Pressure of Loose Soils at Rest between Two Rigid Walls

La Poussée de Sols Pulvérulents au Repos entre Deux Parois Rigides

A. MYSLIVEC Professor of Civil Engineering, Technical University, Prague, Czechoslovakia

SYNOPSIS The pressure of loose soils at rest was deduced for the case of a horizontal surface reaching to great distance. The paper is concerned with the active pressure at rest of a loose consolidated soil and the passive pressure at rest of a fully overconsolidated soil. It determines the distance L_{oa}, in which a loose normally consolidated soil produces maximum active pressure at rest, and the distance L_{op}, in which a fully overconsolidated soil produces minimum passive pressure at rest. Further it determines the active pressure at rest when the surface is not level and when is confined by two rigid walls.

INTRODUCTION

The case is often encountered when the pressure at rest of a normally consolidated soil is to be determined whose surface is not level, and further the pressure at rest of a fully overconsolidated loose soil. In the latter case the pressure at rest usually is 4 to 5 times as high as that of a normally consolidated soil. The paper solves the case showing how the soil behind a rigid wall contributes to the magnitude of the pressure at rest, when variable distance from the rigid wall is considered, or when the surface beyond the wall is subjected to some treatment, or when another rigid wall is built.

MOBILIZED ANGEL OF FRICTION AT REST

When the surface is level /horizontal/, at a depth of h there is a vertical stress $\sigma_z = \sigma_1 = \gamma h$, which is the major principal stress, and a horizontal stress $\sigma_2 = \sigma_3 = \sigma_1 K_o$, which is the minor principal stress in the ellipsoid of stresses. K_o is the coefficient of the pressure at rest, γ is the unit weight of soil. According to Jáky the coefficient of the pressure at rest $K_o = 1 - \sin \varphi_f$. If the principal stresses are known, it is possible to construct a Mohr´s circle over the stress deviator $\sigma_1 - \sigma_3$ /Fig.1/, and its tangent from Point O closes with the horizontal axis the mobilized angle of friction of the loose soil at rest φ_o. According to Fig.1

$$\sin \varphi_o = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} = \frac{\sigma_1 - K_o \sigma_1}{\sigma_1 + K_o \sigma_1} = \frac{1 - K_o}{1 + K_o} \qquad /1/$$

When the value of the coefficient of the pressure at rest $K_o = 1 - \sin \varphi_f$ is substituted in Eq /1/, the mobilized angle of friction of the soil at rest on the planes forming an angle of $/45^o + \varphi_f/2$ with the horizontal plane

$$\sin \varphi_o = \frac{\sin \varphi_f}{2 - \sin \varphi_f} \qquad /2/$$

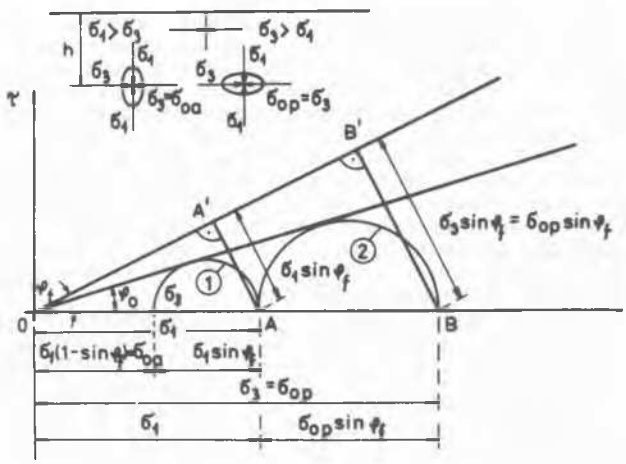

Fig.1 Determination of the mobilized angle of friction at rest.

The angle of friction at rest equals approximately the surface angle of friction: $\varphi_o = \varphi_N$. Within the limits of the angle of soil friction $\varphi_f = 30^o$ and 45^o the mobilized angle of friction in the state at rest is $\varphi_o = 2/3\, \varphi_f$. The pressure of a loose soil at rest, therefore, can be calculated as active pressure, taking into account, however, the mobilized angle of friction at rest φ_o.
The coefficient of the pressure of loose soil at rest is then

$$K_o = \tan^2 /45^o - \varphi_o/2/ \qquad /3/$$

When the angle at rest $\varphi_o = 2/3\, \varphi_f$ is substituted in Eq. /3/, the coefficient of the pressure of loose soil at rest is

$$K_o = \tan^2 /45^o - \varphi_f/3/ \qquad /4/$$

as stated by Wierzbicky.

Table 1. Values of the mobilized angle of friction at rest φ_o

Angle φ_f	5^o	10^o	15^o	20^o	25^o	30^o	35^o	40^o	45^o
Mobilized angle φ_o	$2^o36'$	$5^o28'$	$8^o32'$	$11^o54'$	$15^o34'$	$19^o34'$	$23^o42'$	$28^o14'$	$33^o10'$

The pressure of loose soil at rest can be determined graphically from Mohr's diagram /Fig.1/ by dropping a perpendicular from point A to the tangent leading from point O and closing the angle of friction φ_f.
The distance

$$\overline{AA'} = \sigma_1 \sin \varphi_f \qquad /5/$$

When this distance is subtracted from the major principal stress

$$\sigma_1 - \sigma_1 \sin\varphi_f = \sigma_1(1 - \sin\varphi_f) = \sigma_3 = \sigma_o \qquad /6/$$

the pressure at rest of loose soil is obtained at a depth of h where there is a vertical stress σ_1.

THE PRESSURE AT REST OF A OVERCONSOLIDATE SOIL!

In an overconsolidated soil lateral stress σ_3 increases and can attain, in an extreme case, the value for which the angle of friction at rest φ_o is also mobilized. The pressure at rest of a fully overconsolidated loose soil can be determined by the construction of a Mohr's circle 2 on the other side from Point A which touches the tangent closing with the horizontal axis the angle of friction at rest φ_o /Fig.1/. The distance OB determines the magnitude of the pressure at rest, when the loose soil is fully overconsolidated. The ellipsoid of stress is situated horizontally /Fig.1/.
The active soil pressure is usually marked as σ_a and the coefficient of active pressure K_a, while the passive pressure is usually marked σ_p and the coefficient of passive pressure K_p. Analogously we shall mark the pressure at rest of a normally consolidated soil as the active pressure at rest σ_{oa} and the coefficient of the active pressure at rest as K_{oa}, and the passive soil pressure at rest σ_{op} and the coefficient of the passive pressure at rest K_{op}.

The stress deviator at the moment when the passive pressure at rest in an overconsolidated soil has been attained is, in accordance with Fig.1,

$$\sigma_{op} - \sigma_1 = \sigma_{op} \sin \varphi_f \qquad /6/$$

Hence the passive pressure at rest of a loose soil is

$$\sigma_{op} = \frac{\sigma_1}{1 - \sin \varphi_f} = \frac{1}{K_{oa}} \qquad /7/$$

It holds that

$$K_{oa} = \frac{1}{K_{op}}, \quad K_{oa} \cdot K_{op} = 1 \qquad /8/$$

Analogous relation holds also for the coefficients of the active and passive soil pressure, viz.

$$K_a = \frac{1}{K_p}, \quad K_a \cdot K_p = 1$$

THE GRAPHIC METHOD OF DETERMINATION OF THE ACTIVE PRESSURE AT REST OF LOOSE SOIL.

When the mobilized angle of friction φ_o of soil at rest is known, the magnitude of the active soil pressure at rest can be determined by calculation or by some graphic method. From the number of the latter the Engesser method is well suited, since it can be used also for uneven surface beyond a rigid wall /Fig.2/. The soil behind a rigid wall is broken up into wedges and that wedge is sought as produces maximum soil pressure when the angle of friction at rest φ_o is mobilized in the soil. The weight of the soil wedge P produces the overall active pressure at rest E_{oa} applied to the wall in horizontal direction and the reaction R on the slanting surface of the soil wedge, deviating from the normal by the angle of friction at rest φ_o.
First the biggest wedge of a weight P_1 is considered, then the other two wedges weighing $P_1 + P_2$, further the wedge comprizing the weights of $P_1 + P_2 + P_3$, etc. The connection of the terminal points showing the magnitude of the active soil pressure at rest E_{oa} yields the curve 1, the vertical tangent to it giving, at the point of contact, the maximum magnitude of the active soil pressure at rest E_{oa}.

Fig.2 shows the graphic solution by the Engesser method of the active soil pressure at rest for a horizontal surface, a rigid wall of a height H 8 m, angle of friction $\varphi_f = 35^o$ and the mobilized angle of friction at rest $\varphi_o = 23^o34'$. The active pressure at rest per 1 m of wall width /Fig.2, Curve 1/ is E_{oa} = 282 kN/m.

Let us determine the active soil pressure at rest by calculation:

$$E_{oa} = \gamma H^2 K_{oa}/2 = \gamma H^2 (1 - \sin \varphi_f /2) =$$
$$= 20.8^2 . 0.427/2 = 274 \text{ kN/m}$$

The calculated and the graphically determined values of the active soil pressure at rest are in sufficient agreement.

Fig.2 shows that the maximum active soil pressure at rest is produced by the soil wedge OAB whose inclined face closes with the vertical the angle of $45^o - \varphi_o/2$. The base of this wedge has a length of

$$L_{oa} = H \tan (45 - \varphi_o/2)$$

Curve 1 in Fig.2 shows that the soil pressure at rest E_{oa} increases with the increasing weight of the soil wedge and sinks after its maximum value

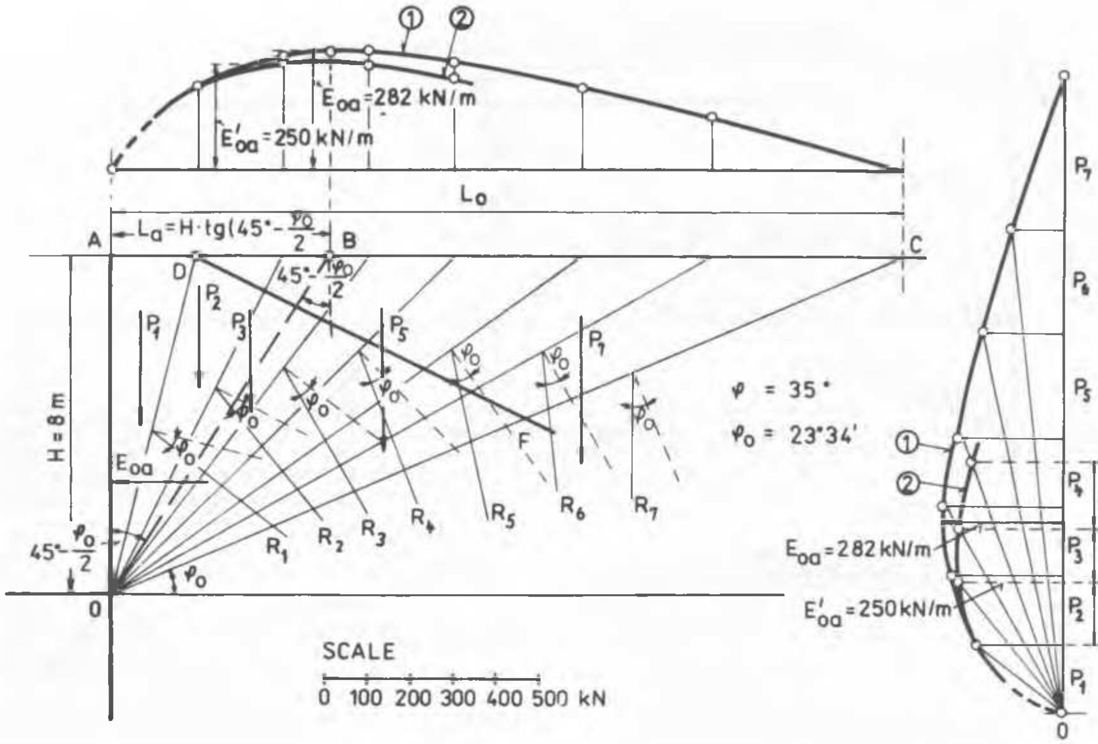

Fig.2. Graphic method of determination of the active pressure at rest of loose soil 1- horizontal surface, 2- inclined surface

has been attained. The lowering of the surface beyond Point B has no influence on the magnitude of the pressure at rest; however, this influence will manifest itself during any works on the surface between Points A and B. For this reason the case was solved in which the surface ADF was inclined /Fig.2/. Maximum value of the overall active soil pressure at rest was E_{oa} = 250 kN/m /Curve 2/.

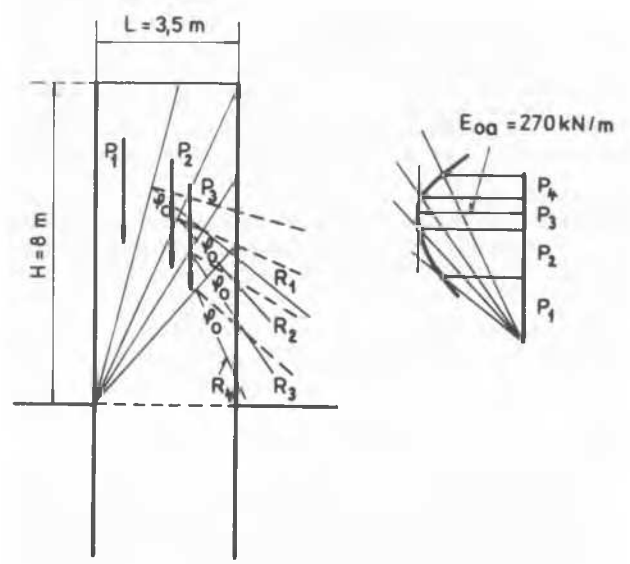

Fig.3. Pressure at rest of a loose soil between two walls

PRESSURE AT REST OF A LOOSE SOIL BETWEEN TWO WALLS

Further the case was solved in which another wall was erected at a distance of 3.5 m from the first underground wall /Fig.3/. The soil pressure at rest was E_{oa} = 270 kN/m.

PRESSURE AT REST OF A FULLY OVERCONSOLIDATED LOOSE SOIL.

Finally another case was considered in which the loose soil was fully overconsolidated by compaction /Fig.4/. The reaction R of the soil wedge deviates from the normal to the inclined wedge face by the angle of φ_o to the other side than it is the case of active soil pressure at rest /Fig.4/. The minimum soil pressure at rest acting on the rigid wall under consideration, which is decisive in this case, is E_{oa} = 1500 kN/m which is 5.3 times as high as

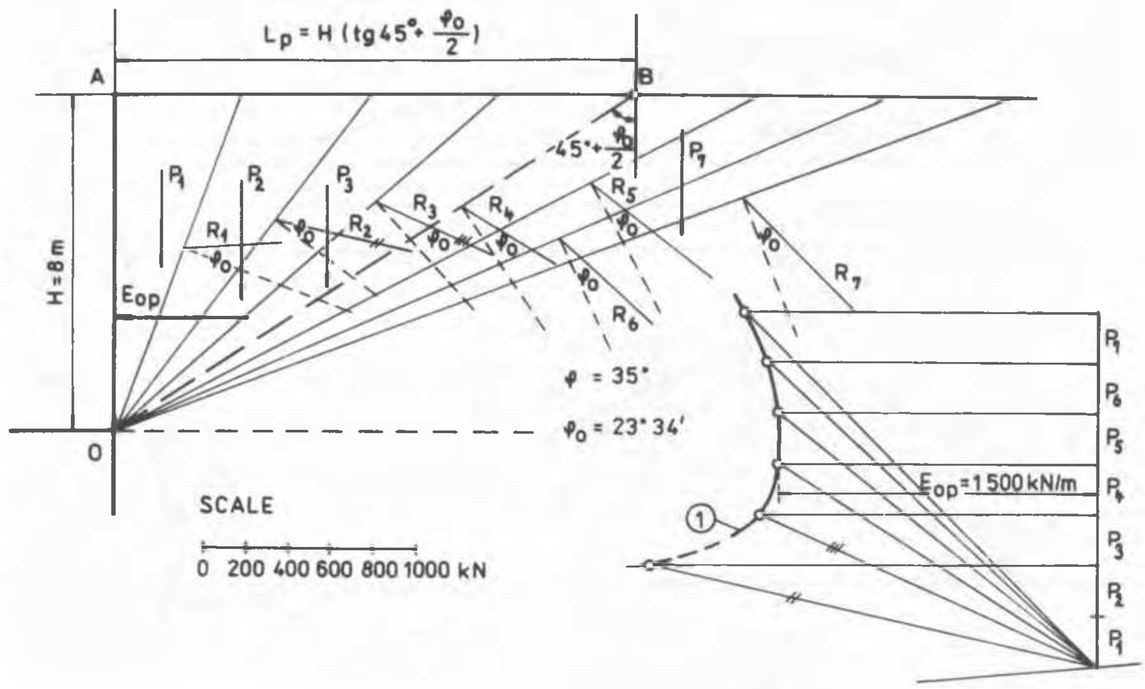

Fig.4. Passive pressure at rest of a loose, overconsolidated soil

the active soil pressure E_{oa}. Consequently, the compaction of the soil is of great importance for the magnitude of the soil pressure at rest. The minimum passive soil pressure at rest is produced by the soil wedge OAB, whose base on the surface has a length of

$$L_{op} = H \tan\ (45^{o} + \varphi_o/2)$$

and whose inclined face closes with the vertical an angle of $44^{o} + \varphi_o/2$. Consequently, any earthworks carried out at a greater distance from the wall than L_{op} would have no influence on the magnitude of the passive soil pressure at rest.

LINE LOADING OF THE SURFACE

If the load on the surface acts on a line, this load is added to the weight of the earth wedge on which the load acts and the pressure at rest is determined grafically by the same way as earlier.

REFERENCES

Kézdi, A. /1962/: Erddrucktheorien. Springer Verlag, Berlin.

Jáky, J. /1948/: Pressure in soils. Proc. 2nd Inter. Conf. on SMAFE, Rotterdam

Myslivec, A. /1972/: Pressure at rest of cohesive soils. 5th Europ. Intern. Conf. on SMAFE, Madrid.

Reinbert, M. et A. /1965/: Mur de soutènement. Eyroles 61, boulevard Saint-Germain. Paris.

Reinbert, M. et A. /1974/: Etudes des butées des ouvrages de foundation. Édutions Eyroles 61, boulevard Saint. Germain. Paris.