# Subgrade Reaction — A Rheological Problem

## La Réaction du Sol — un Problème Rhéologique

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SYNOPSIS In order to effectively assess the deformations of multi-supported deep excavation walls a calculation method is needed that takes into account predeformations and displacements of the supports. Recently, computation with moduli of horizontal subgrade reaction has increasingly become the method of choice. To determine the numerical value of the modulus of horizontal subgrade reaction proves to be difficult, however, as it is not a soil parameter, but depends on the stiffness, the shape of the construction, and the rheological behaviour of the subgrade. The method using moduli of subgrade reaction is based on simplified approximations. These estimates were verified by inclinometer measurements effected on diaphragm walls of the Vienna Underground.

## INTRODUCTION

On account of the lack of space in urban built-up areas it became increasingly necessary to use deep, open or multi-supported excavation walls. Because of the surrounding buildings, sewers, water-pipes, etc., the consulting engineer has to pay attention to the deformation behaviour of soil and construction. Repeated measurements of settlements, heaves and horizontal displacements provide the basis for an evaluation of the activities around the excavation site, i.e. the boundary conditions, which may prove useful, should controversies with the neighbourhood arise. Furthermore, these measurements may assist the consulting engineer in reassessing his assumptions on soil mechanics and, if necessary, take preventive action.

Where sheet piling and Berlinoise walls are concerned, simplified assumptions as to earth pressure distribution and prop displacement are complemented by constant moduli of stiffness covering the total depth of the wall. This method is somewhat difficult to apply in case of diaphragm wall constructions with unevenly distributed reinforcements. Therefore, it is necessary to extend the model and include data on predeformations from the previous excavation stage, on displacements and on deflexions of the wall. The traditional stability verification model should eventually be replaced by a method providing means to distribute the reinforcements according to the actual deformations and load transpositions resulting therefrom.

The method using moduli of horizontal subgrade reaction meets these requirements and proves to be a time saving and efficient tool. It lends itself easily to numerical verification and may be effectively used to assess static conditions around the excavation. The method proves somewhat inadequate when it comes to the determination of the magnitude and distribution of the moduli of horizontal subgrade reaction, as these are dependent not only of the stiffness conditions of the diaphragm walls but also of rheological parameters relating to the excavation.

## SUBGRADE REACTION

The subgrade reaction method is based on a simple static model using a linear-elastic suspended beam with the spring stiffnesses chosen according to the numerical value and the distribution of the moduli of subgrade reaction. Terzaghi (1955) formulates in his comprehensive treatise two fundamental assumptions based on the theory of linear-elastic subgrade reaction.

(i) at every load intensity the modulus of subgrade reaction $k_s$ is independent of the contact pressure p and the corresponding displacement s:

$$k_s = \frac{p}{s} = \text{const.} \qquad (1)$$

(ii) the distribution of the modulus of subgrade reaction for cohesive and cohesionless material can be expressed in the equation

$$k_x = k_s\left(\frac{x}{d}\right)^n \quad n = 0,1,2\ldots \qquad (2)$$

where ratio $k_x$ indicates the modulus of horizontal subgrade reaction activated in the wall/soil contact area in depth x for each different layer.

Fig.1 (a) illustrates the relationship between the actual load determined settlement and the dot-and-dash-marked approximation and clearly shows the error ensuing from the introduction of a constant modulus of subgrade reaction.

It is recommended therefore to use a constant modulus of subgrade reaction only in load ranges of zero up to half of the ultimate bearing capacity p .

Hayashi (1921), Delapierre and Dufour (1980) developed solutions where the non-linear interaction between soil and construction is shown in a hyperbolic load-settlement curve. The results thereby obtained compare better with the in situ measurements than those calculated on the basis of a constant value $k_s$. The error

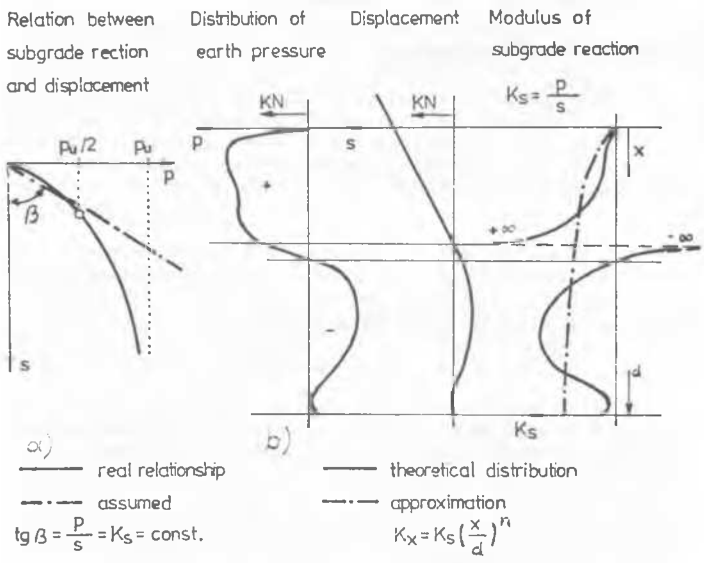



Fig.1 Assumptions of the Theory of Subgrade Reaction

contained in assumption (i) is insignificant, however, as compared to the simplification inherent in assumption (ii).

As can be seen from Fig.1 (b), the correctly calculated distribution of the horizontal subgrade reaction shows points of discontinuity at the zero-points of the load distribution and the deflection curve. Exponent n takes account of two boundary conditions, namely the surface of the excavation and the reducement in utilisation of the horizontal subgrade reaction due to discontinuities. Also, this exponential equation is more convenient for data-processing. Titze (1943), Werner (1970), Sherif (1974) propose a variety of continuous distributions of the horizontal subgrade reaction using one modulus for the total length of the subgraded part of the construction. With the present state of computer technology, it is now possible, however, to introduce also discontinuous distributions of subgrade reaction which considerably improves the determination of differing stiffness conditions of the layers. Still, the accuracy of the results obtained by numerical computation should not be overestimated. When calculating the bending moments for the diaphragm walls, we need to know the approximative value and distribution of the horizontal subgrade reaction, and the possible, time-dependent predeformations resulting from the last excavation stage. Therefore, a third assumption has to be introduced.

(iii) displacement s under load p remains constant over the whole duration of the excavation stage.

This assumption is fully adequate as long as insignificant subgrade reaction occurs. It proves inadequate, however, as soon as the subgrade reaction increases and the displacement curve of the side wall does not remain constant any more. Therefore, when determining the numerical value of the modulus of horizontal subgrade reaction the stress duration must be taken into account. Long excavation stages and the final stages of excavation in subsoils with creeping effects require reduced moduli.

In some sections of the construction site of the Vienna Underground where comparable geotechnical conditions were found, it was considered appropriate to verify input data by comparing them with in situ measurements of the diaphragm walls. Back-analysis of predicted results also was found adviseable.

## INCLINOMETER MEASUREMENTS

In order to determine the value and distribution of the moduli of subgrade reaction, earth pressure and wall displacement have to be measured. The installation of earth pressure cells on the contact face between soil and diaphragm wall proved to be problematic for two reasons: not only had the pressure cell to be protected from being covered with concrete, but the determination of size and stiffness of the cell itself, especially with respect to quarternary and tertiary sediments, presented some problems as well, (Hanna, 1973). Since each sediment responds differently to stress history, and earth pressure on the diaphragm wall varies from one excavation stage to the other, it is not easy to assess the extent of information that may be obtained by means of an earth pressure cell. As a result, direct earth pressure measurements were not used in this case.

Measurements of horizontal wall displacement, however, can be effected satisfactorily prior to and during all excavation stages by means of various inclinometer systems. The procedure is as follows: vertical gauging tubes are installed in test panels of the diaphragm wall; the defelctions of these tubes are supposed to indicate the movements of the wall. The investigation is completed by distometer measurements. The location of the test panels must be chosen in a way that distortion of information on wall displacements caused by spatial aspects of the subsoil's bearing condition is kept to a minimum. As robust instruments were needed at the building site the City of Vienna chose the EASTMAN Multiple Shot inclinometer system. The usual distance brushes were replaced by two pairs of sliding bows, each of them consisting of a spring sliding bow and a rigid sliding bow ( see Fig.2). The pairs may be internally exchanged. As one pair of the sliding bows is connected to the fixing mark of the instrument barrel carrying the coordinated compass angle unit, highly accurate oriented gauging can be effected even in twisted tubes. A pendulum ranging from 0-2 degrees was used. Data obtained were subjected to an error-analysis which distinguished between errors caused by the instrument and systematical errors. Besides these errors the accurate adjustment of the top of the gauging tube to the tunnel direction posed another problem (Martak, 1979).

Fig.3 shows a gauging tube encased in the prefabricated stop ends. Commercial steel or aluminium tubes were used to reduce costs. Despite the high accuracy of the instrument as such, a mean deviation of ±1.6mm related to the 18m

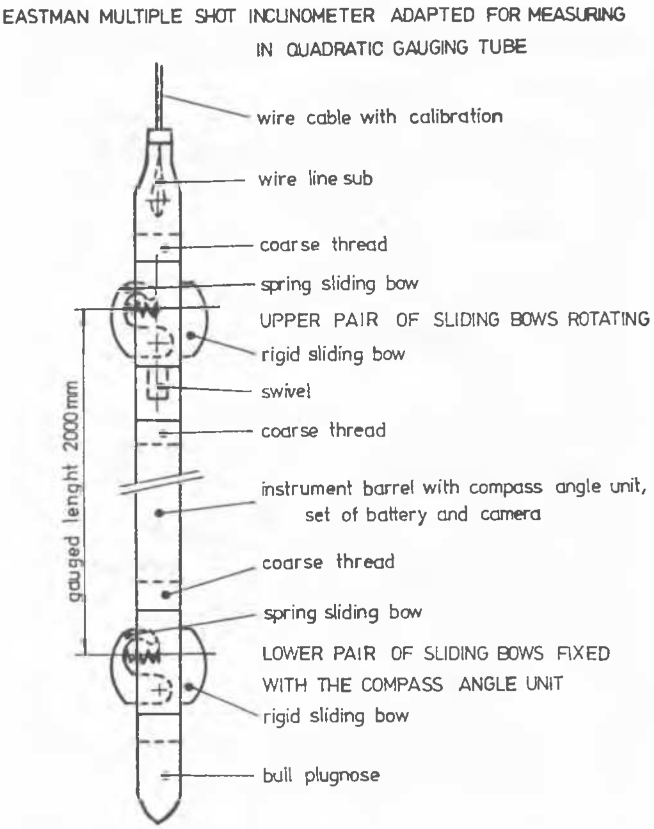



Fig.2 Inclinometer System Used

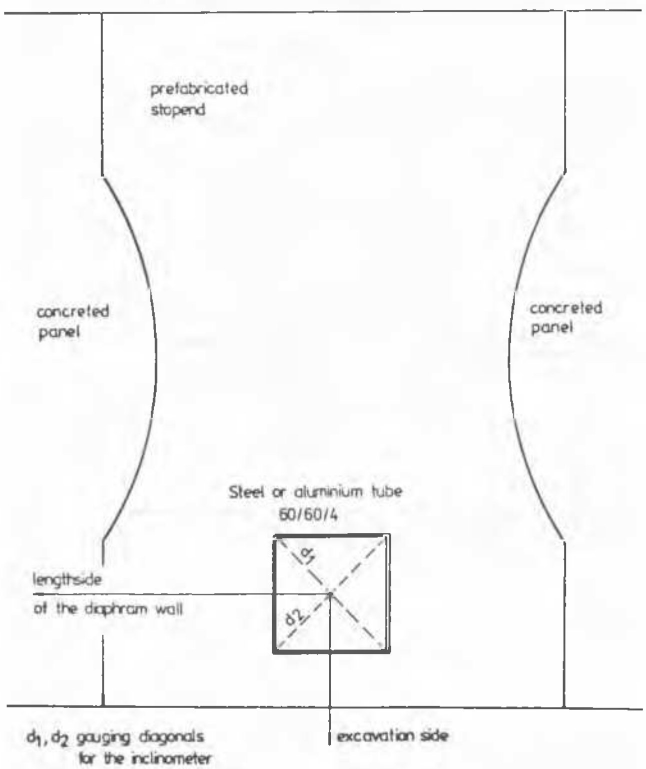



Fig.3 Diaphragm Wall with Prefabricated Stop Ends and Gauging Tubes

deep wall occurred with respect to the displacement of the top of the gauging tube towards excavation (see Fig.4). Repeatedly effected measurements as well as distometer investigations helped to support the error-analysis.

## DISCUSSION OF THE WALL MOVEMENTS

On two building sites, i.e. Praterstern and Mexicoplatz, four instrumentation fields were installed, each equipped with six gauging tubes. Fig.4 shows the soil profile. At first a fill of varying thickness was found consisting of quarternary gravels and sands (alluvions) and tertiary silts and fine sands.The granular size of the sandy gravels ranged from pure middle sands to open gravels with almost no sand. Natural density was between partially compact and loose. Old trunks and big blocks sometimes occurred. The tertiary layers started with a soft yellow-brown oxydation layer overlain by soft to stiff clays and silts as well as silty fine sands with artesian water. Plasticity indices for silts and clays were between 0.65 and 1.00; the fine sands were very compact. Fig.4 shows the displacement lines of an 18m deep diaphragm wall in connection with the different excavation stages and props. Initial gauging started on a preexcavation of three meters. As could be seen from subsequent measurements (1977 04 18 and 1977 05 04) deformation increased strongly up to a depth of 10m. Due to the fact that initial gauging started very late, a displacement of approximately 2mm at the top of the wall had to be taken into account. It could be noted that the typical bent profile appeared only at an advanced stage of excavation, while at first the wall inclined as a whole. This predeformation remained unchanged after installation of the first concrete prop; during the subsequent excavation stages the pro itself became the wall's centre of rotation. Displacements of the diaphragm wall's foot-point were of particular interest as they occurred in opposite directions, i.e.during the second excavation stage away and during subsequent excavation stages towards excavation. After installation of the ground prop and removal of the auxiliary prop the foot-point of the wall moved back again.

These observations could be of general importance (Martak, 1979). Horizontal foot-point displacements of this magnitude evidently cannot only be caused by deformations of the silty clays and fine sands due to earth pressure and water pressure, but also by time-dependent subsoil creeping effects. Furthermore, owing to the loss of soil weight caused by excavation, heaves may appear and stress relief towards excavation occur. These are phenomena known from other building sites of the Vienna Underground. (Martak, 1976). By studying the displacement lines with respect to soil mechanic properties of the soil profile, an impression can be gained of the manifold patterns of interaction existing between soil and construction. This interaction is illustrated in Fig.5. The line connecting the different points of measured displacements is drawn in an attempt to define the time-dependent foot-point behaviour of the diaphragm wall as a rheological response of the soil to both stress and stress relief. Displacement s(ko) corresponds to the subgrade under earth pressure at rest and was calculated on

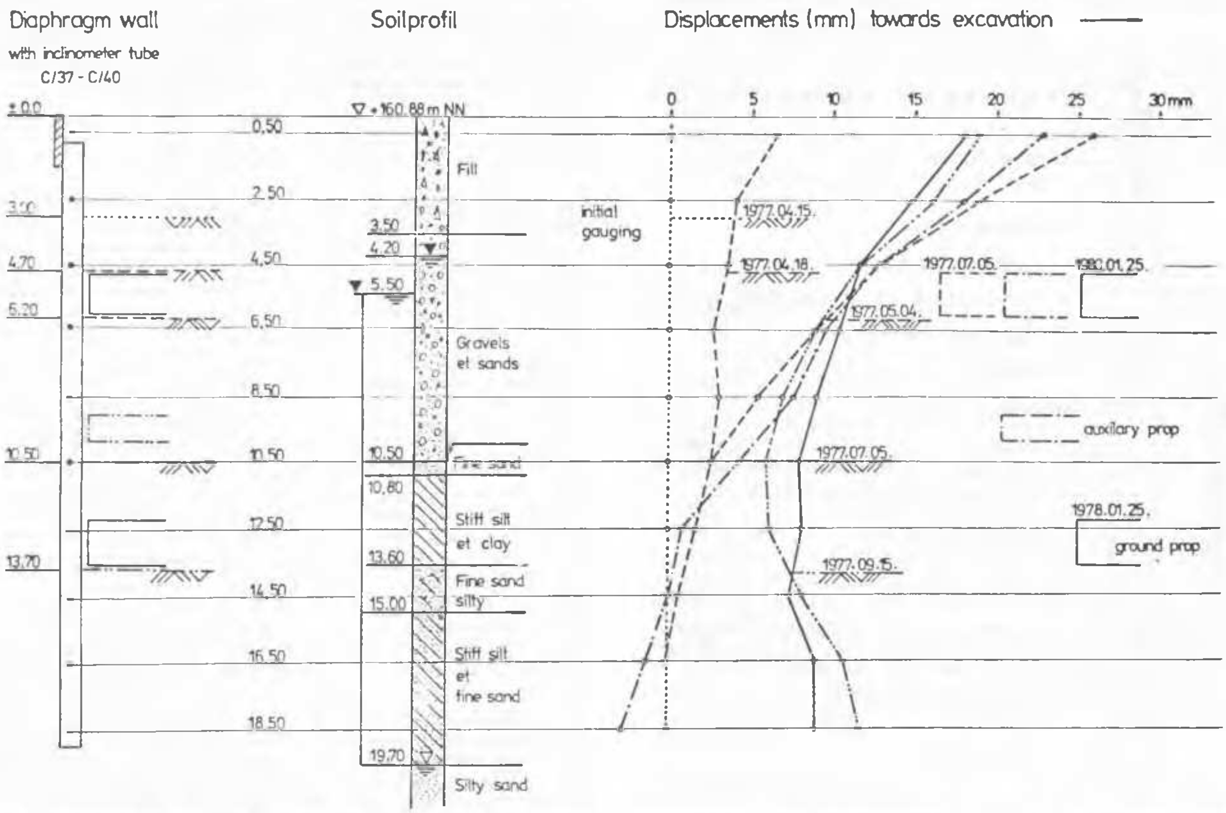



Fig.4 Static System and Measured Displacements of Praterstrasse Instrumentation Field C/37-C/40

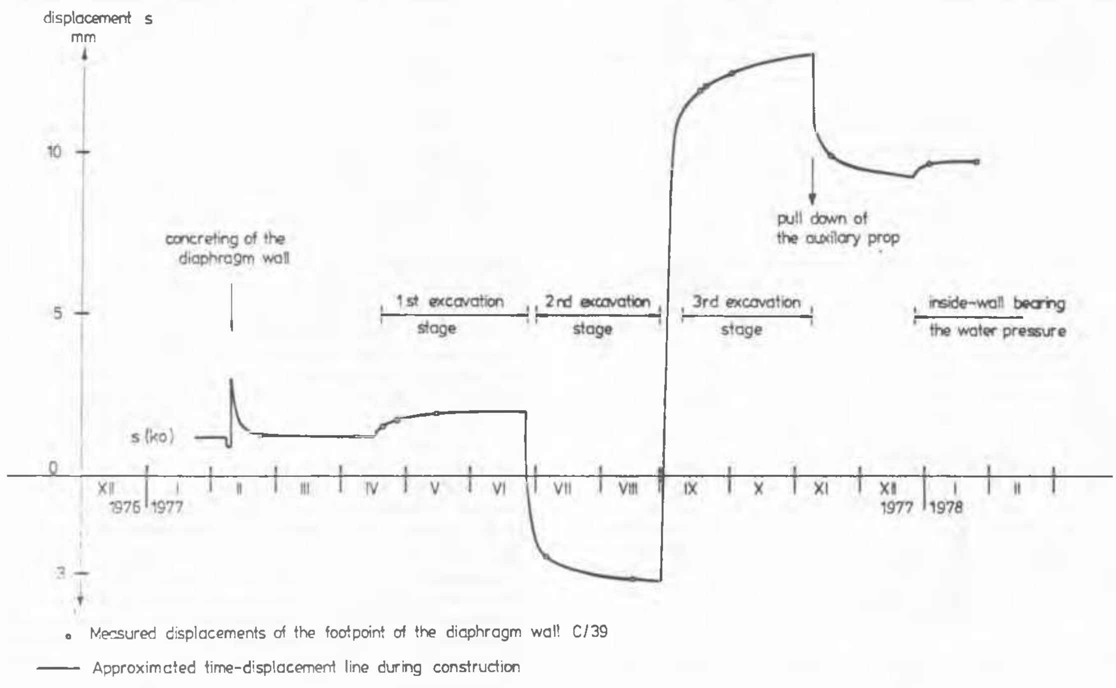



Fig.5 Rheological Behaviour of the Horizontal Subgrade

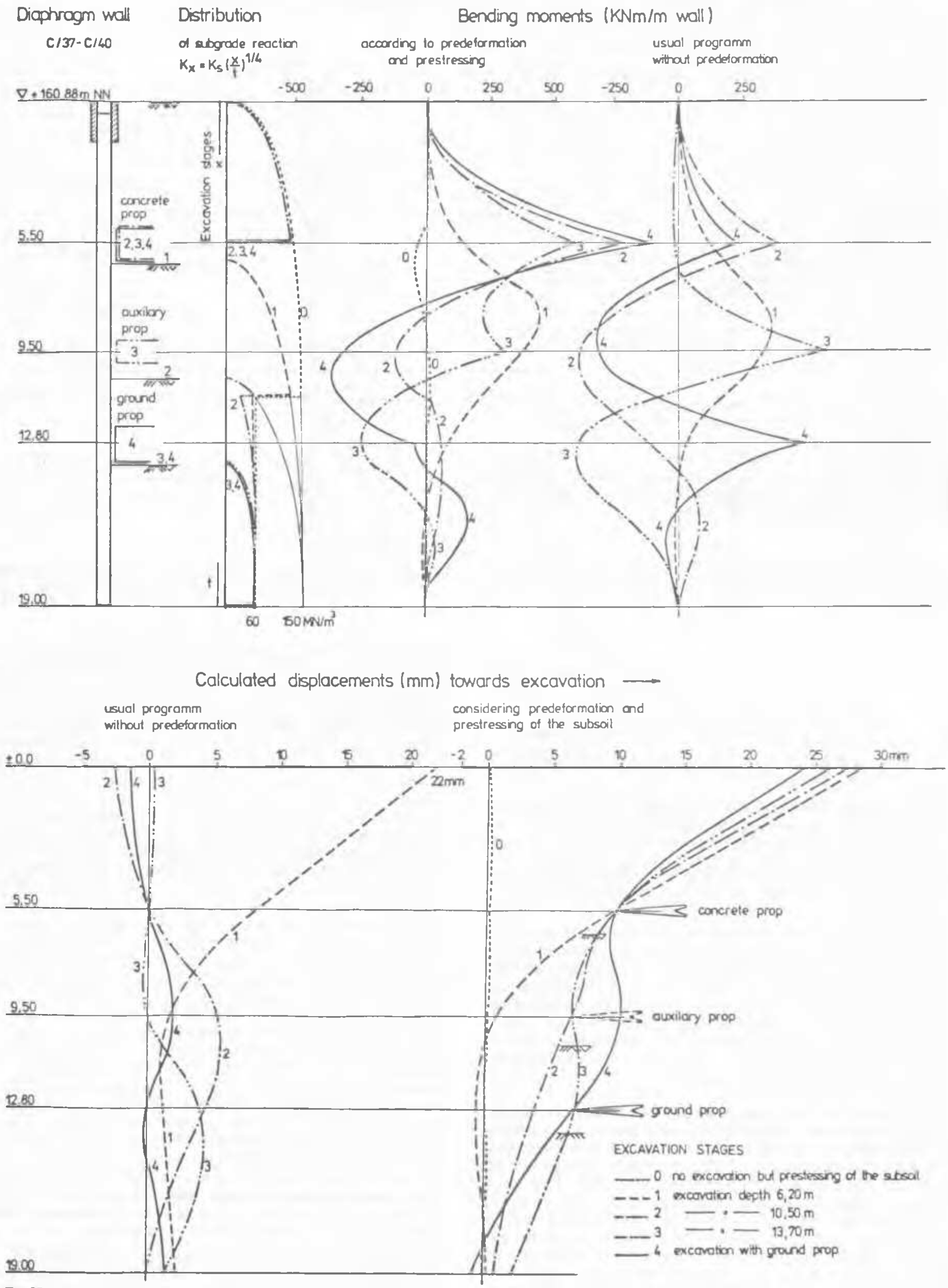



Fig.7 Influence of Predeformation and Prestressing of the Subgrade Reaction For Bending Moments and Displacements

the basis of the design parameters shown in Fig.7. Terzaghi (1955) is right, in principle, when he recommends not to use the theories of subgrade reaction when determining deformations since already minor variations in the stiffness or in the moduli of subgrade reaction may cause significant changes in the deflection lines of the side walls. Yet, even more complicated calculations based on the finite element method show big differences in the deflections gained as soon as the constitutive law of the soil or the element network is only slightly changed. With the present possibilities of data processing still confined to the operation with hypo-elastic and pseudo-elastic constitutive laws (Gudehus, 1979), it proves cheaper and faster to base the computation of diaphragm walls on the simple model of linear-elastic subgrade reaction and to assess the magnitude and distribution of the subgrade reaction moduli from measurements and back-analysis of data relating to similar subsoil conditions (see Fig.7 and 8).

As can be seen from the rheological analysis of the time-dependent deformation line in Fig.5, the alternation between load and load relief may be defined as a combined reaction of simple rheological bodies. In a monophase system approach (Rainer, 1980), the displacement of the wall appears to be dependent on time :

$$s(t) = s(t_0)\cdot e^{-\frac{G_K}{\eta_f}\cdot t} + \frac{\tau_w+\tau_E}{G_M} + \frac{\tau_w+\tau_E-\vartheta}{\eta_{pl}}\cdot t + \frac{\tau_w+\tau_E}{G_K}\left(1-e^{-\frac{G_K}{\eta_f}\cdot t}\right) \qquad (3)$$

where

$$\tau_E = \tau_{0_E}\cdot e^{-\frac{G_{fl}}{\eta}\cdot t} \qquad (4)$$

$\tau_w, \tau_E$ — is the shear strength caused by water pressure and earth pressure

$\vartheta$ — is the shear strength at gradient Io (St.Venant body)

$G_K, G_M, G_{fl}$ — are the moduli of shear deformation of the Kelvin body, Maxwell and Newtonian fluid

$\eta_f, \eta_{pl}, \eta$ — are the viscosities of the rigid, plastic and liquid components

Since earth pressure, contrary to water pressure, constitutes no dead load on the wall, subgrade creeping effects cause a permanent transposition of earth pressure towards stiff layers and parts of the excavation. Thus, each creeping behaviour may be defined as an action of simultaneous relaxation and retarded elasticity accompanied, according to load intensity, by an elastic flow movement (Maxwell flow), Fig.6.

Since instruments for measuring the dynamic subsoil parameters are largely inexistent, a quantitative assessment of the excavation's rheological activities may presently only be done by establishing data on sizeable predeformations and the prestressing of the subgrade springs for each excavation stage based on the boundary conditions of the previous stage. Fig. 7 shows the bending moments and displacements according to a computer programme including data on predeformations and prestressing of the subgrade springs. It also shows the results of a traditional programme for diaphragm wall construction for comparison.

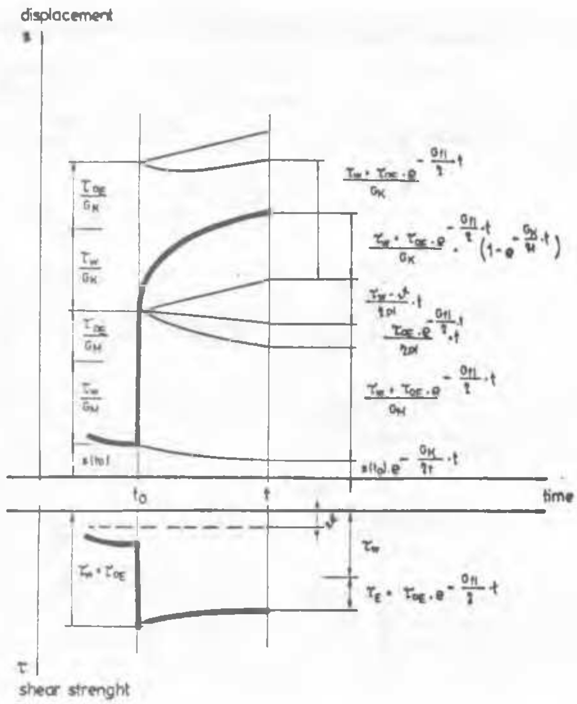
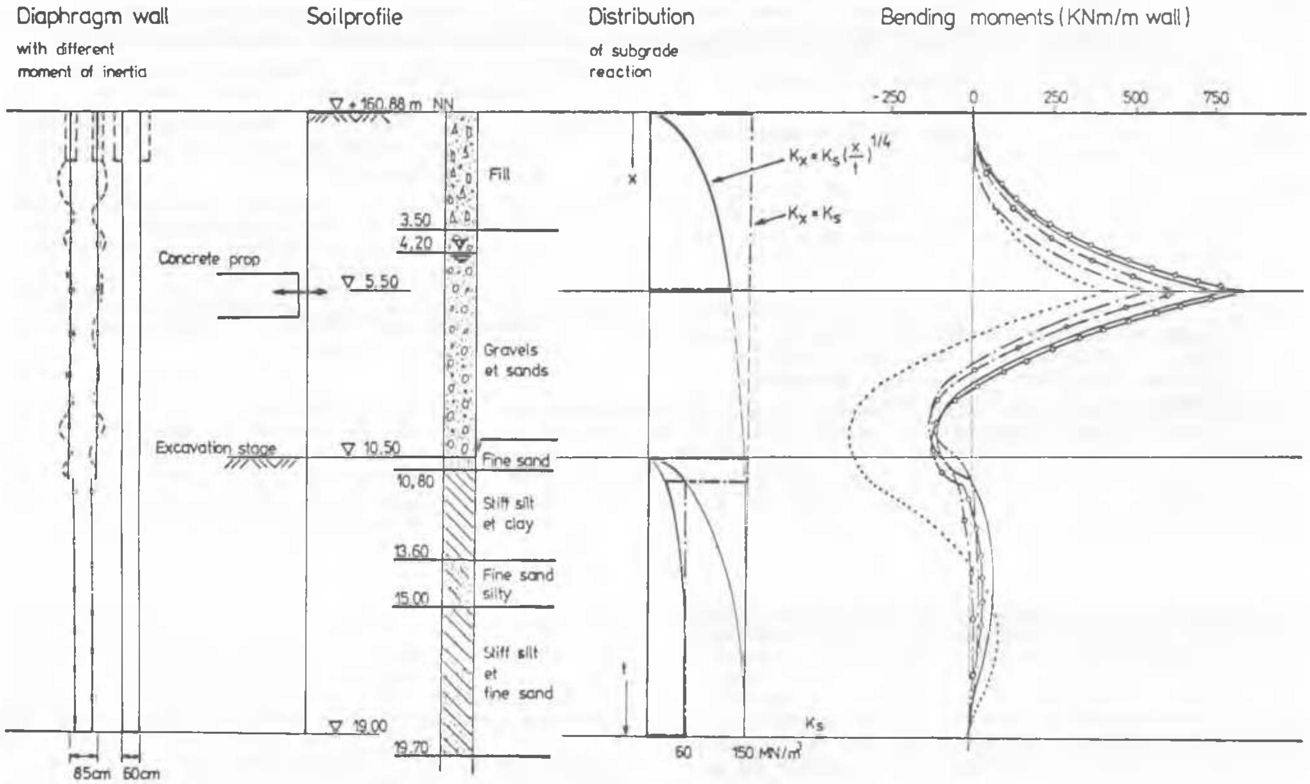



Fig.6 Rheological Stress-Strain Behaviour Under Relaxation and Retarded Elasticity

The presented diaphragm wall programme is built upon the dotted-lined moments and displacements (excavation stage 0) resulting from subgrade prestressing with earth pressure at rest. The peaks of the bending moments at excavation stages 2, 3 and 4 indicate that the second and third prop are significantly less loaded than shown in the traditional programme. The method using data on predeformation and prestressing provides a better means to prevent inadequately distributed reinforcements of both wall and props. The programme is set up in way that the distribution and extent of the horizontal earth pressure and water pressure do not have to be changed from one excavation stage to the next, except in cases of external changes in water level or earth pressure. The transposition of the horizontal load is effected automatically by prestressing the subgrade springs according to the displacement line of the previous excavation stage. The boundary conditions of the new excavation stage determine the static system. The only factor that needs changing is the distribution of the moduli of horizontal subgrade reaction which has to be adapted to each new excavation stage (Martak, 1980). After recalculation of the given test data, the magnitude and distributions of the moduli of horizontal subgrade reaction for sandy gravels and soft to stiff silt were in reasonable agreement with the displacement lines and indicated a high degree of utilization of the possible subgrade with a view to the passive earth pressure. With stiff and overconsolidated silt the determination of the moduli of horizontal subgrade reaction may be particularly difficult, as the problem is not one of failure mechanism but one of adequate deformation, i.e. a rheological problem.



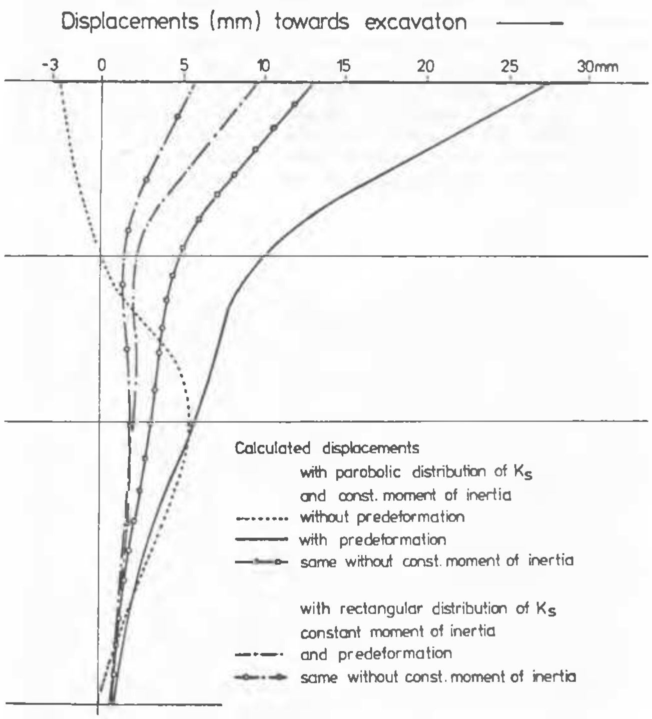




Fig.8 Influence of Moment of Inertia and Distribution of Subgrade Reaction For Bending Moments and Displacements

## CONCLUSION

When computing diaphragm walls with the help of moduli of subgrade reaction, not only magnitude and distribution of the moduli but also the rheological characteristics of the subsoil have to be taken into account. The duration of the individual excavation stages is another factor of importance. In traditional diaphragm wall programmes only approximated values of magnitude and distribution of the horizontal subgrade reaction are used to establish the bending moments. These programmes, however, may involve the risk of inadequately distributed reinforcements. To avoid this, it is necessary to extend the programme and to include data on predeformations occuring during the different excavation stages, particularly all aspects of the subgrade's rheological behaviour including displacements due to creeping effects. An efficient computer programme using moduli of horizontal subgrade reaction should therefore - to provide realistic data on displacements and bending moments - include information on predeformations resulting from the previous excavation stage as well as adequate data on the prestressing of the subgrade springs.

## REFERENCES


Delapierre, J. and Dufour, Ch. (1980). Analysis of horizontal loading test on Franki piles. Ground Engineering, Vol.13, (2), 32-37.

Gudehus, G. (1979). A comparison of some constitutive laws for soil under radially symmetric loading and unloading. 3rd Int. Conf. on Numerical Methods in Geomech., (4), 1309-1323, Aachen.

Hanna, T.H. (1973). Foundation Instrumentation, 5th Ed. Trans Tech Publications.

Martak, L. (1976). Überwachung der Tiefbauarbeiten der Wiener U-Bahn um den Stephansdom aus der Sicht des grundbautechnischen Sachverständigen. Der Aufbau, (3), 30-32, Vienna.

Martak, L. (1979). Horizontalverschiebungsmessungen an Schlitzwänden im Wiener U-Bahnbau. Mitteilungen des Instituts für Grundbau und Bodenmechanik, Technische Universität Wien, (16), Dec.

Rainer, M. (1960). Deformation, Strain and Flow. H.K.Lewis & Co Ltd., London.

Sherif, G. (1974). Elastisch eingespannte Bauwerke. Ernst & Sohn, Berlin.

Terzaghi, v.K. (1955). Evaluation of Coefficients on Subgrade Reaction. Geotechnique, Vol.4, 297-326.

Titze, E. (1943). Über den seitlichen Bodenwiderstand bei Pfahlgründungen. Mitteilungen aus dem Gebiet d. Wasserbaues u.d. Bodenforschung, (14), Berlin.

Werner, H. (1970). Biegemomente elastisch eingespannter Pfähle. Beton und Stahlbeton, (2), 39-43.