Machine Foundations on Layered Soil Deposits

Les Fondations des Machines sur Couches du Sol

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SYNOPSIS A parametric study was carried out to investigate the significance of soil layering on the vertical vibration of circular machine foundations. The soil deposit was idealized by a two-layered system in which the bottom layer was treated as a half-space. Results of the parametric study were illustrated graphically in terms of mass ratio versus frequency ratio at maximum magnification and mass ratio versus maximum magnification for immediate use. The effects of soil layering on wave attenuation were also examined.

A simple analytic model was then developed to approximate the behavior of vertical response of machine foundations on layered soil deposits. To evaluate the method, a machine foundation was analyzed by the simplified model. The results compared favorably with those from a more expensive dynamic finite element method.

INTRODUCTION

Vertical vibrations of machine foundations resting on relatively uniform soil deposits usually can be analyzed satisfactorily by the elastic half-space solution. Soil deposits, however, are often stratified to a degree where the half-space solution is invalid. Available parameter studies on the vertical vibration of machine foundations on layered soils are limited, as they do not cover the ranges in operating frequencies or ratios of layer thicknesses to footing widths that are often encountered in practice (Luco, 1976; Kuhlemeyer, 1969; and Gazetas and Roesset, 1979). The major objectives of this study were to provide improved insight into the vertical behavior of machine foundations on layered deposits and to develop a simplified design procedure for such soil conditions.

The soil deposit for this study consisted of a relatively thin soft surface layer underlain by much stiffer materials whose properties varied with depth, Fig. 1. Such a soil deposit was idealized by a two-layered system, and a parametric study was made to determine the influence that the relative stiffness of the two layers, the ratio of the surface layer thickness to footing width, $2r_o$, and the operating frequency, ω, had on the footing response. The effects of soil layering on wave attenuation were also examined. A simple analytic model was then developed to provide a convenient and economical means to predict the vertical response characteristics of machine foundations on layered soil deposits.

ANALYSIS PROCEDURE

As has been demonstrated by previous studies and exercised by many investigators, footing response can be studied by first obtaining displacement functions for a massless footing and then solving an equation of motion with the footing mass.

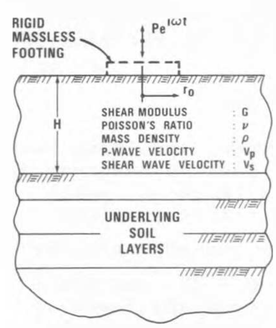

Fig. 1
Analytic Model

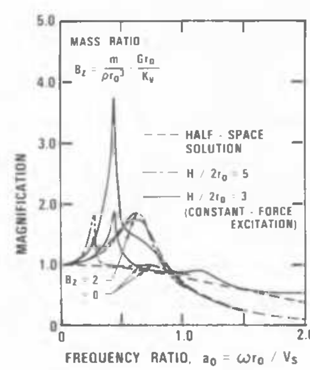

Fig. 2
Footing Response

For a homogeneous elastic half-space, Lysmer and Richart (1966) have shown that the footing response can be characterized by the frequency ratio, $a_o = \omega r_o/V_s$, and the mass ratio, $B_z=(1-\nu)m/4\rho r_o^3$. m is the mass of the footing. These parameters, however, are not convenient for a two-layered system. Typical effects of soil layering on footing response are illustrated for a circular footing on a homogeneous layer with thickness H underlain by a rigid half space, Fig. 2. The first peaks occur near the lowest resonance of the surface soil layer in its vertical mode, and the following peaks are associated with those of the half-space solution. Fig. 2 shows also that the first peaks dominate the footing response for thickness ratios, $H/2r_o$, less than about 3, which is of our present interest. This suggests that the 'resonance' phenomena of the footing on a two-layered system can be more conveniently related to the resonance of the surface soil layer than by the frequency ratio a_o.

Therefore, the frequency was normalized by the lowest resonance frequency of the surface soil layer in its vertical mode, and the mass ratio was redefined accordingly as

$$\bar{a}_o = 2\omega H/\pi V_p \qquad (1)$$
$$\bar{B}_z = (1-\nu)^2\pi^2 m/(8(1-2\nu)\rho r_o H^2 \alpha)$$

where V_p is the P-wave velocity, ν is the Poisson's ratio, and ρ is the mass density of the surface layer; and α is the ratio of the static spring constant for a two-layered system and a homogeneous half-space.

Displacement functions for two-layered systems were obtained using an axisymmetric dynamic finite element method developed by Waas (1972). The method requires a rigid fixed base and fine vertical meshes at high frequencies. The rigid base tends to overestimate the spring constant and underestimate the radiation damping. The effects of the rigid base on footing response were removed by extending the finite element system to a depth where the soil deformations due to a static loading and the Rayleigh wave propagation are negligible.

RESULTS OF A PARAMETER STUDY

In the present analytic model involving two linearly elastic layers, footing response is affected primarily by the thickness ratio, $H/2r_o$, and the relative stiffness of the two layers, G_2/G_1. The Poisson's ratio of the two layers was assumed 0.35, and the material damping of the upper and lower layers were assumed, respectively, 2 and 1 percent throughout this study.

Static spring constants of rigid circular footings on two-layered systems are summarized in Fig. 3; the spring constant,

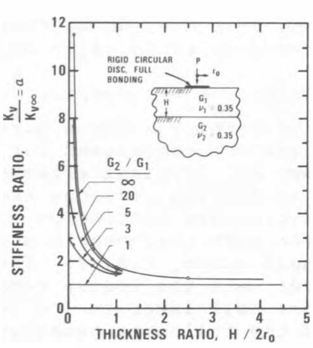

Fig. 3 Static Spring Constants

K_v, is normalized by the spring constant, $K_{v\infty} = 4G_1r_o/(1-\nu_1)$. For thickness ratios less than 1 the stiffness of the lower layer has a large effect on the vertical spring constant, and the layering effects are expected to be large on footing response.

Responses of a footing with various magnitudes of masses resting on two-layered systems were obtained. The frequency ratio at the maximum magnification and the maximum magnification for rotating-mass excitations are shown in Figs. 4 and 5. These figures include also the homogeneous half-space solution ($G_2 = G_1$).

Due to the current definitions of the frequency ratio and the mass ratio in Eq. (1), the half-space solution is not unique and depends on the value of H.

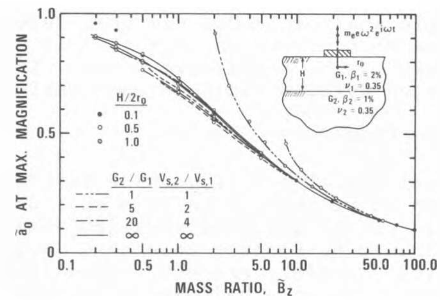

Fig. 4 Footing Response

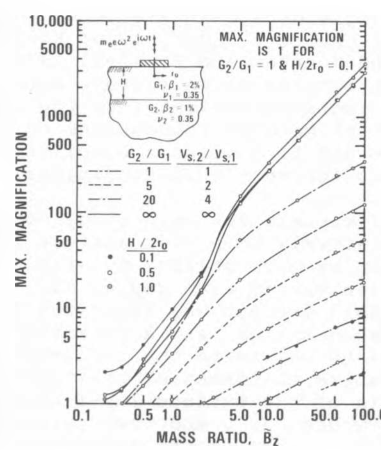

Fig. 5 Footing Response

Major characteristics of the footing response in a two-layered system include: (1) Layering has a large effect on the frequency ratio at the maximum magnification for small mass ratios, and the frequency ratio at the maximum magnification in a two-layered system approaches the half-space solution when the mass ratio becomes large. (2) When G_2/G_1 exceeds 20, the footing behavior becomes similar to that of the case in which the lower layer is infinitely stiff. (3) The frequency ratio at the maximum magnification for a two-layered system is generally less than that of the half-space solution. (4) The maximum magnification cannot be correctly predicted from the half-space theory even for a very large mass ratio, although the half-space solution may yield a reasonable prediction of the frequency ratio at the maximum magnification. Therefore, the layering effects are not necessarily limited to small mass ratios.

Wave attenuation equation with both geometrical and material damping was proposed for relatively uniform soil deposits by Bornitz (1931) as

$$\frac{z}{z_o} = \sqrt{\frac{r_o}{r}} \exp\left[-\gamma(r-r_o)\right] \qquad (2)$$

where z and z_o represent, respectively, the vertical displacements of the ground surface at distances r and r_o from the axis of system symmetry, and γ is the coefficient of attenuation. Based on some experimental data, Richart, Hall, and Woods (1970) suggested a typical range of γ values to be between 0.006 and 0.08 (1/ft), or 0.0197 and 0.262 (1/m).

Major observations from the wave attenuation in two-layered systems include: (1) Wave attenuation is slower for smaller G_2/G_1 values. (2) Eq. (2) with the suggested γ values forms generally the lower and upper bounds of the results of two-layered cases. (3) The rate of wave attenuation changes with distance from a footing. At low frequencies, this break point is located at an r/r_o value ranging from 1 to 2.5. Most wave modes seem to attenuate before the break point, and a Rayleigh wave mode dominates after the break point. (4) At low frequencies, the Rayleigh wave attenuation is slow. The contribution of the Rayleigh wave in the wave attenuation at low frequencies, however, is small. Therefore, the effective wave attenuation at low frequencies is larger than that at higher frequencies.

These results indicate that a conservative wave attenuation field of a two-layered system is provided by Eq. (2) with γ=0.006 (1/ft), or 0.0197 (1/m).

SIMPLIFIED MODEL

A simple damped oscillator analog has been developed by Lysmer and Richart (1966) for the vertical vibration of a circular footing on a homogeneous elastic half-space. A similar model was developed here to approximate closely the footing behavior on a two-layered system. Two key elements were considered in constructing a simplified model: (1) The effective soil resistance to a footing decreases rapidly as the excitation frequency approaches the lowest resonance of the surface layer. (2) The damping in the system is very small at small frequency ratios.

According to the simplified model, an equation of motion of a footing is given as

$$(m + m_1)\frac{d^2 z}{dt^2} + \frac{c_1 \bar{a}_o K_v}{\omega\, g(\bar{B}_z)}\frac{dz}{dt} + k_1 K_v z = f(t) \qquad (3)$$

where $f(t)$ is an exciting force that can vary arbitrarily with time, m_1 represents a virtual mass, c_1 and k_1 are constants, and $g(\bar{B}_z)$ is used to approximate the damping characteristics of a two-layered system. For simplicity, the function $g(\bar{B}_z)$ is assumed as

$$g(\bar{B}_z) = (\bar{B}_z)^{\beta} \qquad (4)$$

and k_1 is assumed unity to guarantee a correct static footing settlement. Thus, the simplified model involves three parameters, $\bar{B}_z^{*}$ (= mass ratio for m_1), c_1, and β, that must be selected to approximate closely the footing response on a two-layered system obtained in the parametric study. Parameter values in the simplified model were determined using the finite element results and are summarized in Fig. 6.

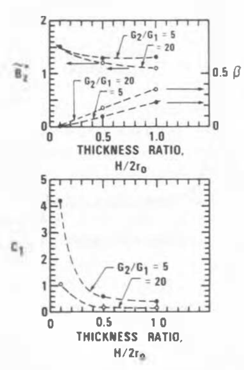

Fig. 6
Simplified Model Parameters

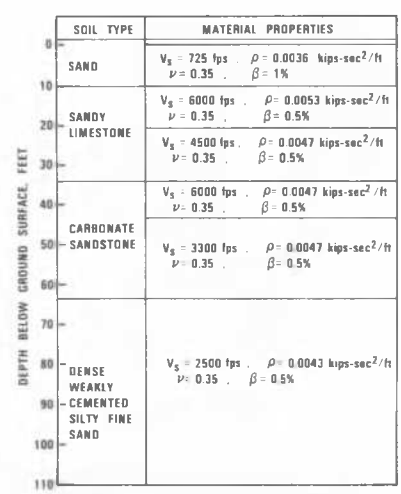

Fig. 7
Case Study Soil Profile

A CASE STUDY

To demonstrate the significance of layering and the applicability of the simplified model to a real problem involving multiple layers, the behavior of a large centrifugal compressor unit was analyzed. The site consisted of a 10-ft (3.05-m) sand layer underlain by soft rock to the terminal depth of the borings, Fig. 7. A strong discontinuity of wave velocities existed between the surface sand layer and the underlying sandy limestone. Operating frequency of the compressor unit was 144 Hz. An equivalent radius of the footing was about 20 ft (6.1 m), and the mass of the foundation unit was 51.06 kips-sec^2/ft (760 kg-sec^2/cm). A finite element analysis was first performed for three different embedment depths; 10 ft(3.05 m), 8 ft(2.44 m), and 6 ft(1.83 m). The side contact between the foundation and the sand layer was neglected.

Responses of the footing with the three different embedment depths are shown in Fig. 8. The frequencies of the full embedment case are normalized by the lowest resonance frequency of the sand layer for the 6-ft (1.83-m) embedment case. The dashed line represents the footing response when the foundation soil is assumed to be a homogeneous half-space with a soil stiffness that yields the correct static settlement of the footing. Fig. 8 shows clearly that the existence of a thin surface sand layer causes a large magnification not found in the half-space solution.

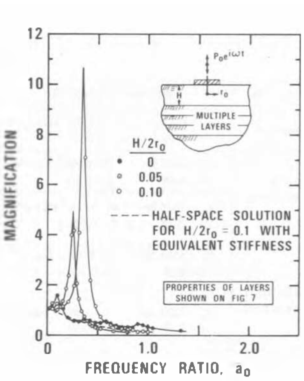

Fig. 8
Case Study

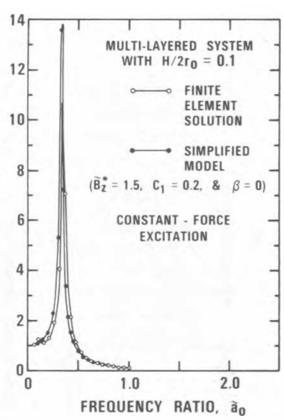

Fig. 9
Comparison

To evaluate the simplified model, the response of the footing with a 6-ft(1.83-m) embedment was analyzed. The thickness ratio is 0.1, and the mass ratio is 7.64, and from Fig. 7, $\bar{B}_z^* = 1.5$, $\beta = 0$, and $c_1 = 0.2$. Fig.9 presents a comparison of the footing response from the simplified model and the dynamic finite element method. The worst agreement between the responses from the two methods is observed near the resonance frequency of the footing-soil system, which can be improved by adjusting the c_1 value. Agreement at other frequencies is very good.

CONCLUDING COMMENTS

The existence of a thin soft surface layer can alter the dynamic vertical response of a footing considerably from that of the half-space solution. A very large magnification may be established due to the layering effects. The simplified model provides a rational and economical means to predict a footing response on a multi-layered system in which a thin surface layer possesses a relatively low wave velocity compared with those of the underlying layers. Use of the results presented in this paper should yield a more reliable prediction on the footing response than the half-space solution in such cases. For final design, more sophisticated analyses should be used to confirm the results obtained from the simplified model developed here.

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