Dynamically Loaded Buildings on the Soil

Bâtiments Sollicités Dynamiquement sur le Sol

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SYNOPSIS   It is shown how we can calculate the effect of the soil on the vibrations of buildings economically. Examples illustrate the method.

INTRODUCTION

Buildings are loaded dynamically either by applied forces or by soil vibrations due to earthquakes, traffic or adjacent vibrating buildings. The properties of the soil affect the behaviour of the buildings. In interaction analyses, the soil is mostly modelled as an elastic body.

THEORETICAL BASIS

The analysis used here is based on solutions for the displacements due to point sources. Such solutions exist for the vertical force acting on a homogeneous half-space (Holzlöhner, 1980). The vertical displacement w is

$$w = \frac{Q}{Gr} f(\bar{r}, \nu) e^{i\omega t} \qquad (1)$$

where Q, G, and r, respectively, denote force amplitude, shear modulus and distance from the load. The factor $e^{i\omega t}$ shows that w is a periodic process with the frequency ω. The function $f(\bar{r}, \nu)$, Fig. 1, describes the behaviour of the half-space. It depends on Poisson's ratio and the parameter $\bar{r} = r \cdot \omega \sqrt{\rho / G}$ where ρ is the mass density.

Solutions for displacements due to point sources of the form (1) can be used to calculate economically the displacements due to distributed loads. By virtue of the reciprocity theorems, the displacement w_P of a surface point P due to a surface stress $q(x,y)$ acting on the area A, Fig.2, can be calculated by

$$w_P = \iint_{(A)} w_1(x,y)\, q(x,y)\, dx\, dy \qquad (2)$$

Here, w_1 is the displacement due to the force Q = 1. If point P lies within the area A, w_1 has a simple pole, see Eq. (1). Therefore, some authors (Wong and Luco, 1976, Kitamura and Sakurai, 1979) supposed difficulties with the application of point source solutions. However, if we use polar coordinates with the position of the pole as origin, the singularity vanishes (Holzlöhner, 1980), and we can easily evaluate the integral (2).

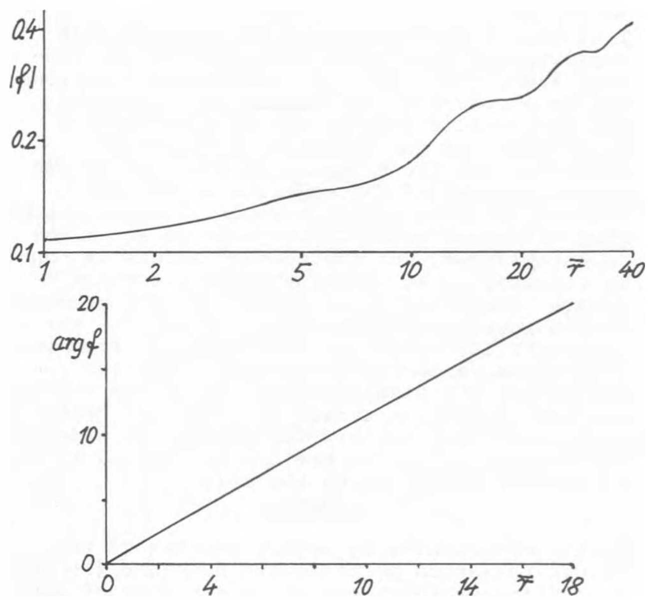

Fig. 1 Vertical surface displacement due to a vertical concentrated load, = 1/3

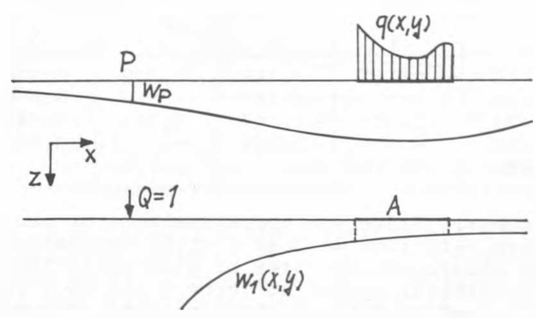

Fig. 2 Calculation of the displacement due to distributed loads by using the point source solution

The method of generating displacements due to stresses by using point source solutions and integrating Eq. (2) can also be applied to other components of displacements, to other systems and if the point source solution has been obtained by a finite element analysis or even by measurements.

INTERACTION ANALYSIS

We can calculate the displacements due to arbitrarily distributed surface stresses by Eq. (2). However, the stress variation in the contact area of vibrating buildings is unknown. It has to be determined such that the produced displacements are compatible with the compliance of the buildings. This is usually done by subdividing the contact area into small rectangular sub-regions. Within each sub-region, the stress is assumed to be constant. (Wong and Luco, 1976, Savidis and Richter, 1977). We can calculate the surface displacements due to a uniformly loaded sub-region economically by evaluating Eq. (2) for q = constant. The values of f need not be calculated each time but can be interpolated from a set of values that is calculated before. This method consumes less computer time than the direct calculation of the displacements due to a distributed load.

Till now, a piecewise constant function has approximated the stress variation on the contact area. Alternatively, continuous functions may be chosen if point source solutions are applied. Evaluating Eq. (2) by Gaussian quadrature, we can formulate a system of equations for the stresses at the integration points instead of the stresses at the sub-regions. In this paper, however, I apply a somewhat different method. No system of equations is established, but the contact stress is found by iterations. At first, a dynamic contact stress distribution is estimated. The produced displacements will generally not satisfy the compatibility condition sufficiently. From the deviations, a corrective stress field is generated. The displacements due to the original and the corrective stress will fit the compatibility condition better. The process is repeated if necessary. This method can be applied to one or several buildings on the soil.

TABLE I

Static compliances by approximations of the stress variation by piecewise constant (1) and by continuous functions (2) n = number of sub-regions or integration points, respectively.

n	C^Q 1	C^Q 2	D^M 1	D^M 2
16	0.157	0.149	0.209	0.176
36	-	0.147	-	0.170
64	0.151	0.147	0.182	0.167
100	-	0.147	-	0.168
256	0.148	-	0.171	-

To compare different approximations of the stress variation we load a rigid foundation with square contact area of side 2a in two ways: by a vertical symmetric force Q and by a rocking moment M. Table I shows the static vertical compliance $C^Q = G \cdot a \cdot W/Q$ and the static rocking compliance $D^M = G \cdot a^3 \cdot \alpha/M$ for $\nu = 1/3$, α is the angle of rotation. W and α are complex amplitudes. Especially in the case of rocking, much more sub-regions than integration points are necessary for a given degree of accuracy.

APPLICATIONS

Two massless rigid foundations of sides 2a are considered. Their center points have a distance of d from one another, see Fig. 3. Either a vertical force Q or a rocking moment M_y acts at foundation 1. Foundation 2 is not loaded directly. The load produces the vertical displacements W_1 and W_2 and the angles of rotation α_1 and α_2. The compliances are defined by

$$C_1^Q = GaW_1/Q \qquad C_2^Q = GaW_2/Q$$
$$D_1^Q = Ga^2\alpha_1/Q \qquad D_2^Q = Ga^2\alpha_2/Q \qquad (3)$$
$$C_1^M = Ga^2W_1/M \qquad C_2^M = Ga^2W_2/M$$
$$D_1^M = Ga^3\alpha_1/M \qquad D_2^M = Ga^3\alpha_2/M$$

The superscript Q or M, respectively, indicates the actual load case. In all examples, ν is equal to 1/3. The shear stresses on the contact areas have been assumed to be zero.

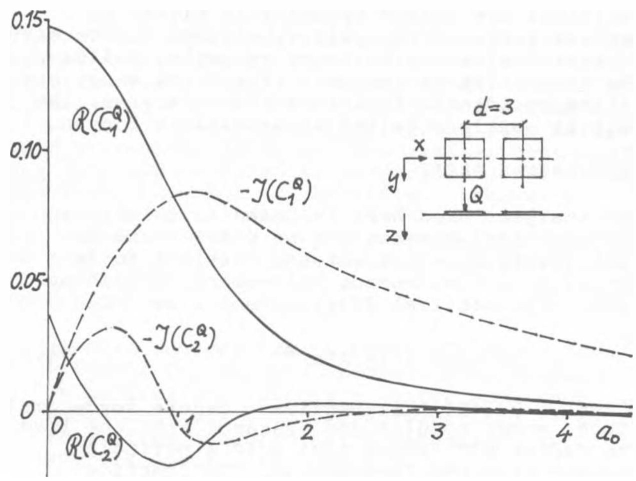

Fig. 3 Compliances of a system of two foundations, ν = 1/3.

Fig. 3 shows the compliances C_1^Q and C_2^Q as functions of $a_0 = a\,\omega\sqrt{\rho/G}$, where a = 1 and d = 3. Fig. 4 includes $|D_1^Q|$ that represents the reaction of the unloaded foundation to the loaded one. The compliance D_1^Q indicates that a building founded on the area 1 and loaded by a vertical symmetric force also moves in a rocking mode even if the mass of the adjacent rigid unloaded building is zero.

Normally, both buildings have some mass. Then, inertia forces act also in the contact area of the "unloaded" building. If the masses of the two buildings are of equal order it is reasonable to compare the quantities D_1^Q and D_2^Q as has been done in Fig. 4. The fact that $|D_1^Q|$ is only about 10 % to 20 % of $|D_2^Q|$ leads to a simplified interaction analysis which I call "analysis without reaction".

I consider now two systems: One system consists of foundation 1 alone. It is used to calculate C_1^Q and the free-field vibrations in the contact area 2. We calculate C_2^Q and D_2^Q with the second system that consists only of foundation 2 excited by the free-field motion. The compliances C_1^Q, C_2^Q and D_2^Q calculated without reaction hardly differ from the corresponding "exact" quantities. The only relevant difference shows up in D_1^Q, which is equal to zero in the analysis without reaction. As $|D_1^Q|$ is only small compared to $|D_2^Q|$ we conclude that the interaction analysis without reaction can be applied even for a small distance between the two foundations. The investigation of two foundations, one being loaded by a moment M_y, leads to the same conclusion.

We can further simplify the analysis by replacing the displacement quantities W_2 and α_2 in Eqs. (3) by averages of the free-field displacements W_f in the area A_2 of the unloaded foundation.

$$\bar{W}_2 = \frac{1}{A_2}\iint_{(A_2)} W_f\,dxdy \qquad \bar{\alpha}_2 = \frac{\iint_{(A_2)} W_f\,x\,dxdy}{\iint_{(A_2)} x^2\,dxdy} \tag{4}$$

Now, the kinematic condition of the rigid contact area 2 is no longer satisfied. I call this method "analysis by averaging". It has been suggested as a simple method in earthquake analyses (Holzlöhner, 1972, Scanlan, 1976). The same problem of two interacting foundations is treated by the analysis by averaging. Fig. 4 shows the compliances $\bar{C}_2^Q$ and $\bar{D}_2^Q$ calculated by averaging in comparison to C_2^Q and D_2^Q that have been obtained by the exact analysis. Up to $a_o = 2$, the compared quantities are similar. Beyond this, only the order of magnitude can be provided by averaging. The governing quantity appears to be the maximum phase differences of the free-field displacements within the area 2 which is about 270 degrees for $a_o = 2$. Of course, these results are valid only in the case of vibrations induced by vertical forces.

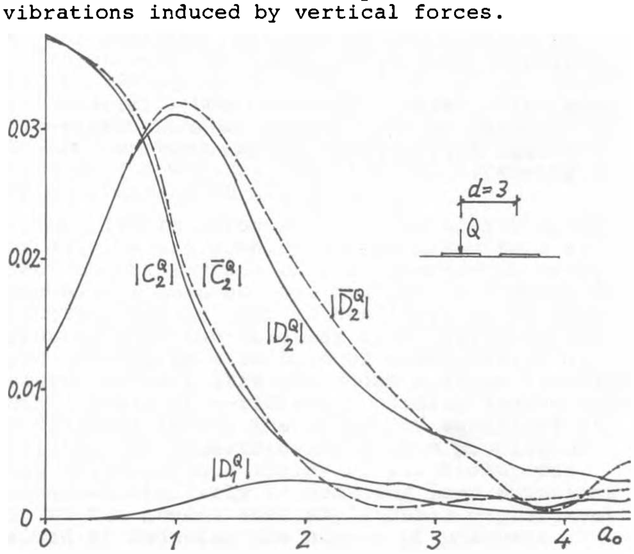

Fig. 4 Vertical (C) and rocking (D) compliances by the exact analysis (full lines) and by averaging (dashed lines), $\nu = 1/3$.

By applying the analysis by averaging, we can calculate the dependence of the compliances on the distance of the two foundation without much effort. At first, we calculate the surface vibrations due to the excited rigid foundation 1. Then, we average the displacement within several contact areas that have different distances from the excited foundation. Fig. 5 shows above the compliance $\bar{C}_2^Q$ normalized to C_1^Q for distances d = 2,3,4,5, and 6. For comparison, the normalized amplitude of the free-field displacement $^*W^Q$ is included in Fig. 5. The ratio of the absolute values $|^*C_2^Q| = |\bar{C}_2^Q|/|C_1^Q|$ remains about 20 % below the corresponding values of the free-field displacement. This is due to the phase difference within the contact area, which is about 2.5 ≙ 145° (Fig. 5). The argument of $^*C_2^Q$ nearly coincides with that of the free-field displacement in the center point of contact area 2.

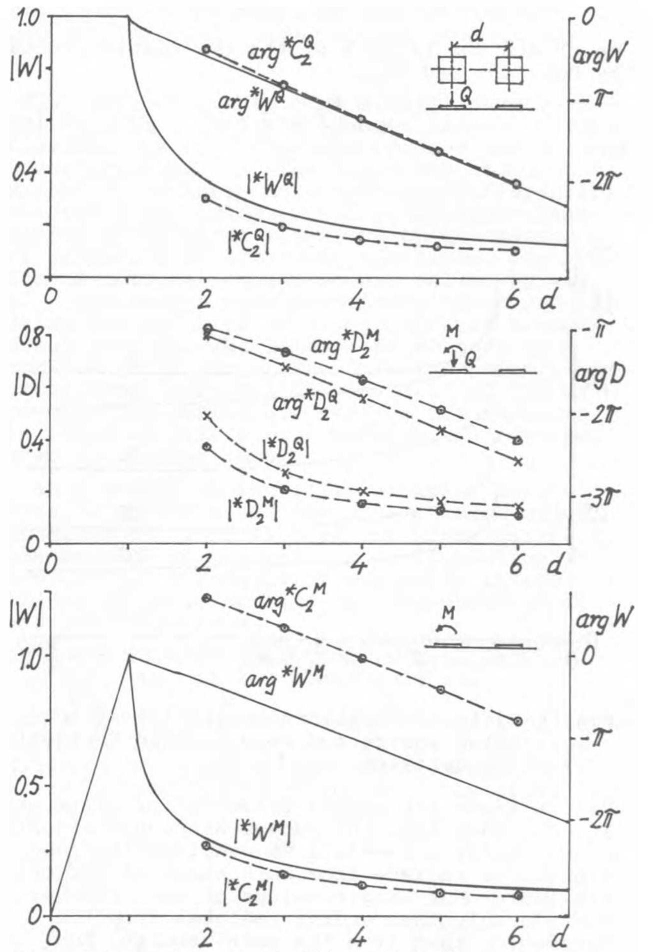

Fig. 5 Normalized compliances and corresponding free-field displacements versus distance. $\nu = 1/3$, $a_o = 1$.

Fig. 5 includes the other compliances of two interacting foundations. A direct comparison with free-field displacements is helpful only with one other compliance, namely $\bar{C}_2^M$. The presented compliances $\bar{C}_2^Q$, $\bar{D}_2^Q$, $\bar{C}_2^M$, $\bar{D}_2^M$ essentially have the same values as those of the corresponding quantities calculated by the exact analysis.

VIBRATIONS RADIATING FROM POINT SOURCES AND FROM RIGID LOADED AREAS.

An exact interaction analysis consumes much time. It would be desirable, therefore, to present obtained results such that we can solve practical problems without much additional calculation. For this scope, it is useful to compare the vibrations produced by a vertical point source to those produced by a vertically excited rigid foundation.

Eq. (1) yields

$$W_Q = \frac{Q}{G\,r}\,f\,(\bar{r}) \tag{5}$$

where W_Q is the complex displacement amplitude due to a point source. The displacement W_F produced by an excited foundation has the form

$$W_F = \frac{Q}{G\,a}\,g(\frac{r}{a},\,a_o) = \frac{Q}{G\,r}\,\frac{r}{a}\,g(\frac{r}{a},\,a_o) \tag{6}$$

We obtain the ratio k of the two displacements by Eqs. (5) and (6):

$$k = \frac{W_F}{W_Q} = \frac{g}{f}\,\frac{r}{a} \tag{7}$$

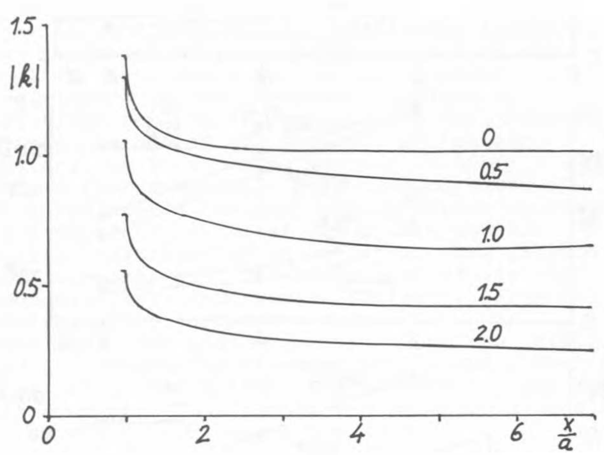

Fig. 6 Ratio of displacements radiated from a point source and from a rigid foundation for different a_o. $\nu = 1/3$.

Fig. 6 shows |k| versus $x/a \approx r/a$ for different a_o. The fact that the curves display horizontal tangents for $x/a \to \infty$ lets us conclude that the vibrations radiate from both kinds of sources similarly. For larger values of a_o , however, considerably less energy radiates from the foundation than from the point source. Eqs. (6) and (3) show that the compliances C_1^a and C_2^a are quantities of the same type as g. The values of |k| for x/a = 1, Fig. 6, are identical to $|C_1^a|$, the absolute value of the compliance of the excited foundation.

From Figs. 1 and 6, we can read the absolute value of the free-field displacement due to a vertically excited foundation. The argument is not so important. For $x/a \geq 2$ we can use arg f of Fig. 1 for arg g. The free-field displacement excites arbitrary foundations. We can calculate the produced vibrations by averaging.

Fig. 6 shows an important fact. As W_Q does not depend on the lengths of the foundation and |k| is almost constant for $x/a \geq 2$, the radiated energy of a vibrating foundation only depends on a_o. For a given force amplitude Q that enters the soil with a fixed frequency ω, we can reduce vibration radiation by increasing a, which means increasing a_o.

CONCLUSIONS

The use of point source solutions saves computer time compared to other interaction analyses. The stress variation in the contact area should rather be described by continuous functions than by piecewise constant functions. Some examples show that the interaction analysis may be simplified without much loss of accuracy. By comparing the vibrations that radiate from a point source and from a rigid foundation we can present the results in a form that is useful for applications.

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