Stresses and Settlements due to Pile Groups

Contraintes et Tassements dus à des Groupes de Pieux

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SYNOPSIS A method to compute vertical stresses due to pile groups is presented. The method uses Geddes's integration of the Mindlin equation and applies it in a systematic way. A comparison with approximate methods that imply a similar amount of manual work shows that these can give misleading results. Application of the method to an actual case in Mexico City showed that Geddes's hypotheses may be used, and that computed results are in good agreement with the observed facts.

INTRODUCTION

Theoretically it is possible to determine the value of the vertical stress increment applied in any point of an elastic semi-infinite mass by a group of friction piles. Mindlin's (1936) solution to the problem of a vertical force acting inside the elastic mass is the tool with which computations can be made.

In practice it is difficult to make routine use of a quite involved expression as Mindlin's. Various early and recent efforts in connection with this problem shuned the use of the available mathematical tools, and instead, simplified approaches were tried.

Whitaker (1976), assumed the total load acting on the group applied at a depth of 2/3L, over an area equal to that of the foundation. He also assumed that the soil and the piles above 2/3L behave as an incompressible pier. Only the soil below the plane of application of the load is compressed, according to the assumptions, and the stress increments are confined within the frustrum of a pyramid with a 30° slope in its sides.

Terzaghi and Peck (1967) pioneered the notion that the soil above 2/3L depth does not compress and considered the load applied by an equivalent raft, below which stresses are spread according to a Boussinesq case.

Other workers attempted to add such factors as pile spacing, soil type, etc. but always using pyramidal or Boussinesq equivalences (Dunham, 1962; Tomlinson, 1963; Zeevaert, 1972; Girault, 1972; etc.).

Meanwhile attempts were being made to avoid the use of such approximations and Hooper (1973) and Poulos (1968) tried with success the finite element method and a superposition of deformations, respectively. More recently Auvinet and Díaz (1977) have presented a computer program to grapple with the Mindlin equation and León and Reséndiz (1979) a sophisticated Mindlin-to-Boussinesq method.

Geddes (1966) achieved the integration of the Mindlin equation for three cases: end bearing, uniform skin friction and linear variation of skin friction. The stress increments due to a single pile are expressed by a very simple equation:

$$\Delta\sigma_z = K_z \, P/L^2 \qquad (1)$$

where P is the load carried by the pile, L its length and K_z an adimensional influence coefficient, function only of Poisson's ratio, r and z (Fig. 1).

Unfortunately the algebraic expression of K_z is even more involved than Mindlin's equation. To sidestep this inconvenience Geddes provided tables of values for K_z, where the arguments are ν, $m = z/L$ and $n = r/L$.

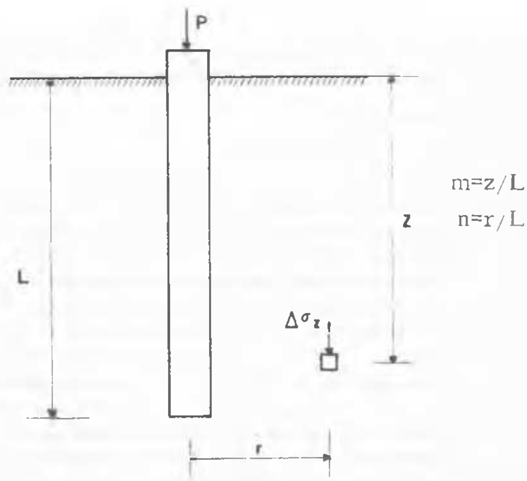

Fig. 1 Action of one pile on the adjacent soil

METHOD

Once the ν value has been determined or chosen, the only variables in the Mindlin or Geddes equations are z and r - or m and n. If as usual it is desired to determine $\Delta \sigma_z$ for a given depth or a series of fixed depths, the only variable is r.

The obvious procedure to apply Geddes's solution to a group of piles, pile by pile, is too laborious, especially in large groups. Instead a short-cut way to obtain the K_z* of the group starts with a plan of the group drawn at scale. Then a point or axis is fixed, where the increments $\Delta \sigma_z$ are to be investigated.

With the point or the trace of the axis as center, several concentric circles are drawn; such that the radius of each represents the average distance of a number of piles to the center. It is desirable that for piles close to the center a few of them are assigned to each circle, in order to minimize the effects of dispersion; but since the farther the piles lay the lesser their influence in the K_z* value, the outward circles can represent a large number of piles, even if these are not very close to the circumferences (Fig.3).

A table is then prepared with the radius of each circle and the number of piles assigned to it. The n values are also tabulated and the K_z's read off or interpolated from Geddes's tables. The number of piles times K_z gives the influence of each circle, and adding up these values the K_z* of the group results.

APPLICATION

In Mexico City, where thick deposits of essentially the same clay exist, Geddes's assumption of a linear skin friction variation does not entail serious difficulties. This can be shown using empirical results from instrumented piles, for example some of Reséndiz's (1964) tests.

The load carried by the lower end of a pile with linear skin friction variation, from the tip to depth z is:

$$P_z = \frac{p\,a}{2\,L}\,(L^2 - z^2) \qquad (2)$$

where p is the perimeter of the section, L the total length and a the maximum adhesion, occurring at the tip. This equation describes a parabola; and in the case of the actual field test for one of Reséndiz piles the values of the parameters are: p = 1.1 m, - - L = 22.5 m and a = 8.24 ton/m^2 at failure.

Fig. 2 shows the field curve of load distribution, obtained with an instrumented pile. Superimposed is the parabola that results when the parameters of the pile are inserted in eq. (2). Fit is excellent, showing that the linear skin friction assumption is valid in this case.

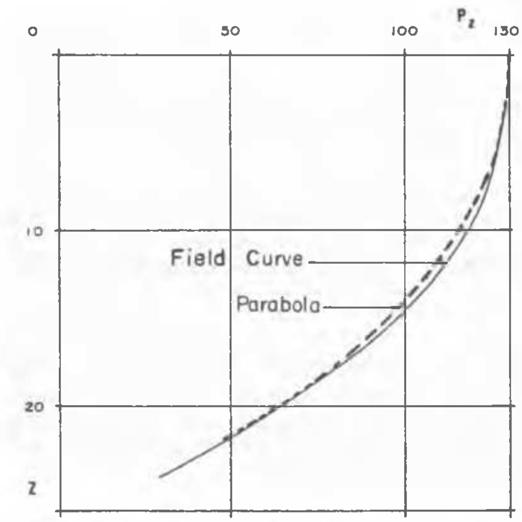

Fig. 2 Fit between field curve and parabola

Case History

It is a ten story high building with a total mean pressure at foundation mat level of 10 ton/m^2. Stratigraphical conditions are depicted in Fig.4 b). There is a 3.1 ton/m^2 compensation due to a 2 m deep box foundation structure. The remaining 6.9 ton/m^2 were assigned to a 21 pile group of friction piles. Each element was 21 m long, 50 cm in diameter and designed to carry an ultimate 70 ton load.

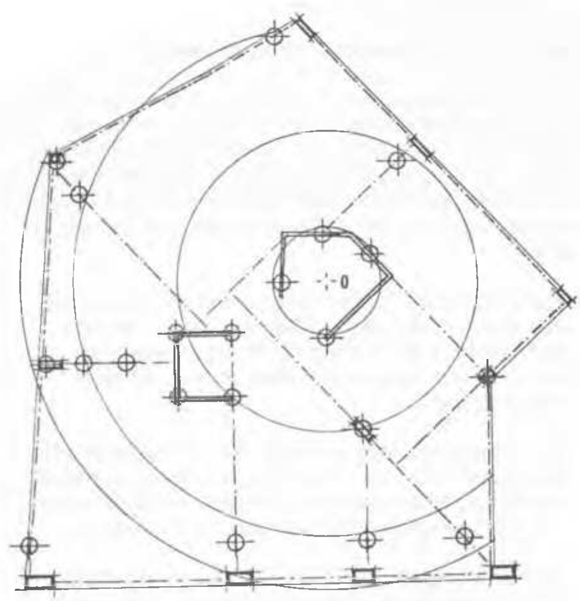

Fig. 3 Plan of foundation and friction piles

For stress increment calculations, it was assumed that the excavation floor defines the upper soil boundary. Point O in Fig.3 was selected for determination of stress increments with depth.

TABLE I

Circle Characteristics

Circle No.	Radius r	No. of Piles	n=r/L
-	m	-	-
1	1.75	4	0.083
2	4.55	7	0.216
3	8.55	6	0.407
4	10.05	4	0.479

Each circle was treated as a single pile and the influence value interpolated from Geddes's table.

TABLE II

K_z Values for ν = 0.3 and Linear Skin Friction

m \ n	0.083	0.216	0.407	0.479
0.2	-0.440	-0.038	0.033	0.032
0.4	-0.654	-0.090	0.075	0.079
0.6	-0.481	0.032	0.139	0.131
0.8	0.357	0.379	0.219	0.183
1.0	2.342	0.678	0.270	0.216
1.2	1.157	0.650	0.297	0.237
1.4	0.552	0.438	0.273	0.228

To determine K_z* at any of the chosen depths on axis O, each influence value in a row is multiplied by the number of piles in the respective circle and then all added up. K_z* is inserted in equation (1) and the value of the stress increment obtained.

TABLE III

Stress Increments vs. Depth*

Depth	Stress Increment
m	ton/m^2
6.2	-0.27
10.4	-0.39
14.6	-0.05
18.8	0.97
23.0	2.63
27.2	1.89
31.4	1.24

* Relative to ground level

These results are shown in Fig.4 a), where it can be seen that up to a depth of 15 m there are no positive increments of stress, but instead small extensions. If one considers that 2/3L lies at depth 16 m, the early assumption of an equivalent pier or raft with no compression of the soil above 2/3L is valid for this case. On the other hand the stress distribution below 2/3L is quite different as one compares the breast shaped curve of Fig.4 a) with what results from Whitaker's assumptions, for instance.

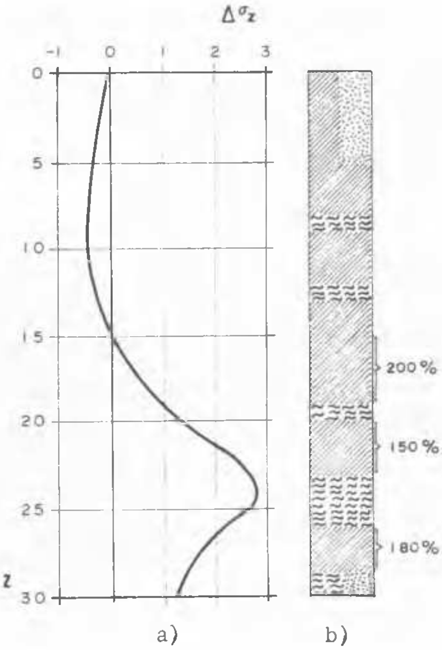

Fig. 4 a) Stress increment in ton/m^2 vs. depth
b) Schematic stratigraphical conditions

Time-settlement curves of the early structure's life are available and the average curve for the three nearest observed points to axis O is shown in Fig.5. Based on this curve it is estimated that total settlement due to primary consolidation came to 10 cm.

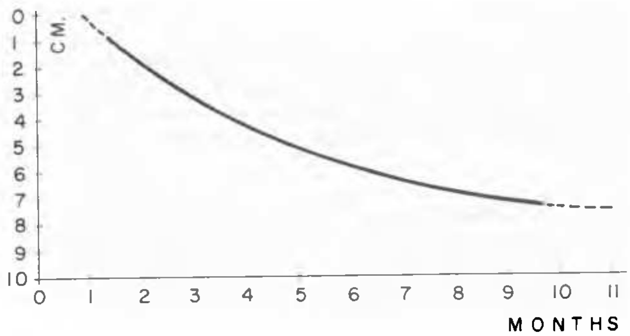

Fig. 5 Average time-settlement curve

Below 15 m depth the compressible layers that can be considered as contributing to the settlement are the following:

From 15 to 19 m a 200% natural water content layer with an average stress increment of 0.5 ton/m^2.
From 20 to 23 m a 150% water content layer, suffer-

ing a net stress increment of 2.1 ton/m^2. From 26 to 28.5 m the water content is 180% and the stress is at 1.9 ton/m^2. An allowance is made for the compression of non clay soils and for the existence of compressible soils below the maximum explored depth; so it is assumed that the contribution to settlement of the three described layers is of only - - 8.5 cm.

The average modulus of volume reduction, $\bar{m}_v$, is then computed from the following equation

$$8.5 = \bar{m}_v(400x0.05 + 300x0.21 + 250x0.19)$$

$$\bar{m}_v = 0.065 \text{ cm}^2/\text{kg}$$

Marsal (1959) has presented regression curves for the mechanical properties of Mexico City clays with the water content as argument. For the zone where the building exists, Marsal's values of the initial void ratio e_o vary between 4 and 5. There are also values of the compressibility coefficient a_v in the recompression, preconsolidation and virgin ranges. In this case the applicable range is the recompression one, due to the fact that water pumping from deep wells has ceased in the central urban zone of the city and some preconsolidation has remained. For the water content ranges encountered, Marsal's a_v fluctuates between 0.25 and 0.45 cm^2/kg in the average. For this particular case, using the abovementioned values of $\bar{m}_v$ and e_o, a_v is determined to fall between 0.33 and 0.39, thus producing a very good agreement between the observed and expected values.

CONCLUSSIONS

1.- A method to use Geddes's tables in a systematic way to analyze groups of piles has been presented, which produces accurate results with a very reasonable work load.

2.- The sress magnitude and distribution that result applying the method are quite different from what is obtained with the use of approximate methods, at least at depths close to the tips, both above and below them.

3.- Since it is very simple to compute stresses with the available tools, it seems unnecesary to continue using approximate methods. Besides there are Geddes's tables for point loads, making it possible to analyze cases where combinations of end bearing with positive or negative skin friction exist.

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