Consolidation due to Lateral Loading of a Pile

Consolidation avec une Force Latérale sur un Pieu

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SYNOPSIS The response of a single pile embedded in a clay soil and subjected to a lateral load may be time dependent. This paper outlines a method of analysis for the time dependent response of such piles due to consolidation of the surrounding elastic medium.

INTRODUCTION

The problem of the response of a single, cylindrical pile embedded in a homogeneous elastic soil and subjected to a lateral load has received the attention of several investigators (e.g. Poulos, 1971; Randolph, 1977). These studies looked at the load-displacement behaviour of the pile in either the short or the long term. However, if these piles are embedded in a saturated clay, then upon loading the instantaneous deformation will cause excess pore pressures to develop within the surrounding soil. These excess pore pressures will dissipate with time after the application of load, and as this occurs the soil around the pile will consolidate (some of it may swell). Further deflection of the pile will be observed. It is the aim of this paper to outline a method of analysis which will allow predictions to be made of the time dependent lateral response of the pile due to consolidation within the surrounding soil.

ANALYSIS

A cylindrical coordinate system (r,θ,z) is adopted, with the z axis corresponding to the axis of the pile. The symbol t is used to represent time.

Fourier Representation

The analysis is based on the assumption that field quantities such as displacement (u_r,u_θ,u_z) and excess pore pressure (p) can be expressed in the following form

$$\begin{aligned} u_r(r,\theta,z,t) &= U_r^{(n)}(r,z,t)\cos(n\theta+\varepsilon_n) \\ u_\theta(r,\theta,z,t) &= U_\theta^{(n)}(r,z,t)\sin(n\theta+\varepsilon_n) \\ u_z(r,\theta,z,t) &= U_z^{(n)}(r,z,t)\cos(n\theta+\varepsilon_n) \\ p(r,\theta,z,t) &= P^{(n)}(r,z,t)\cos(n\theta+\varepsilon_n) \end{aligned} \qquad (1)$$

where $n = 0,1,2,\ldots$ and ε_n is used to establish a reference point for the measurement of θ.

Superposition

Solutions of type (1) can be superimposed to obtain more general solutions of the form

$$u_r(r,\theta,z,t)=U_r^{(o)}+\sum_{n=1}^{N} U_r^{(n)}\cos(n\theta+\varepsilon_n), \quad \text{etc} \qquad (2)$$

The solution to any general problem thus reduces to one of finding the Fourier coefficients $U_r^{(n)}$, etc.

For a pile subjected to pure axial load it is necessary to find only the first of these terms, i.e. $U_r^{(o)}$ etc (n=0); for a pile subjected to pure lateral load it is necessary to find only the second of these terms, $U_r^{(1)}$ etc (n=1).

Constraints at r = 0

At the centreline of the pile, r=0, all field quantities must be single valued. By considering the cartesian components of displacement, it may be shown that certain conditions need to be met. These conditions are -

For n = 0 $\quad U_r^{(o)} = U_\theta^{(o)} = 0 \qquad (3)$

For n = 1
$$U_r^{(1)} + U_\theta^{(1)} = 0, \quad U_z^{(1)} = 0, \quad P^{(1)} = 0 \qquad (4)$$

For n > 1
$$U_r^{(n)} = 0, \quad U_\theta^{(n)} = 0, \quad U_z^{(n)} = 0, \quad P^{(n)} = 0 \qquad (5)$$

Strain Components and Hooke's Law

In a cylindrical coordinate system the strain-displacement relations are

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \gamma_{r\theta} = \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}$$
$$\varepsilon_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}, \quad \gamma_{\theta z} = \frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta} \qquad (6)$$
$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z}, \quad \gamma_{zr} = \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}$$

The increment of stress σ' is related to the increment of strain $\underset{\sim}{\varepsilon}$ by Hooke's law, viz.

$$\underset{\sim}{\sigma}' = D\,\underset{\sim}{\varepsilon} \qquad (7)$$

where $\underset{\sim}{\sigma}' = (\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \tau_{r\theta}, \tau_{\theta z}, \tau_{zr})^T$

$\underset{\sim}{\varepsilon} = (\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{zz}, \gamma_{r\theta}, \gamma_{\theta z}, \gamma_{zr})^T$

$$D = \begin{bmatrix} \lambda+2G & \lambda & \lambda & 0 & 0 & 0 \\ & \lambda+2G & \lambda & 0 & 0 & 0 \\ & & \lambda+2G & 0 & 0 & 0 \\ & & & G & 0 & 0 \\ & & & & G & 0 \\ \text{SYMMETRIC} & & & & & G \end{bmatrix}$$

λ and G are the Lamé parameters for the soil under fully drained conditions.

Fluid Flow and Darcy's Law

It is assumed that the movement of pore fluid through the elastic soil is governed by Darcy's law, viz.

$$\underset{\sim}{v} = -\left(\frac{k}{\gamma_w}\right) \underset{\sim}{\nabla} P \qquad (8)$$

where $\underset{\sim}{v}$ = superficial velocity of the pore fluid

k = isotropic permeability coefficient

γ_w = unit weight of pore water.

Finite Element Approximation

The equations which govern the consolidation of an isotropic elastic medium were developed by Biot (1941). These combine the complexities of an elastic deformation with a consolidation process. An approximate solution to these equations may be obtained using the finite element method (e.g. Booker and Small, 1975).

If the continuous values of the coefficients $U_r^{(n)}$, $U_\theta^{(n)}$, $U_z^{(n)}$, $P^{(n)}$ can be adequately represented by values at selected nodes, i.e. $\underset{\sim}{\delta}_r^{(n)}$, $\underset{\sim}{\delta}_\theta^{(n)}$, $\underset{\sim}{\delta}_z^{(n)}$, $\underset{\sim}{q}^{(n)}$, then we may write

$$U_r = \underset{\sim}{N}_r^T \cdot \underset{\sim}{\delta}_r, \text{ etc.} \qquad (9)$$

For convenience the superscript n has been dropped in equations (9) and in the following.

It can be shown that for the time interval Δt, from t_o to t, the finite element equations for consolidation have the matrix form

$$\begin{bmatrix} K & -L^T \\ -L & -\beta\Delta t\Phi \end{bmatrix} \begin{bmatrix} \Delta\underset{\sim}{\delta} \\ \Delta\underset{\sim}{q} \end{bmatrix} = \begin{bmatrix} \Delta\underset{\sim}{F} \\ \Delta t\Phi\underset{\sim}{q}(t_o) \end{bmatrix} \qquad (10)$$

where $\Delta\underset{\sim}{\delta} = (\Delta\underset{\sim}{\delta}_r, \Delta\underset{\sim}{\delta}_\theta, \Delta\underset{\sim}{\delta}_z)^T$

$= \underset{\sim}{\delta}(t) - \underset{\sim}{\delta}(t_o)$

$\Delta\underset{\sim}{q} = \underset{\sim}{q}(t) - \underset{\sim}{q}(t_o)$

and $\Delta\underset{\sim}{F} = \underset{\sim}{F}(t) - \underset{\sim}{F}(t_o)$

= increment of applied nodal forces.

For the Fourier representation of the field quantities used in (1) and (9), we have

$$K = \int_V B^T.\hat{D}.B\; r\,dr\,dz$$

$$B = \begin{bmatrix} \frac{\partial \underset{\sim}{N}_r^T}{\partial r} & 0 & 0 \\ \frac{\underset{\sim}{N}_r^T}{r} & n\,\frac{\underset{\sim}{N}_\theta^T}{r} & 0 \\ 0 & 0 & \frac{\partial \underset{\sim}{N}_z^T}{\partial z} \\ -\frac{n}{r}\underset{\sim}{N}_r^T & \frac{\partial \underset{\sim}{N}_\theta^T}{\partial r} + \frac{\underset{\sim}{N}_\theta^T}{r} & 0 \\ 0 & \frac{\partial \underset{\sim}{N}_\theta^T}{\partial z} & -n\,\frac{\underset{\sim}{N}_z^T}{r} \\ \frac{\partial \underset{\sim}{N}_r^T}{\partial z} & 0 & \frac{\partial \underset{\sim}{N}_z^T}{\partial r} \end{bmatrix}$$

$$\hat{D} = D\,.\,\underset{\sim}{I}$$

$$\underset{\sim}{I} = (I_c, I_c, I_c, I_s, I_c, I_s)^T$$

$$I_c = \int_0^{2\pi} \cos^2(n\theta+\varepsilon_n)\,d\theta = 2\pi\cos^2\varepsilon_n, \; n = 0$$
$$= \pi, \; n = 1,2\ldots$$

$$I_s = \int_0^{2\pi} \sin^2(n\theta+\varepsilon_n)\,d\theta = 2\pi\sin^2\varepsilon_n, \; n = 0$$
$$= \pi, \; n = 1.2\ldots$$

$$L^T = I_c \int \begin{bmatrix} \left(\frac{\partial \underset{\sim}{N}_r}{\partial r} + \frac{\underset{\sim}{N}_r}{r}\right) . \underset{\sim}{N}_p^T \\ n\,\frac{\underset{\sim}{N}_\theta}{r} . \underset{\sim}{N}_p^T \\ \frac{\partial \underset{\sim}{N}_z}{\partial z} . \underset{\sim}{N}_p^T \end{bmatrix} r\,dr\,dz$$

$$\Phi = \int \begin{bmatrix} \frac{\partial \underset{\sim}{N}_p}{\partial r}, & -n\,\frac{\underset{\sim}{N}_p}{r}, & \frac{\partial \underset{\sim}{N}_p}{\partial z} \end{bmatrix} . \hat{k} . \begin{bmatrix} \frac{\partial \underset{\sim}{N}_p^T}{\partial r} \\ \frac{-n\underset{\sim}{N}_p^T}{r} \\ \frac{\partial \underset{\sim}{N}_p^T}{\partial z} \end{bmatrix} r\,dr\,dz$$

$$\hat{k} = \left(\frac{k}{\gamma_w}\right) \cdot \begin{bmatrix} I_c & 0 & 0 \\ 0 & I_s & 0 \\ 0 & 0 & I_c \end{bmatrix}$$

A solution for the nodal displacement and pore pressure coefficients, U_r etc, may be obtained at a discrete number of times by solving equations (10) and using a marching process. The solution for the nodal displacements and excess pore pressure can then be found using equations (1). For stability of the marching process $\beta \geq 1/2$ (Booker and Small, 1975).

SOME TYPICAL SOLUTIONS

Solutions have been obtained for two different vertical piles embedded in the same saturated, elastic soil mass. Both piles are subjected to a horizontal load P_h only. The following cases were considered

(a) $\ell/r_o = 40$

(b) $\ell/r_o = 20$

where ℓ = pile length and r_o = pile radius. In both cases the shear modulus of the soil is G_s and the drained Poisson's ratio is ν_s'. Each pile has a Young's modulus of E_p and each is considered to be impermeable. For the particular soil and piles studied the following numerical parameters were adopted:

$$\frac{E_p}{G_s} = 10^3, \quad \frac{P_h}{G_s r_o^2} = 1, \quad \nu_s' = 0.4$$

Solutions were obtained using two-dimensional, 8 node, isoparametric finite elements. Predictions of the lateral displacement of the pile head ρ in the direction of the applied force P_h are plotted against time in Fig. 1. A non-dimensional time is plotted as the abscissa, using

$$T = \frac{ct}{r_o^2} \qquad (11)$$

where $c = \left(\frac{k}{\gamma_w}\right) \cdot 2G \cdot \left(\frac{1 - \nu_s'}{1 - 2\nu_s'}\right)$

Fig. 1 shows that in both cases ρ increases with time from some immediate (undrained) response ρ_u to some final (drained) response ρ_d. At all times the pile with $\ell/r_o = 20$ exhibits a slightly stiffer lateral response than the more slender pile with $\ell/r_o = 40$. Poulos (1971) has previously demonstrated this trend for the immediate and final response. The drained solution for the pile with ℓ/r_o = 40 is also in good agreement with a finite element solution obtained, independently, by Randolph (1977).

Lateral displacements for both case (a) and (b) are again compared in Fig. 2, where the degree of displacement, defined as

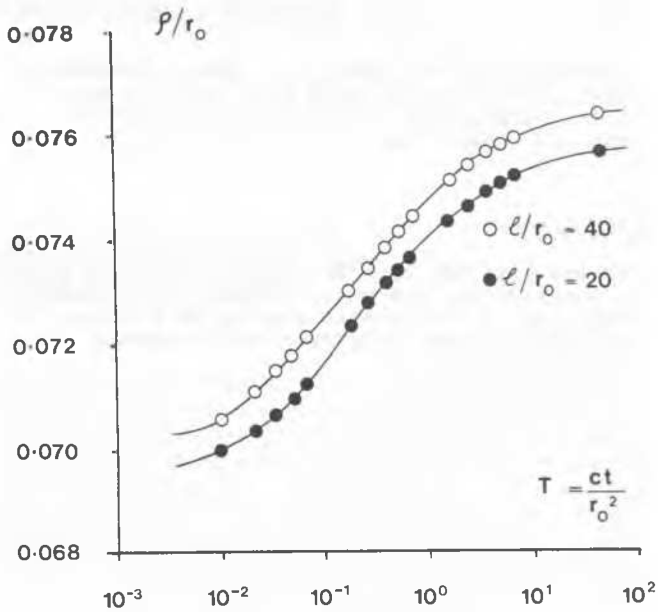

Fig.1. Lateral displacement of pile head v time; $E_p/G_s = 10^3$, $P_h/G_s r_o^2 = 1$, $\nu_s' = 0.4$

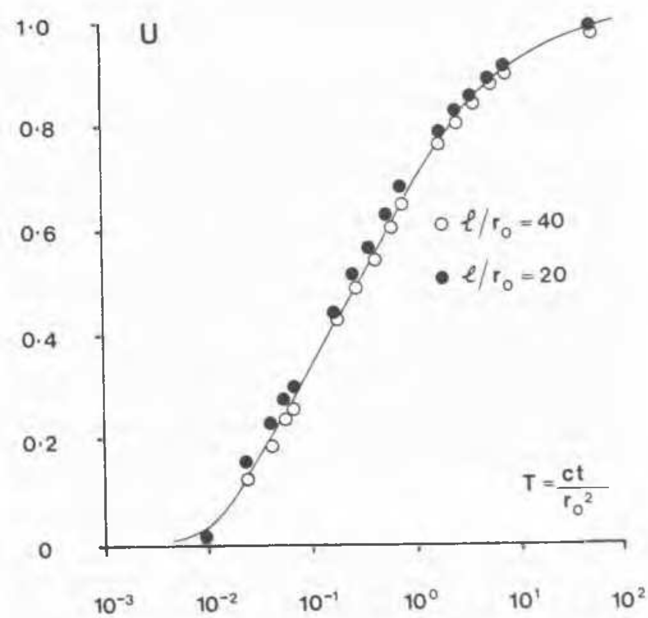

Fig.2. Degree of lateral displacement v time; $E_p/G_s = 10^3$, $P_h/G_s r_o^2 = 1$, $\nu_s' = 0.4$

$$U = \frac{\rho - \rho_u}{\rho_d - \rho_u} \qquad (12)$$

is plotted as the ordinate. When presented in this form it may be seen that there is very little difference in the time response for both of these piles.

CONCLUSIONS

An analysis has been presented which will allow a solution for the consolidation of an elastic soil due to the lateral loading of a pile. The method has been illustrated with several examples.

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