various lengths. 3-D shape and soil cohesion taken into account resulted in bringing together theoretical and experimental data.

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