# A Simplified Analysis of Piles with Lateral Loads

## Analyse Simplifié des Pieux avec des Charges Latérales

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SYNOPSIS . A simplified method to resolve the problems originated by horizon tal static and dynamic loads on piles are described. The purpose of this method is estimate the horizontal displacement at the head and the maximum bending momend in a simple pile or in a pile group. Basically, the analytical model used is a prismatic beam fixed at an un know depth. Several rules for determine this point are recommended.

INTRODUCTION

This present study tries to solve the problem of the practical calculus of piled foundations when they are subjected to horizontal loads situated on the surface of the soil or above it. That is to say, an endeavour has been made to dispose of a simple system of analysis which makes it possible to determine two of the most significant values that define the behaviour of a pile in the practical design: the horizontal displacement at the head and its maximum bending moment.

With this purpose a simplified method was developed to resolve the problem originated by horizontal static loads, in service state (far from failure). The mentioned method has been widely used in Spain for the design of single piles and pile groups under static loads. Recently, the above method has been elaborated to resolve the case of dynamic loads. In this paper both cases are presented together with its most recent adjustments.

This simplified method is based on an analytical system which is described in the following pages.

BASIS OF THE PROPOSED SIMPLIFIED METHOD

Of the methods of calculus currently used to determine the stresses and deformations in a pile group subjected to horizontal static or dynamic loads, those based on the elastic theories are the ones -with all its limitations- that result more correct in order to analyze pile groups. Therefore, in this study it has been supposed that the soil is an elastic, homogeneous and isotropic half-space.

Due to the fact that the static problem is a particular case of the dynamic problem, only the dynamic question will be considered.

The study of the free length of the pile (fig. 1a, $\overline{AB}$) has been substituted by that of the prismatic and elastic beam, in which the bending moment, $M_f$, is a function of the coordinate z, of the external actions H and M, of the inertia forces, Q, and of the loads of structural damping, R.

The pile (fig. 1.b) -considered as a prismatic beam- is found subjected to the exterior loads and to the reactions of the soil ($\sigma$), while the soil is submitted by these same reactions with inverse signs. The effects of inertia and damping of the soil-pile system have been represented by means of additional forces in each segment of the pile: Q and R respectively. The problem is resolved by supposing that the horizontal displacements of the soil are the same as the ones of the pile.

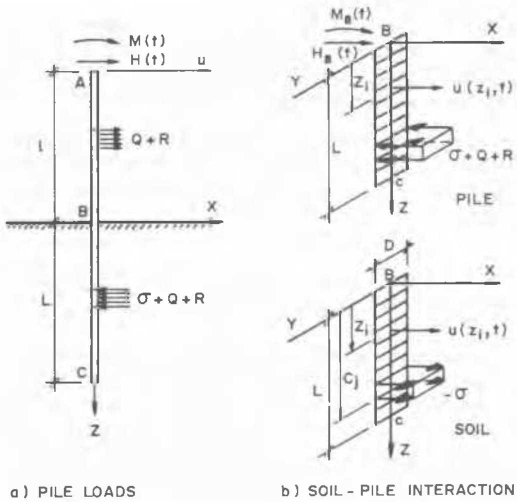


Fig. 1. - Theoretical analysis

$$U_{pile} = \int_o^z \left( \int_o^z \frac{M_f}{E_p I_p} dz \right) + \theta_B z + u_B$$

$$U_{soil} = \int_o^L dc \int_{-D/2}^{D/2} \sigma(c) . K(y, z, c) dy \qquad (1)$$

$$M_f = \phi ( \sigma, R, Q)$$

To find the horizontal displacements of the soil it is necessary to integrate the MINDLIN solution into the middle plane of the pile. This solution corresponds to the case of concentrated force on an elastic half-space and parallel to the surface of the half-space and in the kernell K of the second of the above equations is introduced; $E_p \cdot I_p$ is the bending stiffness of the pile. Also, is necessary to considere the equilibrium and compatibility conditions.

With the intention of arriving to useful solutions for practice, and since an arbitrary function can be expressed as the sum of harmonic functions, it has been supposed that the external loads applied are of senoidal form (dynamic case). It is necessary for to resort the above equations to use numerical methods (Oteo, 1972; Valerio, 1978).

The analysis of a pile group introduces, regarding the study of the single pile, two problems: the "group effect" and the one of "restriction" which the heads of the piles suffer when united by a cap.

The analyze the "group effect" the technique used has been to undertake the study of any group from the principle of superposition and according to the results obtained in the resolution of the six elemental cases: transversals, longitudinals and diagonals, depending on the position of the piles, symmetrical and antisymmetrical, accoding to the position of the horizontal loads. (Oteo, 1972; Valerio, 1978).

Once the theoretical problem is solved in the indicate way, with the help of a computer, it is possible to calculate any desired case. But to simplify, the method described in the following pages has been developed. Essentially, it is based on substituting the pile or pile groups for an equivalent prismatic beam, fixed at an unknown depth. This beam is subjected at the head to the same loads as the pile, and it has the same bending stiffness. The depth of the fixed point is determined in such a way that the beam and the pile embedded at the elastic half-space have equal deformations at the head.

With this system bending moments and displacements can be calculated in a rapid way, only by using formulations derived from the Strength of Materials, as shown as follows.

## SIMPLIFIED METHOD FOR THE ANALYSIS OF PILES UNDER HORIZONTAL STATIC LOADS

As previously indicated, the single pile is similar to a beam or prismatic beam, of analogous mechanical characteristics, fixed at a certain depth under the surface of the soil (fig. 2). To determine the position of the ficticious fixed point the criterion of equivalence in deformation between the pile and the beam has been followed.

For homogeneous soil, namely, that in which the modulus of deformationof the soil, $E_s$, is constant with the depth (the same as the transversal modulus, G), it is possible to take (Oteo, 1973)

$$L' = 1{,}2\ L_e = 1{,}2\ \sqrt[4]{\frac{E_p \cdot I_p}{G}} \qquad (2)$$

where $L_e$ is the elastic length of the soil-pile system. This case corresponds to non-overc onsolidated cohesive materials.

In the case of non-homogeneous soil, the value of $L'$ has been determined, based on the theoretical solutions developed by Banerjee and Davies (1978). Defining the ground by the variation of the modulus of deformation with the depth: $E = E_o + m\ Z$, in which $E_o$ is the modulus on the surface and m the gradient of variation.

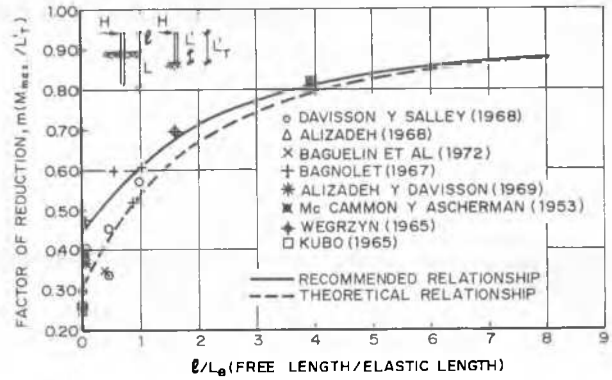


Fig 2. - Reduction factor of the maximum bending moment

To simplify, it is possible to take the parameter "degree of heterogeneity", X, relation between $E_o$ and the modulus at the head of the pile, $E_L$, as reference:

$$L' = 1{,}2\ f \sqrt[4]{\frac{E_p\ I_p}{E_L/3}} = 1{,}2\ f\ L_{eL} \qquad (3)$$

$$f = \begin{cases} 1.70 & (X = 0.0) \\ 1.25 & (X = 0{,}5) \\ 1{,}00 & (X = 1{,}0) \end{cases} \qquad X = \frac{E_o}{E_L}$$

If the pile has a free length, $\ell$, the total length of the equivalent beam will be: $L'_T = \ell + L'$ and the displacements, $Y_H$, and rotations, $\theta_H$, at the head, subjected to a load H and withhinged head, will be:

$$Y_H = \frac{H \cdot (L'_T)^3}{3E_p\ I_p} \quad ;\quad \theta_H = \frac{H \cdot (L'_T)^2}{2E_p\ I_p} \qquad (4)$$

The real maximum moment can be obtained supposing that it is the one that appears at the ficticious fixed point. However, it should be affected by a reduction factor, m, since the equivalence has been established in displacement and not in stresses. That is to say:

$$M_{max} = m \cdot H \cdot L'_T \quad \text{(Hinged head)} \qquad (5)$$

In the fig. 2, the value of m obtained from the theoretical solution has been represented (dotted line). In the same figure actual measurements have been represented which indicate values of m larger than the theoretical ones. Therefore another law of m similar to the theoretical one has been drawn (unbroken line), but with larger values, which is the recommended one. In this way the irregularities of the soil, its non-elasticily, etc., are tried to be taken into account.

In the fig. 3, the value of the maximum moment recommended is compared with the ones obtained according to different theories, for the case of $\ell = 0$. This maximum moment has been drawn according to the length of the pile referring to the elastic length, which is, in our opinion, the most representative of the parameters of the problem. For the usual piles in practice ($2{,}5 < L/L_e < 5$) the influence of the variation of the length is insignificant; the different theoretical values being similar enough to the theoretical ones of Oteo (1972). The recommended value comes out to be larger for the reasons already commented.

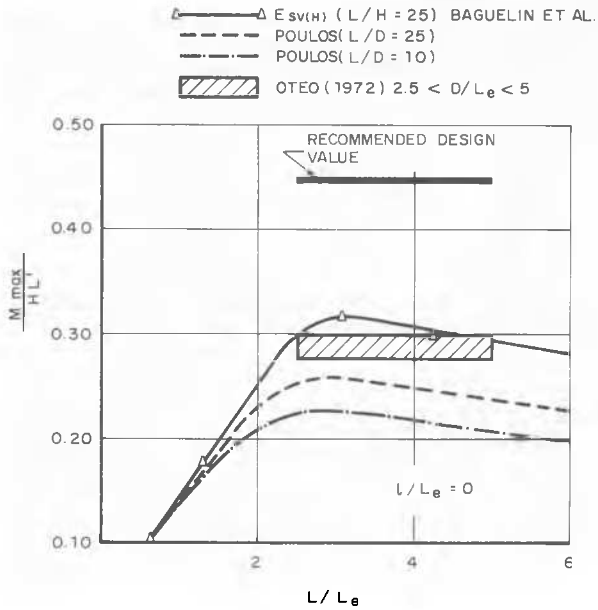


Fig. 3. - Variation of maximum bending moment with pile length

The group effect can be taken into account by a similar procedure. The pile behaves as if the equivalent ficticious fixed point were found situated at a greater depth than the one estimated for the single pile. That is:

$$L'_g = L' \times \alpha \qquad (6)$$

$L'_g$ being the depth of the equivalent fixed point for a pile group, $L'$ being the one obtained in the case of a single pile and $\alpha$ the factor of the group effect.

Considering the average characteristics of the problem ($L/L_e$ of 2,5 to 5 and $D/Le \simeq 0,3$), $\alpha$ has been determined, according to the type of group and the relative spacing between axis of the piles ($S/Le$). The parameter has been represented in fig. 4. It is possible in this way to easily determine the displacement, $Y_{Hg}$, of a pile group:

$$Y_{Hg} = \frac{H\,(\ell + L'_g)^3}{3E_p\ I_p} \quad \text{(Hinged head)} \qquad (7)$$

The maximum bending moment of a pile group can be taken as the one of a single pile, with equal load at the head, enlarged in 10%.

## SIMPLIFIED METHOD FOR DYNAMIC LOADS

In this case the aim of the approximate method is to estimate, with ease, the order of magnitude of the natural frequencies and of the amplification factors of bending moment and deformations of a pile foundation submitted to horizontal dynamic loads, with senoidal variation and maximum value $H_o$.

The calculus of the natural frequencies can be done comparing the pile to a beam and using the formula of free lateral vibration of prismatic beams (Valerio, 1978):

$$P = \frac{\beta^2}{2\pi (L'_T)^2} \sqrt{\frac{E_p\ I_p}{M}} \qquad (8)$$

in which p is the natural frequency in c.p.s.; M is the mass per unit of length of the equivalent beam; and $\beta$ a parameter which depends on the type of restriction at the head; its value is: 1,875 for "hinged heads with translation", and 2,365 for "fixed heads with translation". As the value of the total length of the equivalent beam $L'_T = \ell + L'_d$ will be taken, $L'_d$ being the depth of the equivalent fixed point.

The value of $L'_d$ can be deduced from the fig. 5. In it $L'_d$ has been represented for single piles and group of four piles, with different restrictions at the head. The term "hinged head" implies liberty of rotation and horizontal displacement without limitation. The term fixed head indicates incapacity of rotation and horizontal displacement without restriction.

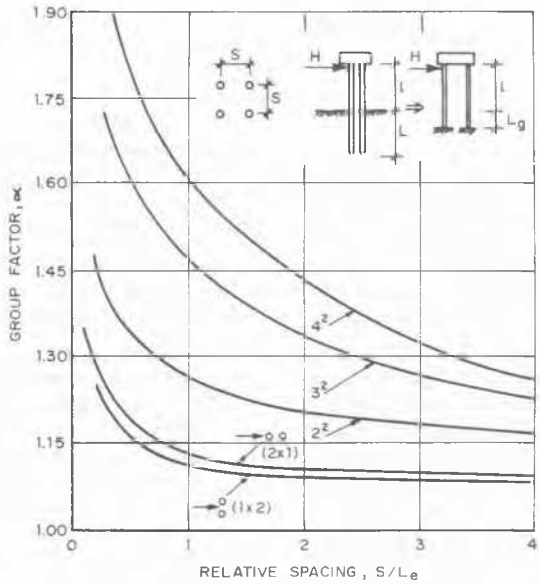


Fig. 4. - Group effect for static loads

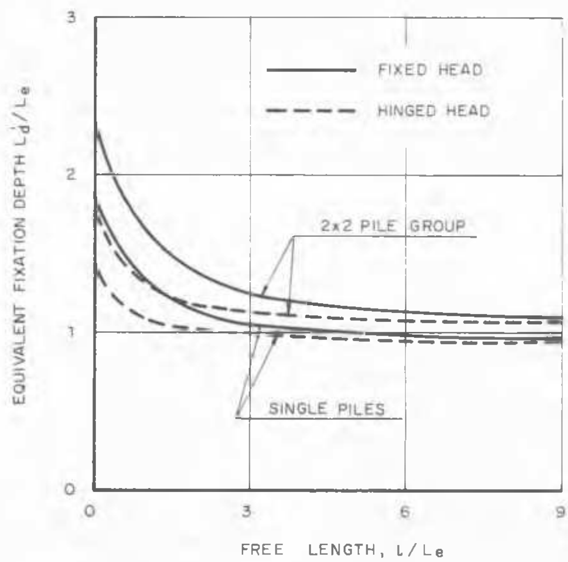


Fig. 5. - Equivalent fixation depth for the natural frecuency determination

If the foundation doesn't have free length ($\ell = 0$), then it is necessary to estimate frequencies of an order superior to the fundamental one. For that, the same formula (8) is used. However, in this case, the values of $L'_d$ must be deduced from the fig. 6, according to whether a single pile or a square group of four piles are involved.

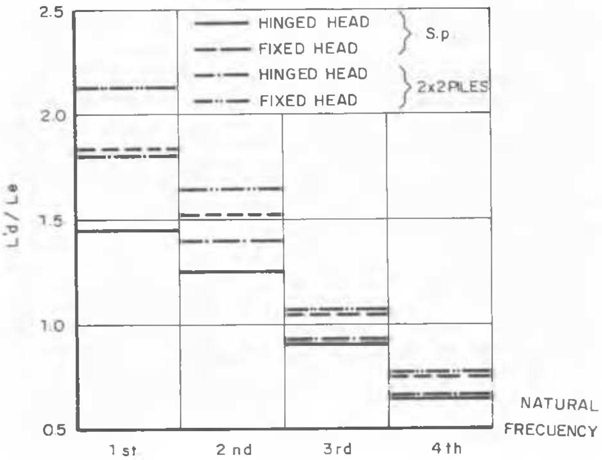


Fig. 6. - Length of equivalent beam for fully embedded piles

For estimate the maximum stresses and deformations produced in forced vibrations, also is necessary to whether it has free length or not must be taken into account.

For piles fully embedded, the approximate method is an extension of the recommendations given in the previous part of the paper for the static case: It is understood that the piles have a ficticious fixed point at a certain depth, $L_V$. But it must be taken into account that the maximum displacements at the head ($y_{H,max}$) are produced in the first or second frequency ($p_1$ or $p_2$), while the bending moments ($M_{max}$) and the rotation at the head ($\theta_{H\,max}$) are originated in the third or fourth natural ($p_3$ or $p_4$).

For the same reasons as in the static case it is necessary to correct the value of the maximum moment calculated in the ficticious fixed point. In the dynamic case the factor m is substituted by another $K_M$ that enlarges $M_{max} = H_o \cdot L'_V \cdot K_M$. This factor $K_M$ is a function of the damping factor, $\xi$ in which it is the relation between the damping of the soil-pile system and the critical of the relation $L_V/L_e$, of the type of restriction, etc. If the pile is fully embedded, the values of $K_M$ and of $L'_V/L_e$, which are shown below, can be taken.

| $\xi$ | $K_M$ | $L'_V/L_e$ |
|---|---|---|
| 0,05 | 1,85 | 2,70 |
| 0,10 | 1,29 | 1,94 |
| 0,15 | 1,05 | 1,59 |

for both single piles and groups of four piles hinged at the head. For the group with fixed head the maximum moment can be taken as 28% less than the case of the hinged head.

If the foundation for piles has free length, the pile is substituted by the beam that vibrates laterally. The amplification factor, $\eta$, of the beam, to find both the moments and the displacements, can be approximated by the one of the systems with one degree of freedom. Therefore, it can be written that:

$$\eta = \frac{1}{2\,\xi_{eq}\sqrt{1-\xi_{eq}^2}} \tag{9}$$

where $\xi_{eq}$ is the equivalent damping factor of the beam. Its value can be deduced from the figure 7. In it $\xi_{eq}$ takes the same value to find the maximum displacement at the head as to calculate the maximum bending moment (the real difference is of $\pm$ 5%).

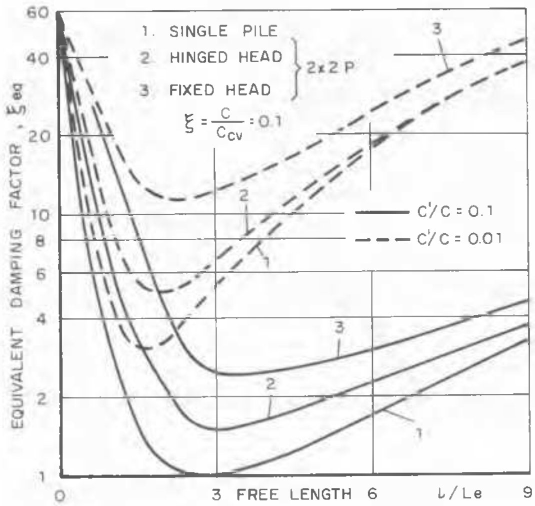


Fig. 7. - Equivalent damping factor for piled foundations

To calculate the displacements and moments of the beam, embedded at the depth $L_d$ (the same as the one that was determined when the natural frequencies were found) it is sufficient to apply the amplification factor, $\eta$, to the obtained values as in only a static load equal to $H_o$ acted.

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REFERENCES

Baguelin, F; Carayannacou-Trézos, S. and Frank, R. (1978). Reaction laterale des pieux: Effets de forme et effets tridimensionnels. Bulletin de Liaison des Lab. des Ponts et Chaussées. No. 104, pp. 33-48.

Banerjee, P.K. and Davies, T.G. (1978). The behaviour of axxially and laterally loaded single piles embedded in non homogeneous soils. Geotechnique 28, No. 3. pp. 309-26.

Oteo, C. (1972). Resistencia y deformación de los grupos de pilotes verticales con especial consideración de los esfuerzos alternativos. Tesis Doctoral. Univ. Politécnica de Madrid.

Oteo, C. (1973). Recomendaciones prácticas para el cálculo de pilotes solicitados por cargas horizontales Bol. de Inf. L.T.M.S. nº 103, pp. 3-33.

Poulos, H.G. (1971). Behavior of laterally loaded piles: I-Single piles. Journal of the Soil Mech. and Found. Div., A.S.C.E., Vol. 97, SM5, pp. 711-31.

Valerio, J. (1978). Análisis de pilotajes con longitud libre sometidos a esfuerzos dinámicos horizontales. Tesis Doctoral. Univ. Politécnica de Madrid.

Valerio, J. and Oteo, C. (1980). Dynamic behaviour of a group of free-standing piles. Int. Symp. on Soil under Cyclic and Trans. Loading. Swansea. pp. 627-638.