Nonlinear Finite Element Analysis of Piles in Cohesionless Soils

Analyse aux Elements Finis de Pieux en Terrains Incohérents

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SYNOPSIS A finite element procedure is presented to simulate the load transfer mechanism between pile and soil for cohesionless soils. The procedure is based upon isoparametric elements non-rigidly connected to their corner nodes in order to model the nonlinear transfer function of the soil-pile interface. An experimentally defined transfer function is shown, together with the formulation of the special element and the solution algorithm. Finally the results of a numerical application are discussed.

INTRODUCTION

It is substantially admitted that, in cohesionless soils, the load transfer through the lateral surface of a pile is due to relative displacements between soil and pile. The phenomenon can be analitically described by a nonlinear function, said transfer function.
However, the analytical simulation of complex structures is impossible and usually the problem is solved by means of numerical procedures, such as finite element techniques, in which the transfer function itself is taken into account. To model the above described situations some standard finite element codes provide "gap-elements" of suitably defined properties. This procedure leads, in general, to good results but, in some instances, the characteristics of the interface elements loose their physical meaning thus reducing its applicability.
An alternative finite element model is presented in this paper, based on special elements connected to their nodes by springs allowing, along one or more degrees of freedom, relative displacements to take place thus giving a more simple representation of the physical phenomenon of the load transfer from pile to soil.
A first application of such an element was presented in a previous paper (Dalerci and Del Grosso, 1977) where, by means of a linear two-dimensional model, the experimental determination of the transfer function for r.c. piles in sand was reproduced.
In the present paper, an isoparametric element for axisymmetric problems with nonlinear nodal springs is developed.
The transfer function to be simulated by the nodal springs will be discussed first.

TRANSFER FUNCTION

The experimental evidence shows that, in cohesionless soils, the load transfer mechanism is governed by the relative displacement between soil and pile. In a first stage the equilibrium is maintained by the lateral stresses roused by the sliding and, as the load increases, the load transfer involves more and more the bottom end of the pile.
From the literature, one can find many formulas proposed to represent the above described phenomenon. Among these formulas, the more realistic seems to be the one given by Kezdi (1957), a modification of which was already presented by the Authors (1977) as a result of an interpretation of the experiments carried out by Bocci (1964). This formula can be written as :

$$\frac{\tau}{\sigma} = tg\delta_{lim} \frac{d}{d+d^*} \qquad (1)$$

where :

d is the relative displacement between soil and pile

$tg\delta_{lim}$ is the soil-pile friction coefficient for $d \to \infty$

d* is the ratio $tg\,\delta_{lim}/tg\alpha$, being $tg\alpha$ the initial tangent to the curve.

The degree of approximation of this law is highly dependent from the evaluation of $tg\,\delta_{lim}$, usually different from $tg\delta_r$, δ_r being the maximum angle of friction between soil and pile. By introducing a mobilization coefficient $R_f = tg\delta_r/tg\delta_{lim}$, Eq. (1) becomes :

$$\frac{\tau}{\sigma} = \frac{tg\delta_r}{R_f} \cdot \frac{d}{d+d^*} \qquad (2)$$

It may be observed that the value of R_f ranges between 0.8 and 1.

ELEMENT FORMULATION AND SOLUTION ALGORITHM

The stiffness matrix of elements non-rigidly connected to the nodes along one or more degrees of freedom can be derived according to a general procedure (Del Grosso, 1974) that will be briefly recalled here for clarity.
Being $\{\phi\}$ the vector of the relative nodal displacements, the components of which will be non-zero only for those d.o.f. non-rigidly connected, and $\{f\}$ the nodal force vector, one can write :

$$\{\phi\} = [S] \; \{f\} \tag{3}$$

The matrix $[S]$ represents the set of nodal spring constants and, for the case at hand, will be derived from the interface transfer function. The equilibrium relationship for the element is therefore :

$$[k] \; (\{\Delta\} - \{\phi\}) = \{f\} \tag{4}$$

where $[k]$ is the stiffness matrix and $\{\Delta\}$ the vector of the absolute nodal displacements. By substitution of Eq. (3) into Eq. (4) and after some algebra one gets :

$$([S][k] + [I])^{-1} [k] \{\Delta\} = \{f\} \tag{5}$$

For the development of the present research, the procedure was applied to axisymmetric rectangular elements with non-rigid connections along the vertical direction on one or both the external nodes. The original elements used were isoparametric elements of the MICS (Mixed Interpolation Collapsable Sides) type, restricted to linear shape functions. As the stiffness matrix is obtained by numerical integration, the procedure (5) can be implemented in a very straightforward manner.
Once the solution is reached for the complete model, the relative nodal displacements and the internal displacement field of each element of this kind are easily obtained.
The terms of the $[S]$ matrix are all zero except the diagonal ones corresponding to non-rigidly connected d.o.f. Calling d_i, τ_i the relative displacement and the shear stress at the node i on the soil-pile interface, respectively, the generic S^e_i for the element e incident upon the node i of the interface is given by :

$$S^e_i = \frac{d_i}{\pi x_i \, \ell^e \tau_i} \tag{6}$$

where x_i is the abscissa of the node i and ℓ^e is the length of the side of the element coincident with the interface.
Because of the nonlinearity of the transfer function a solution algorithm was used as described by the following phases :

I) the total load to be applied at the pile top is subdivided into a number of equal increments; the solutions of each increment are accumulated in order to get the final result.

II) within the n-th load increment the solution is searched iteratively. A first approximation of the complete nodal displacement vector $\{\bar{\Delta}_n\}^0$ is computed by evaluating all the $[S]$ matrices from the tangents to the transfer functions calculated for the solution $\{\bar{\Delta}_{n-1}\}$; that is, with reference to Fig. 1 and to Eq. (2) :

$$(S^e_i)^0_n = \frac{[(d_i)_{n-1} + d^*]^2 R_f}{\pi x_i \ell^e (\sigma_i)_{n-1} \; tg\delta_r d^*} \tag{7}$$

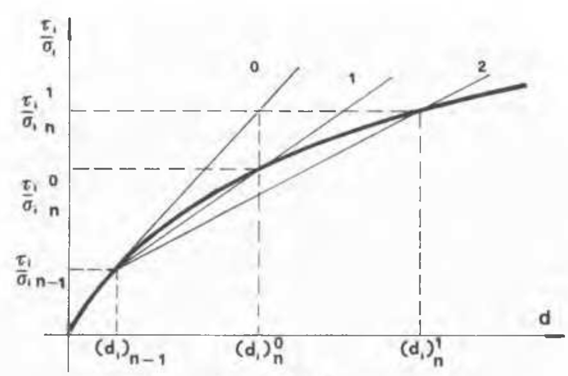

Fig. 1

The relative displacements $(w_i)^0_n$ corresponding to the first tentative solution allow the evaluation of the $[S]$ matrices from the secant (label 1 in Fig.1); that is :

$$(S^e_i)^1_n = \frac{(d_i)^0_n - (d_i)_{n-1}}{\pi x_i \ell^e \left\{(\sigma_i)^0_n \dfrac{tg\delta_r (d_i)^0_n}{[(d_i)^0_n + d^*]R_f} - (\tau_i)_{n-1}\right\}} \tag{8}$$

From these values a new approximation $\{\bar{\Delta}_n\}^1$ is found, than a new secant (label 2 in Fig. 1) is computed and so on up to convergence. This latter can be recognized with the criterion :

$$\max_i \left| \frac{(\tau_i)^{p+1}_n - (\tau_i)^p_n}{(\tau_i)^p_n} \right| \leq \nu \tag{9}$$

being ν a given tolerance.
For the first load increment $\{\bar{\Delta}_0\}$ is zero and the stresses $(\sigma_i)_0$ are taken as the geostatic values. It can be observed that, although each iteration requires a complete reanalysis, the con-

vergence is very fast also for large increments and the total cost of computation can be comparable to the one corresponding to a simple incremental solution.

NUMERICAL EXAMPLE

The validity of the procedure was tested on the following example. A finite element model was studied, representing a r.c. pile of 0.5 m diameter and 16.17 m lenght, together with a soil cylinder having a diameter equal to 36 times the diameter of the pile and a total height equal to 1.5 times the height of the pile.

For the r.c. elements it was assumed E = 20.GPa and $\nu = 0.2$. The behaviour of the soil ($\gamma = 18$ kN/m³; $\phi' = 32°$; c'= 0) was supposed to be linear and the elastic characteristics of the soil elements were derived from the hyperbolic laws :

$$E_t = (1-\lambda_1)^2 \ E_i \tag{10}$$

$$\nu_t = \frac{G-F\log\left(\frac{\sigma_3'}{P_r}\right)}{\left[1-\frac{(\sigma_1'-\sigma_3')}{E_i(1-\lambda_1)}\right]^m} \tag{11}$$

where :

$$E_i = k'P_r\left(\frac{\sigma_3'}{P_r}\right)^n$$

$$\lambda_1 = \frac{R_f(\sigma_1'-\sigma_3')(1-\sin\phi')}{2c'\cos\phi'+2\sigma_3\sin\phi'}$$

being P_r the reference pressure (100 kPa), σ_1' and σ_3 the max and min principal stresses, respectively, and :

G = 0.54   F = 0.24   m = 4
k'= 1500   n = 0.6 R_f= 0.9

Being the soil linear, σ_1' and σ_3 were taken as the geostatic values.

For the computation of the transfer functions it was assumed :

$d^* = 0.0018$ m; $R_f = 0.85$; $tg\delta_r = 0.625$

The effective geostatic stresses were determined by applying to the nodes of a model consisting of the soil elements only, a load distribution equivalent to the dead weight of the soil(case of a bored pile). From this preliminary analysis it was observed that the ratio $k = \sigma_h / \sigma_v$ decreases with depth and, from a value of 1.1 close to the soil surface, reaches a constant value of 0.4 at a depth of 20 diameters. Figs. 2,3,4 and 5, show the results of the test case.

A general comment to the curves shown in the figures is that the behaviour of the pile can be

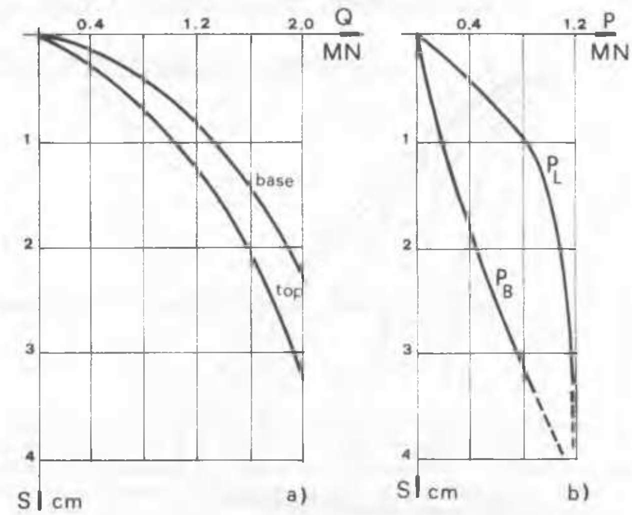

Fig. 2 - Load-settlement curves
a) applied external load
b) lateral and base loads

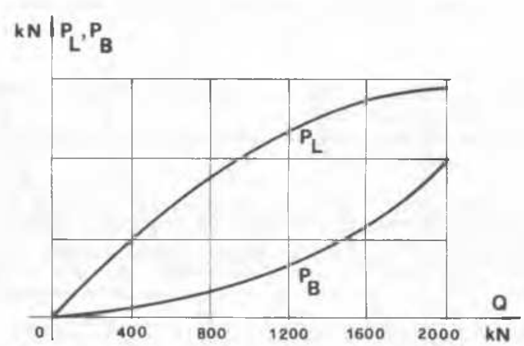

Fig. 3 - Load distribution between lateral surface and bottom of the pile

unrealistic, after the lateral surface has developed its complete bearing capacity, due to the hypothesis of linearly elastic soil.

Up to moderate loads, however, the behaviour is in very good agreement with the experimental knowledge. It should be pointed out that the range of moderate loads is the more interesting in view of technical applications.

Obviously the behaviour of the pile near to the ultimate load can be studied only by introducing a nonlinear soil model. Of course, the computational effort will be, in this case, substantially encreased.

Nevertheless, the plots of Figs. 2,3,4 and 5 are sufficiently clear and representative of the load transfer mechanism already described. Finally it can be noticed that driven piles can be analyzed by introducing a nonlinear model for the soil. The initial state of stress, indeed, is different from the geostatic one and can be computed by taking into account the large deformations due to the penetration of the pile.

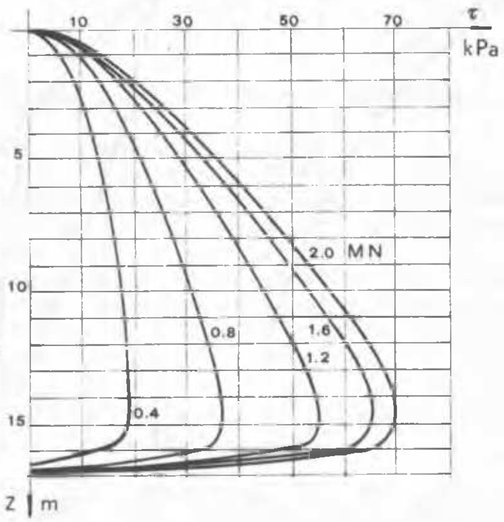

Fig. 4
Shear stresses at soil-pile interface

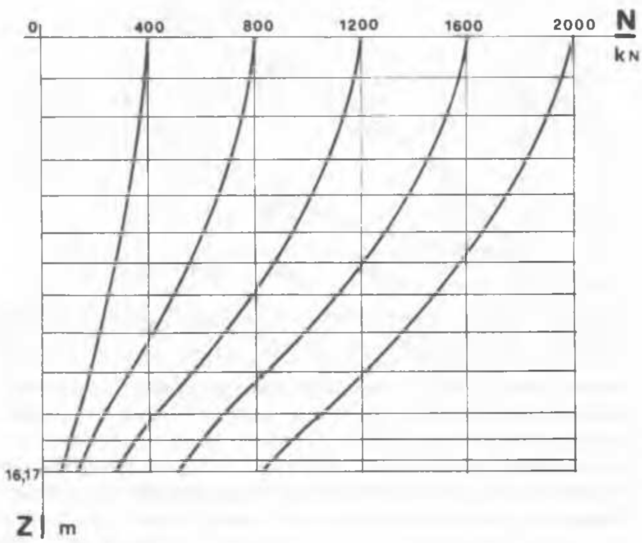

Fig. 5
Axial force in the pile

The program was run on an IBM 370/158, taking approximately 45 minutes of total computer time for the complete analysis (five load increments), and having the model 420 degrees of freedom, 167 standard elements and 13 special elements.
Within each load increment, the convergence was always reached in the maximum limit of three iterations.

CONCLUSIONS

In the paper, a finite element technique was presented to simulate the load transfer mechanis between pile and soil in the case of cohesionles soils.
The technique is based on special isoparametric axysimmetric elements connected to the corner nodes by nonlinear springs. The nodal spring properties are derived from an experimentally defined transfer function.
The numerical application, carried out on a testcase represented by a bored pile in sand, has shown the validity of the procedure within the limits of the hypothesis of linearly elastic soil. In particular, the results seem to be in very good agreement with the experimental knowledge until the lateral surface of the pile has developed its full bearing capacity.
To investigate the behaviour of the pile near to the ultimate load a nonlinear soil model should be introduced.

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