Pile Embedded into a Nonlinearly Elastic Medium

Pieu dans un Milieu Non-Linéairement Elastique

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SYNOPSIS This work shows a procedure to analyze the behaviour of an axially loaded foundation pile, embedded into a nonhomogeneous, nonlinearly elastic soil, under drained conditions. The analysis allows to obtain, for every load step, the pile displacement, its elastic strain and the state of stress in the surrounding soil. The axial load is transferred to the soil by skin friction as a function correlating the lateral shear with the amount of slip between pile and soil. The procedure takes into account the initial conditions in the soil, the variations introduced by the pile contruction, the variation of the confining stress applied from the soil to the pile due to the stress transferred from the pile to the soil. The state of displacement is computed regarding to a continuously variable modulus of deformation. Results are shown for drilled shafts and, moreover, useful graphs, valid for diameters between 0,6÷1,0 m, are provided for a fast design of these piles.

1. INTRODUCTION

1.1. This paper presents an analytical approach allowing for a single pile having embedded length L, diameter D and subjected to axial load P_0, (see fig. 1) to evaluate :
- the vertical displacement S of any point located at the pile axis, S_0 of the top, S_B of the base, (see fig. 2-a).
- the distribution of the vertical load along the pile allowing to separate the mobilized shaft friction P_L from the mobilized base resistance P_B, (see fig. 2-b).
- the distribution of the axial load $P_j(z)$ and the tangential stresses $\tau(z_j)$ along the pile shaft as a function of depth and applied working load P_0.
- the complete stress system induced by P_0 into soil adjacent to the pile.
- the variations of soil stiffness around the pile as function of the applied load. The modulus E' is function of $(\sigma'_1 - \sigma'_2)$, $(\sigma'_1 - \sigma'_3)$, σ'_3.

1.2. This approach has been worked out adopting the following assumptions :
- Pile materials obey laws of the elasticity theory and have elasticity modulus E_p.
- The soil response to applied loading in terms of effective stress under fully drained conditions is only considered, assuming linear Mohr-Coulomb strenght envelope having cohesion intercept c'= 0 and effective angle of shearing resistance ϕ'.
- The transfer law relating τ to relative displacement W of the pile-soil interface is function of soil properties and acting radial effective stress σ'_r, the later depending on the applied P_0.

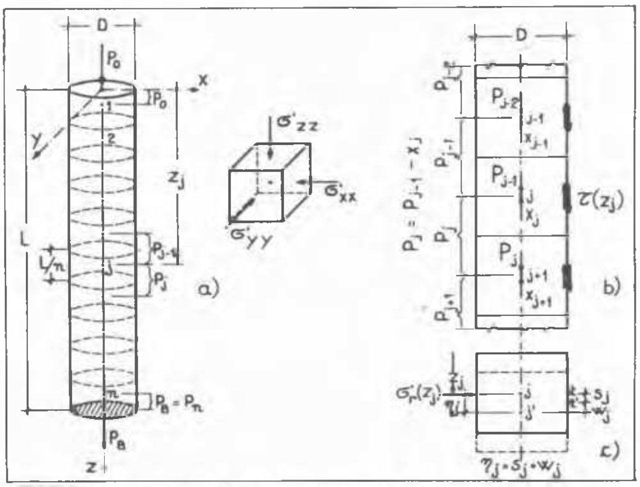

Fig. 1 - Reference scheme

Particularly the presented calculation method allows to take into account :
- the initial soil stresses and their changes caused by pile installation;
- continuous and discrete (= stratification) heterogeneity with respect to both soil strenght and stiffness.

1.3. The above mentioned calculation method has been implemented on high speed digital computer simulating thereafter numerically the behaviour of some instrumented piles installed in well geotechnically documented sand deposits for which the results of load tests are reported in the literature. For further details concerning the here below exposed approach the reader may refer to works by Berardi (1959, 1961) and Berardi et al. (1973, 1975).

2. PILE-SOIL INTERACTION

The pile-soil interaction problem when the former is subjected to the external load P_o may be briefly described as follows ; see also fig. 1:
- Every generic pile segment "j" having lenght L/n is subjected to radial stress $\sigma'_r(z_j)$ to which corresponds mobilized shaft friction $\tau(z_j)$
- The axial load on the considered pile segment "j" is P_j.
- The vertical displacement of the pile segment is η_j and the corresponding vertical displacement of soil adjacent to the pile segment is S_j leading to a relative pile-soil interface displacements $\eta_j - S_j = W_j$
- The axial force transmitted to soil through the pile tip is P_B.

2.1. Under such assumption the solution of the examined problem may be obtained solving n compatibility equations (2) and equilibrium equation (1) leading to the determination of n unknown forces P_j and value of P_B.

$$P_o = P_B + \sum_{1}^{n} X_j ; \qquad (1)$$

$$\eta_j = W_j + S_j \quad ; \qquad (2)$$

$$X_j = \pi D \frac{L}{n} . \tau(z_j); \qquad P_j = P_{j-1} - X_j \qquad (3)$$

2.2. The solution of the problem requires the knowledge of the relationship between τ and W and of the friction coefficient on the pile-soil interface tgδ. Basing on the experience reported in the existing literature the following hypothesis are here adopted, (Kezdi, 1957) (Potyondy, 1961)(Berardi, 1966) (Coyle-Reese, 1966)(Coyle-Sulaiman, 1967)(Clough-Duncan,1971)

$$tg\delta(W) = \frac{W}{W_c} \; tg\delta, \quad \text{per } W \leq W_c \qquad (4)$$

$$tg\delta(W) = tg\delta \quad , \quad \text{per } W > W_c \qquad (5)$$

assuming that the critical relative displacement W_c falls within the range of values between 0.08 cm and 0.4 cm valid for cohesionless materials and may be also rappresentative for those cohesive ones under fully drained conditions. In sands it is generally argued that W_c decreases as relative density D_r of sand and pile smoothness increase. Under these circumstances the equation (3) may be rewritten as follows

$$X_j = \pi D \frac{L}{n} \frac{tg\delta}{W_c} W_j (\sigma'_{ho,j} + \sigma'_{r,j}) \qquad (6)$$

where $\sigma'_{ho,j}$ represents the effective radial stress after pile installation but before application of any external load and the $\sigma'_{r,j}$ corresponds to the effective radial stress induced by load P_o acting on the pile.

2.3. In order to write in explicit form the equations (2) the absolute displacement η_j and S_j are evaluated by means of the well known Mindlin (1966) solution, leading to the determination of the influence coefficients I_j previously tabulated by Berardi et al. (1973) allowing to express (2) in a compact form.
Doing that a clear distinction must be done between the pile segments for which condition $W_j \geq W_c$ is verified corresponding to full mobilization of the shaft friction, from those in which $W_j < W_c$ i.e. only part of the ultimate shaft friction has been mobilized. In this conditions the equations (2) become :

$$P_o(C_{i,P} - N_i) + W_c \sum_{j=1}^{m} (C_{i,j} - V_{i,j}) + \sum_{j=m+1}^{i-1} W_j (C_{i,j} - V_{i,j}) +$$

$$+ W_i (C_{i,i} - V_{i,i} - 1) + \sum_{j=i+1}^{n} W_j (C_{j,j} - V_{i,j}) = 0 \qquad (7)$$

being m number of the pile segments for which $W_j \geq W_c$, the coefficients C_{ij} and V_{ij} may be evaluated by means of formulae given in Appendix A (see fig. 9).
The system of equations (7) is solved by means of iterative process satisfying the compatibility conditions for the shaft friction expressed by means of (4) and (5).

3. NON-LINEAR SOIL STIFFNESS

The evaluation of displacement within soil mass is done supposing a medium having stress-dependent deformability modulus E' and constant Poisson coefficient ν'. In the calculations, the average soil stiffness representative for soil cylinder of height 1.5 L, subdivided in 15 strata, and having diameter 10 D, subdivided in 13 co-axial annulus is considered. Therefore E' adopted in each computation step is an average of E' in 175 soil elements weighted with respect to induced vertical stress σ'_{zz} acting in each element.
In order to evaluate the soil stiffness E'_t during each computation step in every soil element the hyperbolic stress-strain relationship(see Kondner (1963), Duncan and Chang (1970)), has been postulated, leading to the following equa-

tion :

$$E'_t = E'_i(1-f_d)^2 \qquad (8)$$

in which f_d correspond to the coefficient of mobilization of the ultimate soil strenght :

$$f_d = \frac{T - T_o}{T_{max} - T_o} \qquad (9)$$

being : T = actual stress level in considered the soil element;
T_o = initial stress level in the considered soil element;
T_{max} = stress level at failure

where T is defined according to Lade (1977,78) by means of :

$$T = (I_1^3/I_3 - 27)(I_1)^q \qquad (10)$$

as function of the first (I_1) and third (I_3) stress invariants, being T_{max} and exponent q inherent sand properties measured in the appropriate triaxial tests.
The use of the equation (9) when evaluating f_d allows to take into consideration in every soil element, the complete stress state ($\sigma'_1, \sigma'_2, \sigma'_3$) and the resulting principal plane rotations. The initial soil stiffness E'_i before each calculation step has been evaluated from the well known Janbu (1963) empirical relationship :

$$E'_i = mp_a\left(\frac{\sigma'_3}{p_a}\right)^{\alpha} \qquad (11)$$

in which the value of σ'_3 resulting from the previous computation step has been introduced.

4. CALCULATION RESULTS

4.1. The above briefly described approach has been applied in order to carry out a parametric study of a large number of piles with the aim to obtain for different soil conditions a set of information useful for design concerning the behaviour of a single pile in sand under the working load.

TABLE I

SAND	e	n %	γ kN/m	D_r %	φ' (°)	δ (°)	K_o	T_{MAX}	q	W_c (cm)	m	α	
1	0,61	0,379	16,324	100	41	39	0,344	80	0,230	0,100	1213	0,42	SACRAMENTO
2	0,71	0,415	15,382	78	38	36	0,384	53	0,188	0,125	778	0,50	RIVER SAND
3	0,87	0,465	14,068	38	34	33	0,441	28	0,093	0,175	347	0,65	(Lee-Seed, '67)
4	0,75	0,412	14,911	82	40	38	0,357	40	0,100	0,120	558	0,45	CHATTAHOOCHEE
5	1,03	0,505	12,851	20	32	32	0,470	18	0,000	0,200	173	0,54	RIVER-SAND
													(Vesic-Clough'68)

Due to the limitation of space only bored cast in situ piles embedded in sand are here considered keeping in mind the fact that particularly for these piles the sharing of P_o between side and base resistance and pile head settlement under working load are of great interest to engineers.

TABLE II
N_{SPT} values

Depth (m) \ sand	1	2	3	4	5
5	60	34	8	38	3
10	80	45	10	50	4
20	115	66	15	73	5
40	190	110	25	120	9

As the reference, two geotechnically well documented sands have been taken into consideration adopting in piles calculation the relevant geotechnical parameters shown in Tab. I. From this table it may be argued that for the same D_r the SR sand is much more stiff than CR sand probably because due to their different mineralogical composition and particularly due to the presence of the micaceous particles in the latter sand.
The sand heterogeneity has been simulated by means of the continuous change of N_{SPT} vs depth i.e. the variation of D_r with depth being the latter computed according to Gibbs and Holtz (1957) and Bazaraa (1967). The N_{SPT} values assumed for five types of sand at different depth are given in Tab. II.
The first i.e. the beginning of calculation value of E' has been obtained by means of (11) introducing in it the value of σ'_{ho} .
As piles concern four C=L/D ratios have been considered with values of 10,20,33 and 50 which correspond to circular concrete full section piles with $0.6 \leq D \leq 1,0$ m.

4.2. Figs. 2 and 3 show the results of calculations for pile having D = 0.762 m embedded 25 m in sand n.2 subjected to gradually increasing load up to 800 MN. From these figures it may be deduced that :
- Almost all shaft friction (333MN) is mobilized when P_o = 397MN to which corresponds S_o = 1.53 cm and S_B = 0.89; when exceeding such value of P_o almost all the additional load is transferred to the pile tip causing very pronunced increase of base settlement S_B.

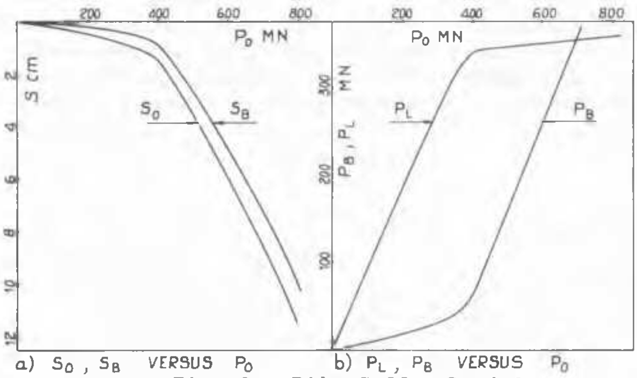

Fig. 2 - Pile C=33, Sand 2

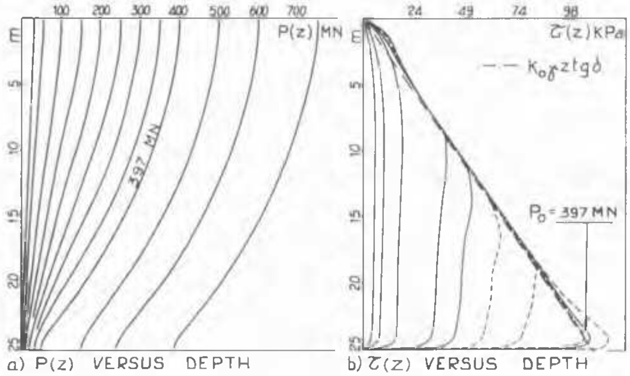

Fig. 3 - Pile C=33, Sand 2

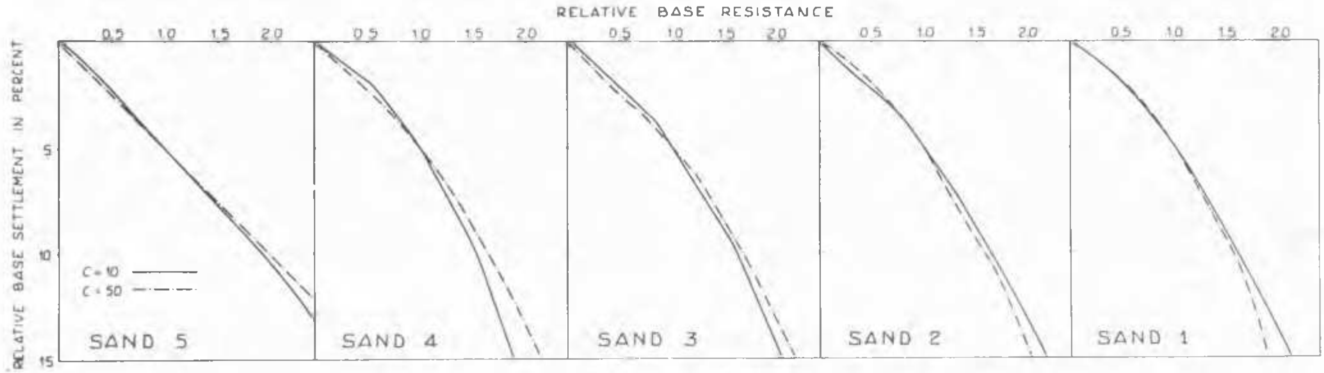

Fig. 4 - $q_B/q_{B,ULT}$ versus S_B/D

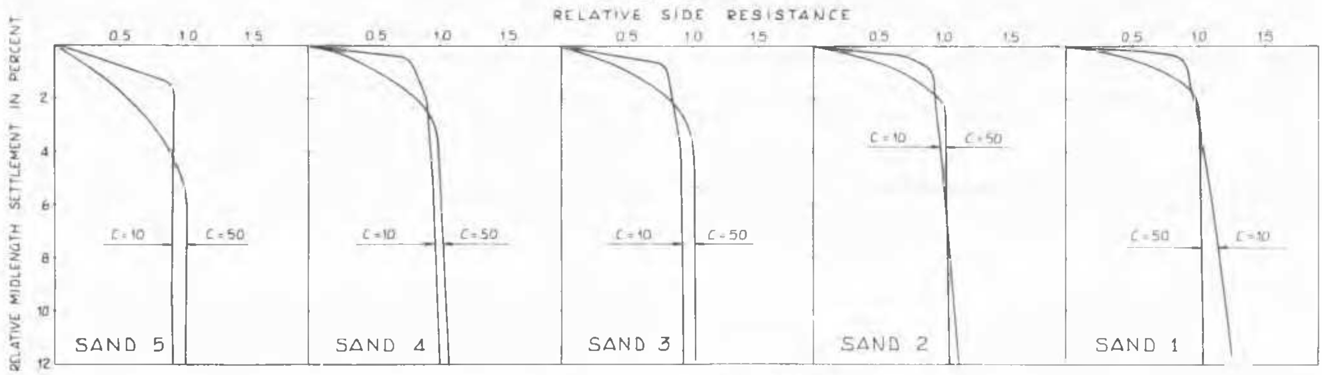

Fig. 5 - $P_L/P_{L,ULT}$ versus S_M/D

- Fig. 3-a shows a very typical distribution of axial load along pile while fig. 3-b reports how shearing stress, due to skin friction, is rapidly mobilized in higher zone of the pile and how, on the contrary, steadies on the lower zone, increasing here with P_o.

4.3. Fig. 4 shows the ratio of $q_B/q_{B,ULT}$ versus relative base settlement S_B /D, assuming $q_{B,ULT}$ corresponding to limit of the serviceability base resistance which is mobilized when S_B/D = 0.05.
In fig. 5 the variation of ratio $P_L/P_{L,ULT}$ versus S_M/D is reported, being S_M the settlement of the pile mid-height and assuming that the ultimate shaft friction is equal to

$$P_{L,ULT} = \pi D \frac{L^2}{2} k_o \gamma tg\delta \qquad (12)$$

In this equation it was conventionally assumed, Meyerhof (1976) that for bored piles the ratio of horizontal to vertical effective stresses in proximity of the pile shaft is equal to k_o=1-sen ϕ'.
It may be deduced that the side resistance P_L weakly increases with P_B ($\Delta P_B \simeq \Delta P_o$) owing to the increase of the stress σ'_r , see eq. 6. Such behaviour is considerable for lower ratios L/D and for higher relative density of sand; nevertheless this increasing become important when S_B/D>0.05 so has not interest in design criteria. In fig. 6 are shown the values of $q_{B,ULT}$ as function of N_{SPT} at depth z = L and σ'_{vo} .

It may be observed that $q_{B,ULT}$ vs N_{SPT} deduced from axial load tests on the instrumented bored piles in sands as proposed by Reese (1978) falls within the field embraced by computation results and covering very large range of σ'_{vo}.

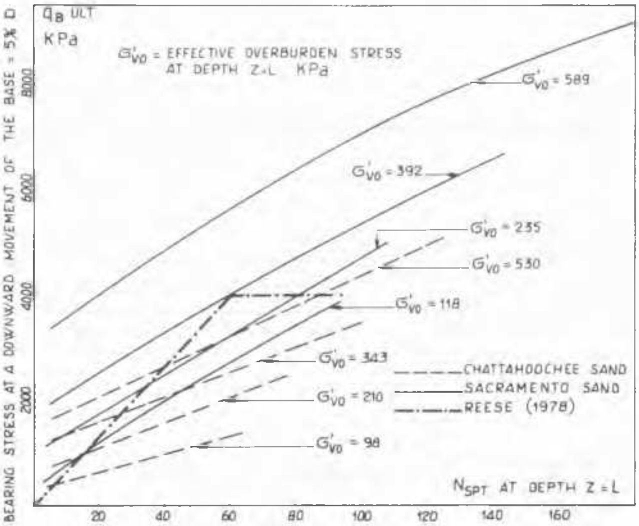

Fig. 6 - $q_{B,ULT}$ versus N_{SPT}

Further sample of information which is possible to obtain from the briefly discussed here calculation approach are shown in : Figs. 7 and 8 reporting normalized mobilized shaft friction $P_L/P_{L,ULT}$ as function of S_M/D.

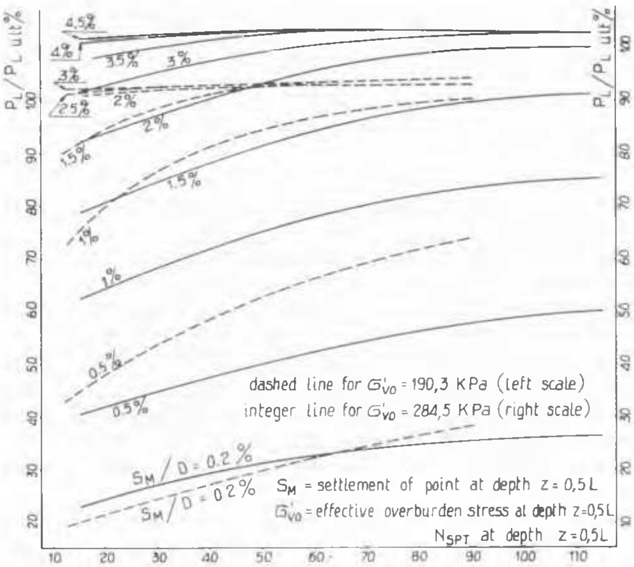

Fig. 7 - $P_L/P_{L,ULT}$ versus N_{SPT}

Finally in the Tab. III the normalized pile settlements S_o/D, S_M/D and S_B/D as function of stress $\sigma_c = 4\,P_o/\pi D^2$ applied to the pile head are reported believing that such set of data may be of great interest when designing large bored cast in situ pile or widely spaced piles

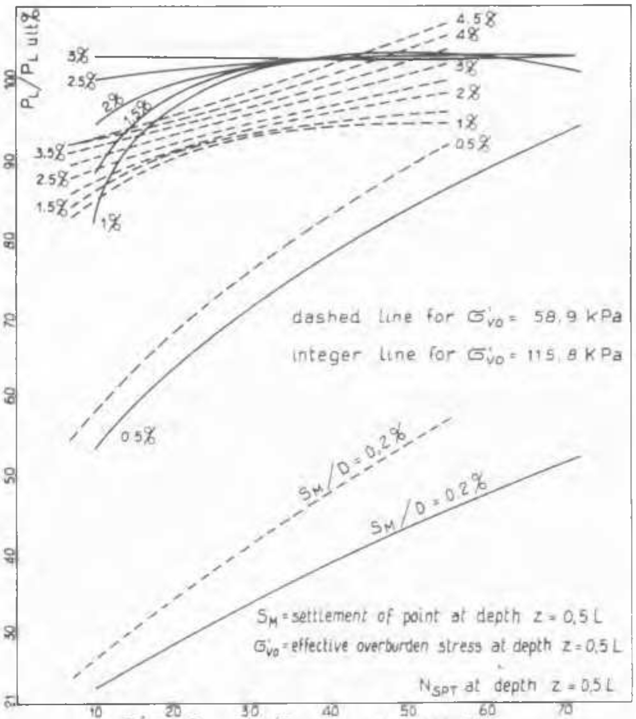

Fig. 8 - $P_L/P_{L,ULT}$ versus N_{SPT}

group in sand; the design of which is governed by the service limit state rather than by the ultimate bearing capacity considerations.
Due to the limitation of space only SR sand has been plotted in figs. 6-7 and Tab. III.

4.4. The failure of a single pile is assumed to conceed with the relative base settlement $S_B/D = 0.05$. Under such assumption the safety factor of a pile is given by :

$$F = \frac{P_{ULT}}{P_o} = \pi\left(\frac{D^2}{4}\,q_{B,ULT} + P_{L,ULT}\right)/P_o \qquad (13)$$

5. DESIGN EXAMPLE

This paragraph illustrates how above exposed computation results may be used in design.
As example the pile embedded in sand n.2 subjected to a working load P_o = 3237 KN is considered.
Assuming maximum allowable σ_c = 6.38 MPa and imposing as the design criterion S_o = 1 cm, one deduces D ≃ 0.8 m, i.e. S_o/D = 1.25%.
For this set of data from Tab. III one obtains : C=33 i.e. L=26 and S_M/D = 0.73%, S_B/D = 0.55 %. Introducing such data into Tab. I, one gets :

Z=L/2: σ'_{vo} = 200 KPa and Z=L: σ'_{vo} = 400 KPa

Therefore from Tab. II it results : $N_{SPT}(L/2)$ = = 51 blows/foot and $N_{SPT}(L)$ = 79 blows/foot. From fig. 4 it may be seen that to this latter N_{SPT} corresponds $q_B/q_{B,ULT}$ = 17.5% which leads to (fig. 6) $q_{B,ULT}$ = 4750 KPa and $P_{B,ULT}$ = 2386 KN.

TABLE III

Relative settlements versus $4P_o/\pi D^2$ (MPa)

σ_c	Sand 1 C=50	3,33	4,41	6,57	8,73	10,88	Sand 2 C=50	3,33	4,41	6,57	8,73	10,88	Sand 3 C=50	3,33	4,41	6,57	8,73	10,88
S_o/D		0,36	0,50	0,81	1,20	1,50		0,42	0,58	0,92	1,31	1,74		0,57	0,77	1,22	1,72	2,30
S_M/D		0,13	0,17	0,29	0,43	0,60		0,18	0,18	0,39	0,57	0,79		0,31	0,42	0,67	0,97	1,33
S_B/D		0,06	0,09	0,14	0,21	0,28		0,10	0,14	0,22	0,33	0,45		0,22	0,30	0,48	0,70	0,95
σ_c	Sand 1 C=33	3,33	4,41	6,57	8,73	10,88	Sand 2 C=33	2,25	3,33	4,41	6,57	8,73	Sand 3 C=33	2,25	3,33	4,41	6,57	8,73
S_o/D		0,39	0,54	0,91	1,44	2,99		0,31	0,47	0,66	1,13	2,01		0,47	0,72	1,02	1,80	4,42
S_M/D		0,20	0,29	0,52	0,91	2,31		0,19	0,29	0,41	0,73	1,47		0,34	0,53	0,76	1,40	3,88
S_B/D		0,14	0,20	0,35	0,63	1,90		0,14	0,21	0,31	0,55	1,18		0,29	0,45	0,66	1,21	3,56
σ_c	Sand 1 C=20	2,25	3,33	4,41	6,57	7,65	Sand 2 C=20	1,72	2,25	3,33	4,41	5,59	Sand 3 C=20	1,18	1,72	2,25	3,33	4,41
S_o/D		0,35	0,66	1,63	4,39	6,10		0,35	0,49	1,06	2,75	4,85		0,42	0,65	0,85	2,81	6,90
S_M/D		0,27	0,54	1,47	4,13	5,81		0,28	0,41	0,93	2,58	4,64		0,37	0,59	0,87	2,69	6,74
S_B/D		0,23	0,47	1,36	3,94	5,57		0,26	0,37	0,86	2,47	4,48		0,35	0,56	0,83	2,61	6,62
σ_c	Sand 1 C=10	0,64	1,18	1,72	2,25	3,33	Sand 2 C=10	0,64	1,18	1,72	2,25	3,33	Sand 3 C=10	0,64	1,18	1,72	-	-
S_o/D		0,24	0,54	1,25	2,11	4,16		0,36	0,99	2,27	3,81	7,56		0,79	3,88	9,79	-	-
S_M/D		0,23	0,52	1,22	2,07	4,10		0,35	0,96	2,24	3,76	7,49		0,78	3,85	9,75	-	-
S_B/D		0,22	0,51	1,19	2,03	4,04		0,34	0,95	2,21	3,73	7,43		0,77	3,84	9,75		-

As shaft friction concerns : $P_{L,ULT}$ = 3.14 . . 0.8 . 26 . 200 . 0.384 tg 36° = 3644 KN, from fig. 7 :$P_L/P_{L,ULT}$ = 78% and therefore mobilized P_L= 2842 KN.

Finally it results that the working load P_o is shared between shaft friction P_L = 2842 KN and base resistance P_B = 0.175.2386 = 418 KN, i.e. P_o= 3260 KN.

The safety factor as formulated by means of (13) results : F = (3644+2386)/3260 = 1,85

$$C_{i,p} = C_{b,p} + \frac{L}{20 E_p A}(21-2\cdot i)$$

$$C_{b,p} = \frac{1}{E'L}\, I^{a_L, P_B}$$

$$C_{i,j} = C_{b,j} - \frac{L}{20 E_p A} R_j (21-2\cdot i)$$

$$C_{b,j} = \frac{R_j}{LE'}\left(I^{a_L,P_j} - I^{a_L,P_B}\right)$$

$$N_i = \frac{1}{E'L}\, I^{b_i,P_B}$$

$$V_{i,j} = \frac{R_j}{LE'}\left(I^{b_i,P_j} - I^{b_i,P_B}\right)$$

$$C_{i,i} = C_{b,i} - \frac{L}{20 E_p A} R_i (21-2\cdot i)$$

$$V_{i,i} = \frac{R_i}{LE'}\left(I^{b_i,P_i} - I^{b_i,P_B}\right)$$

$$C_{j,j} = C_{b,j} - \frac{L}{20 E_p A} R_j (21-2\cdot j)$$

$$R_j = \beta_j \left[(\sigma_{ho})_{3,j} + \sigma_{r,j} \right]$$

$$\beta_j = \pi D \frac{L}{10} \frac{tg\, \delta_j}{W_c}$$

Fig. 9 - Appendix A

REFERENCES

Bazaara, A.R.S.S.(1967).Use of the SPT for Estimating Settlements of Shallow Foundations on Sand. Thesis presented to Univ.of Illinois.

Berardi G.(1961). Sul comportamento del palo di fondazione in terreno incoerente. Atti U.Pisa (85).

Berardi G., Dalerci G., & La Magna A.,(1973). Stato di tensione-deformazione, in fase di esercizio, del palo in terreno incoerente. Proc.11th CAGI-MI

Clough G.W., Duncan J.M. (1971). Finite Element Analyses of Retaining Wall Behaviour. ASCE J. SMF Div., 8583, Dec., 1657-1673.

Coyle H.M., Reese L.C.(1966). Load Transfer for Axially Loaded Piles in Clay. ASCE J.SMF Div., 4702 Mar., 1-26.

Coyle H.M., Sulaiman I.H.(1967).Skin Friction for Steel Piles in Sand. ASCE J. SMF Div., 5590, Nov. 261-278.

Lade P.V.(1977). Elasto-Plastic Stress-Strain Theory for Cohesionless Soil with Curved Yeld Surfaces.I.J.Solids Structures(13),Nov1019-1035

Lade P.V.(1978). Prediction of Undrained Behaviour of Sand. ASCE J.Geot.Eng.Div.,13834,June,721-735

Lee K.L., Seed H.B.(1967). Drained Strength Characteristics of Sand. ASCE J.SMF Div.,5561,Nov.,117-141

Meyerhof G.G.(1976). Bearing Capacity and Settlement of Pile Foundations. ASCE J.Geot.Eng.Div., 11962, March., 196-228.

Reese L.C.(1978). Design and Construction of Drilled Shafts : The Twelfth Terzaghi Lecture. ASCE J. Geot.Eng.Div., 13503, Jan., 91-116.

Vesic A.S., Clough G.W.(1968).Behaviour of Granular Materials Under High Stresses. ASCE J.SMF Div., 5954, May, 661-688.