Soil Structure Interaction and Volume Change Soils

Interaction Sol-Structure et Sols Gonflants

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SYNOPSIS Mathematical models are provided for load-settlement curves for foundations. Several original methods for the analysis of Soil-structure interaction are presented. Models for expansive and collapsing soils are briefly described. A photo-elastic method was used to check a model on sand. Inter-relationships between moments and vertical loads are described.

INTRODUCTION

State of the art reviews have been provided by Hooper and Scott (1978),Burland and Wroth(1975), Desai and Christian(1977), Bowles(1975) the Institution of Structural Engineers (1978) Teng(1975), and by others such as Little(1961). Early contributions were made by Popov(1951) and by Chamecki(1956).

Most modern references deal with raft foundations (e.g.Hooper 1978) or piling (Poulos 1979) and finite elements (e.g.Desai and Christian). Although this paper contains certain principles which might apply both to frame structures and to continuous raft or wall systems, the writer considers simple frame or truss structures. In particular, emphasis is given to methods which might be suitable for teaching, or for solution by programmable pocket calculators.

MODELLING THE LOAD-SETTLEMENT CURVE

The load-settlement curve for a central vertical load on a foundation is shown in Fig.1. The slope of the initial portion AB is determined mainly by the compressibility of the soil and the area of the footing. However the region D and the maximum limit Q_{max} are affected mainly by the shear strength of the soil and the geometry of the soil-footing system.

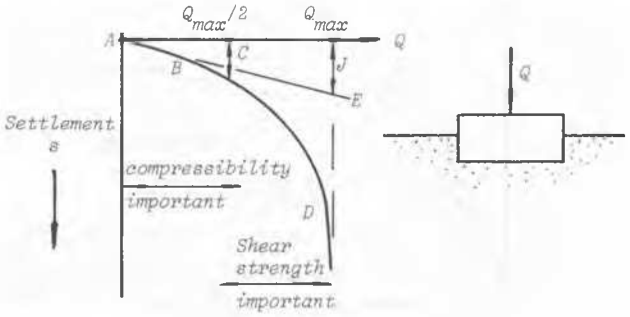


Figure 1

MATHEMATICAL MODELS

Approximately twelve equations for the curve in Fig.1 have been considered. Some of these are also suitable for modelling curves of stress versus strain for soils. In Fig.1, the settlement C occurs at the load $Q_{max}/2$, and J is the intercept made at Q_{max} by the initial tangent AE to the curve ABD.

The following are some of the proposed formulae:

$$s = C\left(\frac{Q}{Q_{max}-Q}\right) \quad ; \quad \text{for which } J=C; \qquad (1)$$

$$s = C.\tan\left(\frac{Q.90°}{Q_{max}}\right); \text{ and } J = 1,57\ C; \qquad (2)$$

$$s = -1,442.C.\ln_e\left(\frac{Q_{max}-Q}{Q_{max}}\right); \ J=1,442\ C; \qquad (3)$$

$$s = 0,8509.C.\sinh\left(\frac{Q}{Q_{max}-Q}\right); J=0,8509\ C; \qquad (4)$$

$$Q = \frac{s.Q_{max}}{C + s} \ ; \text{ and } J = C\ ; \qquad (5)$$

Equations (1) and (5) are identical. Other equations include variations of the above formulae in which the expressions within the equations have been raised to the power n. For example, the portions within brackets, or indeed the whole right-hand side of the equation can be raised to the exponent n where n can be greater than or less than unity. However this type of approach can cause very steep or very flat initial slopes at A on the curve ABD. The equations may however still be usable for solving problems. Two different equations of this type are:-

$$s = C.\left[\tan\left(\frac{Q.\ 90°}{Q_{max}}\right)\right]^n \qquad (6)$$

$$Q = \frac{Q_{max}.s^n}{C^n + s^n} \qquad (7)$$

Practical methods for fitting curves

To use the above formulae, two steps are needed:

(i) Firstly the value of Q_{max} can be estimated by bearing capacity calculations based on shear strength parameters and the area of the foundation. Approximately five methods are suitable for estimating Q_{max}, such as bearing capacity factors, model footing tests on the same soil, or the interpretation of results from dynamic probe tests.

(ii) The intercept J in Fig. 1 can be estimated from typical values of the coefficient of sub-grade reaction for this size of footing founded on this type of soil. The value of C may be found from the value of J as shown in equations (1) to (5). Alternatively, the settlement C at a load of $Q_{max}/2$ can be estimated from compression tests in the oedometer or from a loading test on a smaller footing. Approximately seven methods exist for estimating C or J.

If it is found that one of the first five equations does not provide a suitable fit for the real curve, then use can be made of equations such as (6) and (7).

Expansive and Collapsing Soils

A load Q_B applied to a footing on a fairly dry soil will cause a settlement from A to B along the curve ABD in Fig. 2.

If the soil is expansive and is now wetted, the footing will rise to the position B_1 on the curve $A_1B_1D_1$. Note that the value of Q_{max} may be reduced by the wetting. The new value of Q_{max} and the distances BB_1 and AA_1 will depend on the change in water content as well as other parameters such as the initial water content at state B, the percentage of clay particles, the type of clay, the stresses due to Q_B, the initial void ratio at state B, and the thickness of the soil affected by expansion.

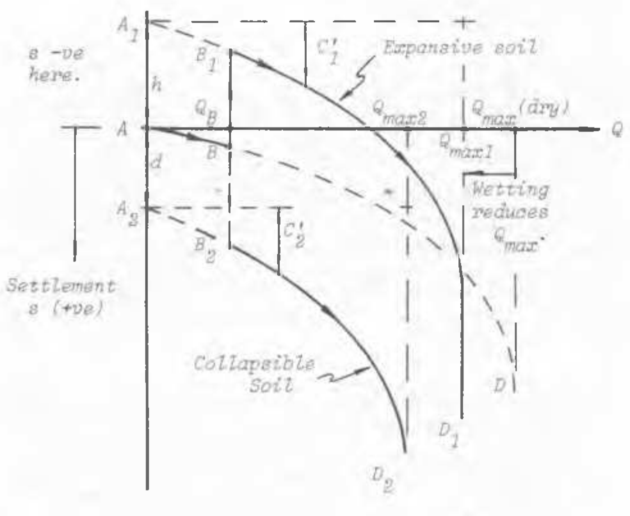


Figure 2

If the soil is collapsible, the footing will settle from B to the state B_2 on the new curve $A_2B_2D_2$. The new value of Q_{max}, and the distances BB_2 and AA_2 will depend on the change in water content in the soil and many other parameters as described above for expansive soils.

A test involving a single sample in a single oedometer ring has been used at UCT for estimating expansive or collapsing strains due to adding small increments of water. This permits a greater understanding of factors which affect the distances BB_1 and BB_2.

Expansive Soils (curve $A_1B_1D_1$)

The value of (s+h) can be used instead of (s) in equations (1) to (7), where h = distance AA_1. Q_{max} is reduced, and C may be modified to C'_1. Equation (1) would be altered to:

$$s = C'_1 \cdot \left(\frac{Q}{Q'_{max} - Q}\right) - h \qquad (8)$$

Collapsible Soils (curve $A_2B_2D_2$)

The value of (s-d) can be used instead of (s) in equations (1) to (7), where d = distance AA_2. Q_{max} is reduced, and C will change to C'_2. Equation (1) would be altered to:

$$s = C'_2 \cdot \left(\frac{Q}{Q'_{max} - Q}\right) + d \qquad (9)$$

Note that h and d are not the heave nor the deflection from state B. Also C'_1 and C'_2 occur at $Q'_{max}/2$ as shown in Figure 2.

Eccentric Inclined Loading

The capacity of a horizontal footing to carry the vertical component P of the applied loading is greatly reduced if the load is inclined from the vertical and if it is applied with a moment M about the centroid of the footing. It is tempting to use two curves of the shape shown in Fig. 1 (i.e. P versus s, and M versus rotation θ). A complex inter-relationship exists between M and P. A tentative suggestion is shown in Fig. 3.

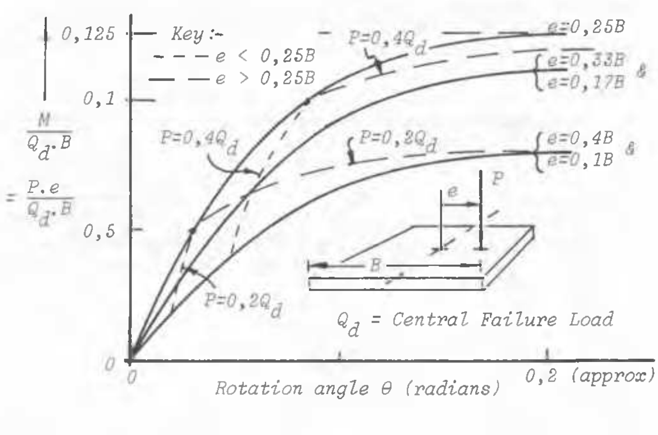


Figure 3 (Model by Sparks for M and P effect.)

Tests indicate that the horizontal scale in Fig.3 changes slightly for different types of soils and for different relative densities. However the general pattern has been confirmed.

Connecting rotations to settlements

Preliminary tests have confirmed the following expression by Sparks for relating the rotation due to a vertical load P_e at eccentricity e, and the central deflection Δ_o due to a central vertical load P_o on a similar footing of width B.

$$\frac{\tan\theta}{\Delta_o} = (\text{Constant } I_M)\left[\frac{e}{B}\right]\left[\frac{P_e}{P_o}\right]\frac{1}{B} \qquad (10)$$

where I_m = 4,55 for square footings on elastic supporting media, and drainage clays.
= $3\pi/2$ for circular footings on clays (D=B)
= 2,4 for long strip footings on clays
= 6 for square footings on loose sand
= (6 to 15) for square footings on dense sand
= 2 or 3 for undrained soft clays.

Further tests are needed to confirm the values of I_m. Equation (10) applies to small rotations and deflections, and not to failure regions. Values of subgrade reactions for uniform loading can be converted to provide rotational stiffnesses via equation (10).

A FEW METHODS

1) Deflections Δ in terms of tensile strains

1a) Δ_b due to bending only (bending strain)

Deflections in beams and walls can be expressed in terms of stresses or strains at the extreme fibre distance (e.g. Burland and Wroth 1975). Hardy Cross (1932) considered different types of beams continuous over two equal spans (Figs. 4a, 4b).

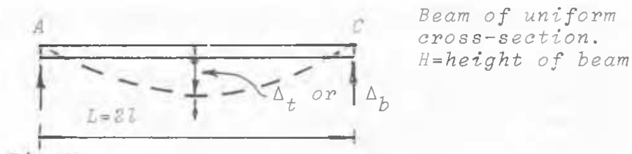


Fig 4a

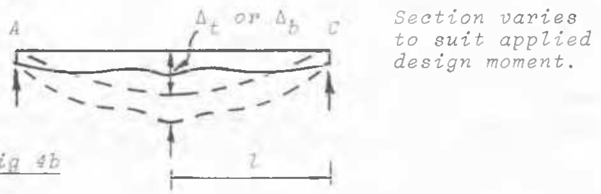


Fig 4b

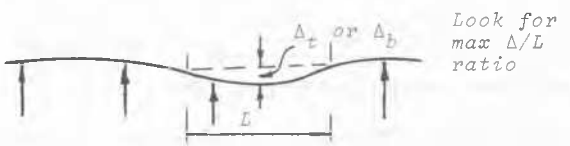


Fig 4c

Hardy Cross concluded that for most of these beams, the same formula applied. A modification of his formula is

$$\frac{\Delta b}{L} = \left(\frac{f_{max}}{E}\right) \cdot \left(\frac{L}{8y}\right) \qquad (11)$$

where y = fibre distance from neutral axis

f_{max} = fibre tensile stress
L = 2ℓ = overall distance AC (Fig.4).

In terms of tensile strains at the extreme fibre distance this becomes (See Fig. 4a, 4b):-

$$\Delta_b/L = (\text{max tensile fibre strain}) \cdot \left(\frac{L}{8y}\right) \qquad (12)$$

And for continuous girders over several spans (Fig. 4c):-

$$\Delta_b/L = \frac{(\text{max tensile fibre strain})}{1,5} \cdot \left(\frac{L}{8y}\right) \qquad (13)$$

In the latter formula L is not the total length of beam. If the neutral axis is at mid-height of the continuous beam (Fig.4c) then y = H/2 and equation (13) yields.

$$\Delta_b/L = \left(\varepsilon_{bmax}\right) \cdot L/6H \qquad (14)$$

where ε_{bmax} = max.tensile fibre strain due to bending.

These equations are similar to those published by Burland and Wroth (1975). The above formulae are based on the deflection Δ_b due to bending and hence do not apply to small L/H ratios where shear deflections are important.

1b) Fibre strain due to Δ_t (caused by bending and shear)

For small L/H ratios it becomes necessary to include deflections due to shear strains.

The deflection Δ_t caused by both bending and shear in a simply supported beam of span L carrying a central load P is given by:

$$\Delta_t = \frac{PL}{8}\left(\frac{L^2}{6EI} + \frac{3}{GA}\right) \qquad (15)$$

where G = the shear modulus.

After expressing P in terms of the maximum fibre tensile strain ε_{bmax} this equation becomes:

$$\Delta_t/L = \varepsilon_{bmax} \cdot \left[\frac{1}{6}\left(\frac{L}{H}\right) + \frac{1}{4}\left(\frac{H}{L}\right)\frac{E}{G}\right] \qquad (16)$$

1c) Diagonal tensile strain

The principal tensile strain ε_{dmax} acts at 45° on the neutral axis of a rectangular beam. For a beam of span L carrying a central point load it can be shown that near the supports

$$\varepsilon_{dmax} = \frac{3}{2}\frac{E}{G}\left(\frac{H}{L}\right)^2\frac{\Delta_b}{L} \qquad (17)$$

where Δ_b = central deflection due to bending.

Substituting the ratio Δ_b/Δ_t from equations (14) and (16) into equation (17) yields

$$\Delta_t/L = 1 + \frac{2}{3}\frac{G}{E}\left(\frac{L}{H}\right)^2 \times \varepsilon_{dmax} \tag{18}$$

1d) <u>Cracking by hogging action</u>

Burland and Wroth suggested that at a hogging section the compressive restraint from the foundation causes the neutral axis to be near the lower edge of the wall.

The above formulae then become :-

$$\Delta_t/L = \varepsilon_{bmax}\left[\frac{1}{12}\left(\frac{L}{H}\right) + \frac{1}{2}\left(\frac{H}{L}\right)\frac{E}{G}\right] \tag{19}$$

and

$$\Delta_t/L = \varepsilon_{dmax}\left[1 + \frac{1}{6}\frac{G}{E}\left(\frac{L}{H}\right)^2\right] \tag{20}$$

1e) <u>General comments</u>

Burland and Wroth use a limiting tensile strain of ε_t = 0,075% for the onset of cracking in most buildings. For reinforced concrete a value of ε_t = 0,04% will be reasonable.

Equations (16), (19) and (20) have been plotted in Fig. 5 for ε_b = 0,06%, E/G = 2,5 and ε_d = 0,03% and 0,06%.

The above theory does not consider tensile strain conditions at sharp corners of windows or doors in a wall. The latter might require lower limits for Δ/L than shown in Figure 5.

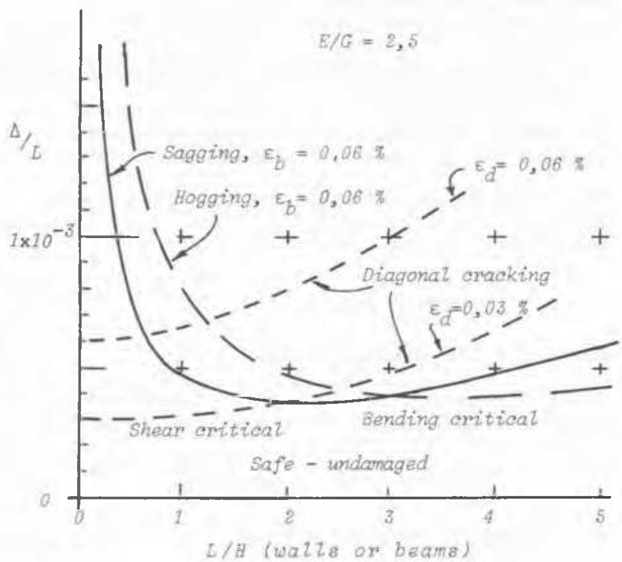


Fig.5 (Using concept by Burland and Wroth)

1f) <u>Open frame structures</u>

Burland and Wroth conclude from results by Webb that the limit of 1/150 for angular distortion is suitable for open frame reinforced concrete buildings.

2) <u>Intersecting characteristics</u>

A cantilever is supported at its free end by a reaction R from a footing on a non-linear support (See Figure 6a).

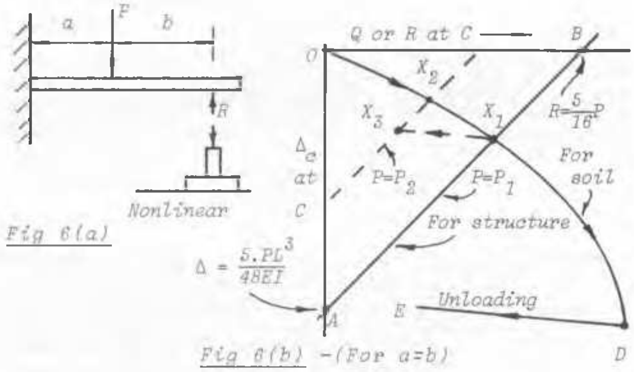


Fig 6(a)

Fig 6(b) -(For a=b)

Consider only the beam structure subjected to P and arbitrary load R. The deflection Δ_c at C will be given by

$$\Delta_c = \frac{L^3}{48EI}(5P - 16R), \text{ if } a=b . \tag{21}$$

If P is constant, this equation yields the straight line AB which describes the behaviour of a structure removed from the soil. (See Figure 6b). Curve OD is the non-linear settlement curve for the soil.

The point X_1 yields the required solution values of Δ_c and R for the beam supported by this footing. If the value of P is subsequently reduced from P_1 to P_2 then the solution is given by X_3, where X_1X_3 is parallel to the unloading curve DE.

The curve OD must be modified for expansive or collapsing soils (See Figure 2).

3. <u>Multiple Supports (Iterative Calculations)</u>

The distribution of moments and forces in the structure (Fig.7) depends on the relative displacements of A, B and C; whereas the actual forces transmitted to the soil depend on absolute deflections at A, B and C.

The reaction at B is

$$R_B = \frac{48}{69,818}(W_1 + W_2) - \Delta_B' \frac{48EI}{L^3} \tag{22}$$

where Δ_B' = relative deflection of B (relative to the new base line through A and C)

Also $R_A = R_C = \frac{1}{2}(W_1 + W_2 - R_B)$ (23)

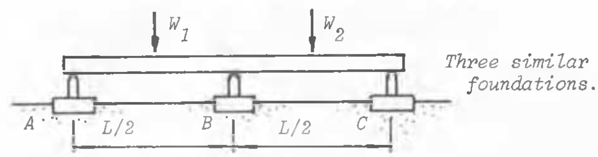


Fig.7 (Two equal spans)

An iterative procedure

a) Start with trial R_B (e.g. for unyielding supports)

b) Enter R_B into equation (22) to find Δ'_B.

c) Find R_A, R_C by using equation (23).

d) Enter curve in Fig.(1) with R_A, R_C to find Δ_A and Δ_C.

e) Calculate $\Delta_B = \Delta'_B + \frac{1}{2}(\Delta_A + \Delta_C)$

f) Enter curve in Fig.(1) with Δ_B to find R_B.

g) Compare R_B (step f) with the trial R_B (step a)

h) Enter step (b) with the new R_B from step (f) and repeat steps (b) to (g) until the R_B value converges.

4) Matching characteristics

The following method is suitable for programming into small pocket computers.

Figure 8 illustrates an iterative procedure for the continuous beam shown in Figure 7 for the symmetrical case ($W_1 = W_2 = W$). R is the reaction on the beam and Q is the same reaction on the ground.

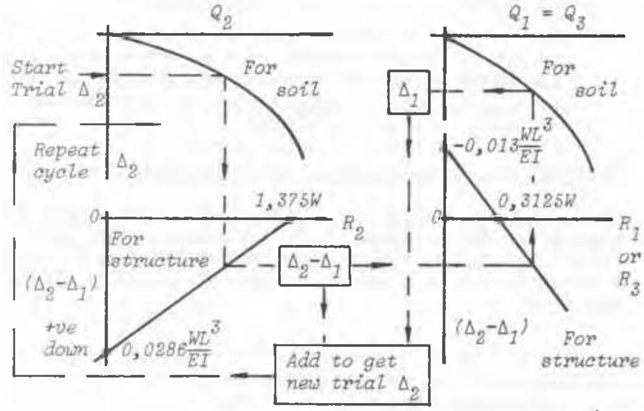


Fig.8 (Symmetrical case $W_1=W_2$, in Fig.7)

It is advisable to enter the calculations with a trial settlement Δ_2 for the foundation B which is closest to failure, rather than with a trial value of Q_2. The method shown in Figure 8 might provide an unusual value for Δ_2 after the first cycle, but it usually converges to the correct solution within four cycles even if footing B has failed on a vertical assymptote on the Q_2 versus Δ_2 diagram. In the case of unequal spans (Fig.9), one must consider the relative deflection δ' of support B from the new base line AC.

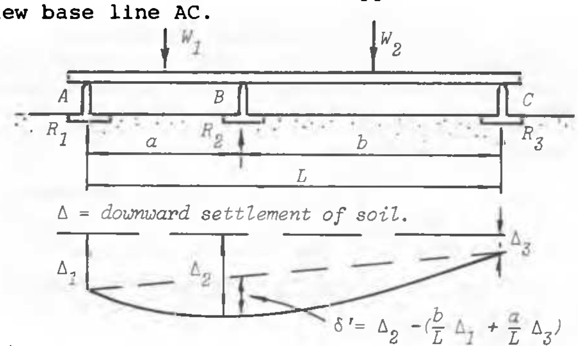
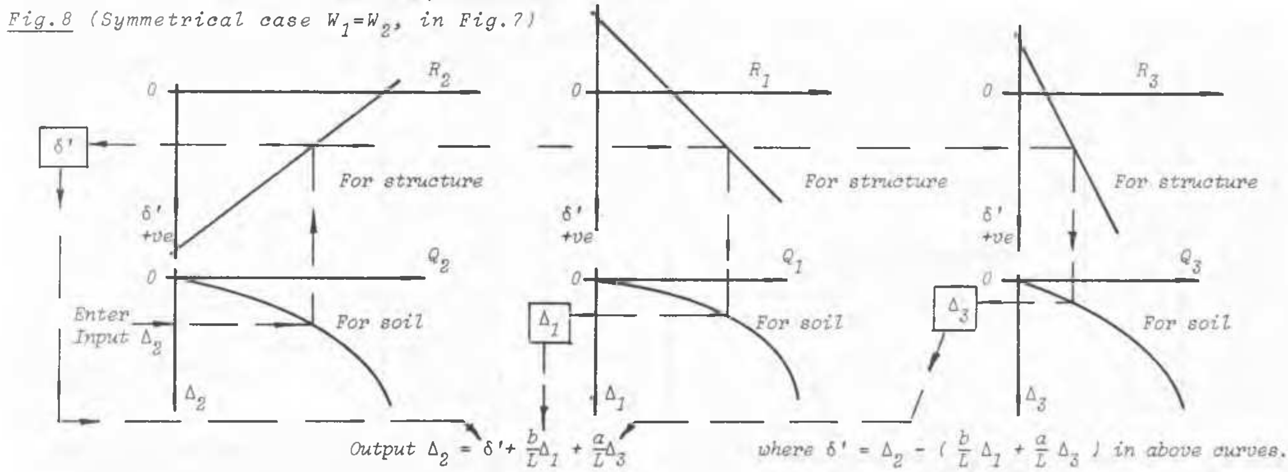


Fig.9 Unsymmetrical spans etc.

The flow chart for one cycle of calculations is shown in Figure 10. Convergence is assisted by averaging the input and output value of Δ_2 for each cycle, and using this average value as the new input for Δ_2 into the next cycle. Solution occurs when the input and output values of Δ_2 are similar for a cycle.

5. Expansive or collapsing Soils

The flow chart shown in Figure 10 can be adapted for the case when footings in Figure 9 display additional movements due to moisture changes in expansive or collapsing soils. The upper three sketches in Figure 10 remain unchanged in this method. One or more of the three curves at the bottom of Figure 10 must be modified as described in Figure 2.



Input Δ_2 for next cycle = Average of (Input Δ_2 and Output Δ_2 of present cycle).

Fig. 10 One cycle of iterative calculation (for beam in Fig.9).

6) Matrix Methods

Matrix methods have been used by Sommer (1965), Cheung(1968), Lee and Brown(1972),Bowles (1977), Desai and Christian(1977), and others.

The following matrix method has been used by Sparks and Retief. For a structure as shown in Fig.11, the matrix equation (25) is produced in Table I. It is convenient to number all the reaction forces and moments as F_1, F_2, F_3...etc., and the displacements and rotations as $\delta_1, \delta_2, \delta_3$...etc. The positive direction of a displacement (e.g. $s_1 = \delta_1$) need not be the same as the positive direction of the corresponding force ($R_1 = F_1$). However, after deciding on the positive directions, one must use these positive conventions throughout the calculation.

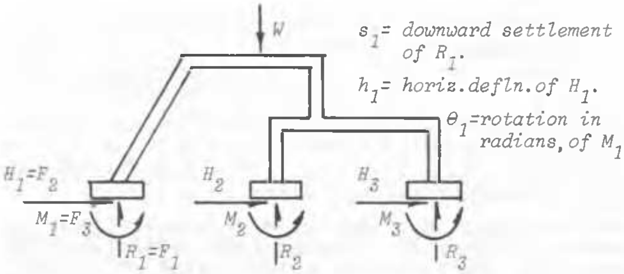


Fig.11 Frame on nonlinear soil supports.

For a uniform beam on three supports (Fig. 7), the matrix equation reduces to:

$$\begin{bmatrix} R_A \\ R_B \\ R_C \end{bmatrix} = \frac{12EI}{L^2} \begin{bmatrix} -1 & 2 & -1 \\ 2 & -4 & 2 \\ -1 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} s_A \\ s_B \\ s_C \end{bmatrix} + \begin{bmatrix} 0{,}3125\ W \\ 1{,}375\ W \\ 0{,}3125\ W \end{bmatrix} \quad (24)$$

where R = reaction (positive upward)
s = settlement (positive downwards).

The diagonal elements in equation (24) are negative because of the different sign conventions for R and s.

In the equation (24), the K matrix has a zero determinant because the structure is symmetrical; hence this K matrix cannot be inverted in this example. However, iterative methods can still be used to solve this problem. In the symmetrical problem, reduction of the unknowns can also be achieved by using $R_A = R_C$, $s_A = s_C$.

Solving The matrix equations

The matrix equations can only be solved if the relationship between the reactive forces and the displacements at the reactions are known. See also the warning listed below.

Simplified relationships for soil supports

A simplified relationship is one in which a displacement (or rotation) at a soil support is assumed to be a function of only one force (or moment) applied to the soil support

(a) The displacement can be expressed in terms of forces [e.g. by equation (1)], in which case the non-linear matrix equation will contain the reactive forces as unknowns; or

(b) The reactive forces can be expressed in terms of displacements [e.g. equation (5)] in which case the non-linear matrix equations will contain the displacements at supports as unknowns.

Warning and general soil relationships(see Fig.3)

If a footing applies a normal reaction R and a moment M, and a shear H to the soil, then any displacement in the column vector in Table I is a function of all these forces R, M and H (e.g. see Fig. 3).

TABLE I - Matrix Equation for Structure (Fig.11)

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \\ F_9 \end{bmatrix} = \begin{bmatrix} R_1 \\ H_1 \\ M_1 \\ R_2 \\ H_2 \\ M_2 \\ R_3 \\ H_3 \\ M_3 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & \cdots & K_{19} \\ K_{21} & K_{22} & \cdots & & \\ K_{31} & \cdots & & & \\ \cdots & & & & \\ & & \text{Square Matrix} & & \\ & & K & & \end{bmatrix} \cdot \begin{bmatrix} s_1=\delta_1 \\ h_1=\delta_2 \\ \theta_1=\delta_3 \\ s_2=\delta_4 \\ h_2=\delta_5 \\ \theta_2=\delta_6 \\ s_3=\delta_7 \\ h_3=\delta_8 \\ \theta_3=\delta_9 \end{bmatrix} + \begin{bmatrix} R_{10}=F_{10} \\ H_{10}=F_{20} \\ M_{10}=F_{30} \\ R_{20}=F_{40} \\ H_{20}=F_{50} \\ M_{20}=F_{60} \\ R_{30}=F_{70} \\ H_{30}=F_{80} \\ M_{30}=F_{90} \end{bmatrix} \quad (25)$$

where K_{ij} = Force F_i in assumed positive direction of F_i caused by a unit displacement δ_j in the assumed positive direction of δ_j (while all other displacements δ are zero).Applies to unloaded structure.

F_{i0} = Force F_i caused by external loadings (e.g. W) when all reaction displacements are zero.

TABLE II - Complementary Energy Method

Member	T = tensile force due to Q & 150 kN	L/AE mm/kN	Length increase D = TL/AE mm	δT due to δQ	D.δT
AB	$\sqrt{2} \times 150 - \sqrt{2}Q$	4×10^{-2}	$4(\sqrt{2}x150-\sqrt{2}Q)x10^{-2}$	$-\sqrt{2}.\delta Q$	$-4(300-2Q)x10^{-2}x\ \delta Q$
BC	$150 - Q$	2×10^{-2}	$2(150 - Q) \times 10^{-2}$	$-\delta Q$	$-2(150-Q)x10^{-2}x\ \delta Q$
BD	$- Q$	$0,\dot{6}x10^{-3}$	$0,\dot{6}(-Q) \times 10^{-3}$	$-\delta Q$	$+0,\dot{6}Qx10^{-3}x\ \delta Q$
DE			Negligible (AE large)		negligible
EF (soil)			For soil	$\Delta.\delta Q$	$= \frac{0,5\ Q}{1-Q/50} \times \delta Q$

δC = sum of last column

i.e. $40(300-2Q)+20(150-Q)-0,\dot{6}Q - \frac{0,5\ Q \times 10^3}{1 - Q/50} = 0$

Provides quadratic. Smallest solution is Q = 17,326 kN

Hence (settlement of E) = 0,5 Q/(1-Q/50) = 13,26 mm

7) Principle of stationary complementary energy

For a framework of m members carrying n loads, the complementary energy C is defined by:-

$$C = \sum_{1}^{m} \int_{0}^{T_i} D_i.dT_i = \sum_{1}^{n} \int_{0}^{F_j} \Delta_j.dF_j \qquad (26)$$

where D_i = Extension of internal member i

T_i = Tension force in internal number i

Δ_i = Displacement of external load j

F_j = Force of external applied load j

A structure deforms so that the complementary energy is stationary with respect to an unknown force Q in an internal member. That is:-

$$\frac{\delta C}{\delta Q} = 0 \qquad (27)$$

The non-linear soil support between E and F (Fig.12) can be regarded as an internal member. In Table II, T and D are due to Q and the applied load (150 kN). But δT is due only to Q. The last column must total zero. Hence the unknown Q, and the settlement are found.

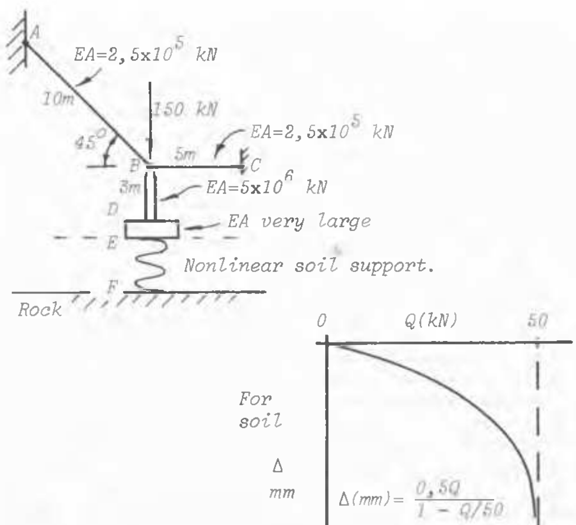


Fig. 12

8) Using mathematical programming for method 7

The condition at which C has a zero slope as Q is changed can be found by search techniques.

9) By sketching deflected shapes

Reasonable results have been obtained by sketching deflected shapes to estimate the intercepts of the straight lines in Figures 6(b), 8 and 10. An iterative method using trial positions for points of contraflexure is suitable for frame structures.

10) Using photo-elastic modes on soil supports

This method has been used at UCT to check positions of contraflexure and to estimate moments and thrusts applied to the footings for model structures such as shown in Figure 11.

11) Finite difference methods (e.g. For rafts).

These have been published by many authors.

12) Finite element methods

These include strain analysis for non-linear materials. See Lee(1974), Desai & Christian(1977) Bowles(1974), Naylor(1978), Majid & Cunnell(1976) King and Chandrasekran(1975) and others.

CONCLUSION

Convergence of iterative methods depends on the order in which the variables are used. The sketching of deflected shapes is worthy of greater use. Photo-elastic models are useful to check and teach principles.

The curve relating a vertical load to the soil settlements (e.g. Fig.1) is not unique. The curve is also a function of the moment M and the shear H which acts on the soil.

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