Prediction of Coupled Vibrations by a New Model

Prédiction des Vibrations Accouplés par un Modèle Nouveau

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SYNOPSIS For the successful design of a machine foundation subjected to coupled modes of rocking and sliding vibrations, one has to predict properly the peak amplitudes and resonant frequencies in rocking and sliding motion. This paper deals with the development of a new mass-spring-dashpot model for the prediction of response of a machine foundation resting on soil surface and subjected to coupled modes of rocking and sliding vibrations. The internal damping in translation and rotation have been represented by independent parameters and the radiation damping alone in translation and rotation by linear viscous dashpots. Analytical solutions have been obtained for the proposed model and they are presented in the form of curves for various values of internal damping factor. Carefully controlled field vibration tests have been carried out at I.I.T., Madras, India, to check the validity of the analytical solutions. It has been found that, the new mass-spring-dashpot model developed is quite satisfactory for the prediction of response of a machine foundation resting on soil surface and subjected to coupled modes of rocking and sliding vibrations.

INTRODUCTION

To predict the response of a machine foundation-soil system, two important parameters, namely, the spring constant and the damping ratio are to be evaluated. The damping ratio represented by a viscous dashpot in the mass-spring-dashpot model represents total damping which is a combination of radiation damping and internal damping. The radiation damping is a function of mass and size of machine and foundation block and eccentric moment whereas the internal damping is a material (soil) property. Therefore a viscous dashpot cannot represent truly the behaviour of internal damping which is taken as a material (soil) property. In this investigation, a mass-spring-dashpot model representing the two degrees of freedom in coupled rocking and sliding motion has been proposed in which, the internal damping in both translation and rotation is taken care of by independent parameters and the radiation damping in translation and rotation by linear viscous dashpots.

THE PROPOSED MODEL

The proposed model with two degrees of freedom for a footing with a circular base, resting on soil surface and subjected to horizontal vibrations (which generate coupled modes of rocking and sliding vibrations), is shown in Fig.1. where,

b_x = coefficient of internal damping in translation.
b_φ = coefficient of internal damping in rotation.
c_x = coefficient of viscous damping in translation.
c_φ = coefficient of viscous damping in rotation.

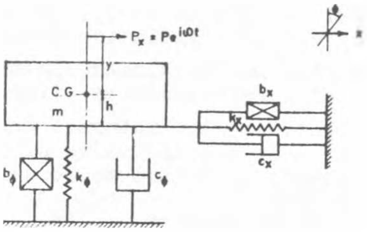

Fig.1. Proposed Mass-spring-dashpot model.

h = height of C.G. from the base of footing.
k_x = spring constant in translation.
k_φ = spring constant in rotation.
m = mass of machine and foundation block.
P_x = applied horizontal force = $Pe^{i\omega t}$
y = height of applied force from the C.G.
ω = frequency of excitation at time, t, in radians/unit time.

From the equations of motion for such a system, analytical solutions have been obtained and they are presented in the form of curves for various values of internal damping factor h_1 (Sreekantiah, 1978). As h_1 represents the internal damping property of soil, the set of curves for a particular value of h_1 can readily be used for prediction purposes.

FIELD TESTS

Carefully controlled field vibration tests have been carried out at I.I.T., Madras site (silty sand), on R.C.C. footings of various sizes and masses and with different eccentric moments. The footings are resting on soil surface and subjected to horizontal vibrations (which generate coupled modes of rocking and sliding vibrations). Each test represents a single field vibration test and the resonant frequencies in rocking and sliding and the peak amplitudes in rocking and sliding have been found for each test.

ANALYSIS

The spring constants, k_x, k_φ, in sliding and rocking respectively of a circular footing of radius r_o havebeen expressed as follows:

$$k_x = k_{sh} \cdot r_o = \frac{8G}{(2-\mu)} r_o \qquad \ldots \quad (1)$$

and

$$k_\varphi = k_{sr} \cdot r_o^3 = \frac{8G}{3(1-\mu)} r_o^3 \qquad \ldots \quad (2)$$

where,

k_{sh} = dynamic soil shear modulus

k_{sr} = dynamic soil rocking modulus

G = shear modulus

μ = Poisson's ratio.

From elastic half-space theory, it has been shown that k_{sr} and k_{sh} are independent of the size and mass of machine and foundation block as well as the eccentric moment and are essentially constants for the given site and are the properties of soil (Sreekantiah, 1978; Rama Sastri, 1975).

Each one of the tests carried out at I.I.T., Madras, has been taken as a single field vibration test (reference test). To obtain D_{rx}, $D_{r\varphi}$, representing radiation damping ratios in sliding and rocking respectively and k_{sh}, k_{sr} of a referencetest from the analytical solutions, the internal damping of the soil at site and the value of l, as defined in equation (3) have to be evaluated.

$$l = \frac{mh^2}{I} = \frac{(h/r_o)^2}{\frac{1}{4} + \frac{1}{3}(h/r_o)^2} \qquad \ldots \quad (3)$$

where,

I = mass moment of inertia of the machine and foundation block about the axis of rotation.

r_o = radius of footing.

The soil at site is silty sand and the value of internal damping factor, h_1 has been chosen as 0.05 (Whitman and Richart, 1967).

EVALUATION OF D_{rx}, $D_{r\varphi}$

For the reference test, the value of l has been found knowing its h/r_o ratio from equation (3). The dimensionless maximum amplitudes $A_{1(max)}$ and $A_{2(max)}$ in sliding and rocking respectively as defined in equations (4) and (5) are then evaluated knowing the peak amplitudes, A_{xr}, $A_{\varphi r}$ observed in sliding

$$A_{1(max)} = \frac{m A_{xr}}{m_e l_e} \qquad \ldots \quad (4)$$

$$A_{2(max)} = \frac{m A_{\varphi r} r_o}{m_e l_e} \qquad \ldots \quad (5)$$

where,

$m_e l_e$ = eccentric moment.

and rocking respectively for the reference test after applying the correction to m, to take care of the mass of soil in phase with vibration. Using the computed values of l, $A_{1(max)}$ and $A_{2(max)}$, the values of D_{rx}, $D_{r\varphi}$ of the reference test have been found from the curves presented in Fig.2 corresponding to $h_1 = 0.05$. It is possible to get two sets of D_{rx}, $D_{r\varphi}$ values of a reference test since there are two dimensionless amplitudes, provided both of them fit into the curves.

EVALUATION OF k_{sh}, k_{sr}

From the values of D_{rx}, $D_{r\varphi}$ and l obtained for the reference test, the values of $\frac{\omega_{rx}}{\omega_{n1}}$, $\frac{\omega_{r\varphi}}{\omega_{n2}}$, are then found from the curves presented in Fig.3 corresponding to $h_1 = 0.05$. Since ω_{rx}, $\omega_{r\varphi}$ are the observed values of the resonant frequencies in sliding and rocking motion respectively which are already known from the test, ω_{n1} and ω_{n2} (representing the undamped natural frequencies in I and II mode respectively of the system) are then computed from the ratios obtained. Knowing ω_{n1}, ω_{n2} and l of the reference test, ω_{nx}, $\omega_{n\varphi}$ (representing undamped resonant frequencies in pure sliding and pure rocking respectively) are then computed from the frequency equation, defined in equation (6), whose positive real roots are ω_{n1} and ω_{n2}.

$$\omega_n^4 - \omega_n^2(\omega_{nx}^2 + \omega_{n\varphi}^2 + l\,\omega_{nx}^2) + \omega_{nx}^2\,\omega_{n\varphi}^2 = 0 \qquad \ldots \quad (6)$$

From the computed values of ω_{nx}, $\omega_{n\varphi}$ and applying correction to m to take care of the mass of soil in phase with vibration, k_{sh}, k_{sr} of the reference test have been evaluated from equations (7) and (8).

$$k_{sh} = \frac{\omega_{nx}^2 . m(1+c)}{r_o} \qquad \ldots \quad (7)$$

and

$$k_{sr} = \frac{\omega_{n\varphi}^2 m(1+c)k^2}{r_o^3} \qquad \ldots \quad (8)$$

where,

c = correction factor.

k = radius of gyration defined in equation, $I = mk^2$.

The typical values of k_{sh}, k_{sr} for the tests carried out at I.I.T., Madras have been presented in Tables I and II. It can be observed from Tables I and II that the values of k_{sh}, k_{sr}, obtained by elastic half-space theory and that by the proposed model are essentially the same and are constants for the given site and hence k_{sh}, k_{sr} are the properties of soil.

PREDICTION OF RESONANT FREQUENCIES AND PEAK AMPLITUDES

To predict the resonant frequencies and peak amplitudes of motion for any other machine

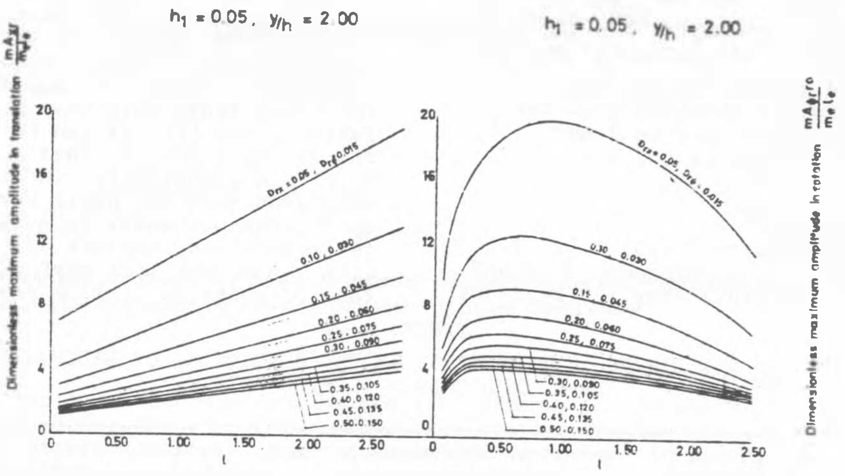

FIG. 2 Relationship between $\frac{mA_{xr}}{m_e l_e}$ and l. Relationship between $\frac{mA_{\varphi r}r_o}{m_e l_e}$ and l

1-1 $D_{rx} = 0.05$ to 0.30, $D_{r\varphi} = 0.015$ to 0.090
2-2 $D_{rx} = 0.35$ to 0.45, $D_{r\varphi} = 0.105$ to 0.135
3-3 $D_{rx} = 0.50$, $D_{r\varphi} = 0.150$

1-1 $D_{rx} = 0.05$ to 0.20, $D_{r\varphi} = 0.015$ to 0.060
2 $D_{rx} = 0.25$, $D_{r\varphi} = 0.075$
3 $D_{rx} = 0.30$ to 0.35, $D_{r\varphi} = 0.090$ to 0.105
4 $D_{rx} = 0.40$, $D_{r\varphi} = 0.120$
5 $D_{rx} = 0.45$, $D_{r\varphi} = 0.135$
6 $D_{rx} = 0.50$, $D_{r\varphi} = 0.150$

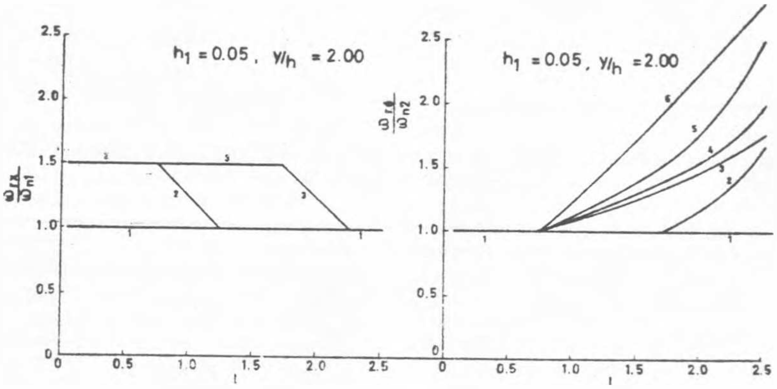

FIG 3 Relationship between $\frac{\omega_{rx}}{\omega_{n1}}$ and l Relationship between $\frac{\omega_{r\varphi}}{\omega_{n2}}$ and l

foundation resting on the same site from a reference test, the spring constants k_x, k_φ and the radiation damping ratios D_{rx}, $D_{r\varphi}$ have to be evaluated. The procedure for prediction has been summarised in the following steps.

1) From the values of k_{sh}, k_{sr} obtained from the reference test, the spring constants k_x, k_φ, for any footing of radius r_o are evaluated using equations (1) and (2).

2) The radiation damping ratios D_{rx}, $D_{r\varphi}$ for any other machine foundation can be obtained from the values of D_{rx}, $D_{r\varphi}$ obtained from the single field vibration test (reference test) taking into account relevant factors influencing D_{rx}, $D_{r\varphi}$ (Sreekantiah, 1978; Rama Sastri, 1975).

3) Knowing l, D_{rx}, $D_{r\varphi}$ for the test conditions for which the prediction is to be made (field conditions), $A_{1(max)}$ and $A_{2(max)}$ have been obtained from the curves presented in Fig.2.

4) From the computed values of l, D_{rx}, $D_{r\varphi}$ obtained from the field conditions, $\frac{\omega_{rx}}{\omega_{n1}}$ and $\frac{\omega_{r\varphi}}{\omega_{n2}}$ have been obtained from the curves presented in Fig.3.

5) ω_{nx}, $\omega_{n\varphi}$ are then computed for the field conditions from equations (7) and (8), taking into account the mass of soil in phase with vibration. ω_{n1} and ω_{n2} have been then computed from the frequency equation defined in equation (6).

6) Finally, the values of A_{xr}, $A_{\varphi r}$ representing the peak amplitudes of motion in sliding and rocking respectively for the field

conditions are obtained from the computed values of $A_{1(max)}$ and $A_{2(max)}$. The values of ω_{rx}, $\omega_{r\varphi}$, representing the resonant frequencies in sliding and rocking respectively for the field conditions are obtained from the frequency ratios computed in step 4 and ω_{n1}, ω_{n2} computed in step 5.

DISCUSSION AND CONCLUSION

Each one of the tests carried out at I.I.T., Madras has been taken as a reference test and the prediction of peak amplitudes and resonant frequencies in rocking and sliding motion has been made for the other tests. Typical test results showing the errors in the prediction of resonant frequencies and peak amplitudes, predicted from the reference tests, for other tests have been presented in Tables I and II. It can be observed from the Tables I and II, that the errors are small and acceptable. Therefore, it has been concluded that the newly developed mass-spring-dashpot model is quite satisfactory to predict the response of a machine foundation resting on soil surface and subjected to coupled modes of rocking and sliding vibrations.

TABLE - I

TEST DATA, VALUES OF k_{sr} AND ERRORS IN PERCENTAGE IN THE PREDICTION OF RESONANT FREQUENCY AND PEAK AMPLITUDE - ROCKING MOTION

Test No.	Equivalent radius in cm.	I in kg.sec^2.cm.	Eccentric moment $m_e l_e$, in kg.sec^2	Observed $f_{n\varphi}$, in cycles per sec.	Observed $A_{\varphi(max)}$ x 10^{-4} in radians	Average k_{sr} from el. half-space kg/cm^2.	Average k_{sr} from model kg/cm^2.	Errors in prediction of $f_{n\varphi}$			Errors in prediction of $A_{\varphi(max)}$		
								Ref. Test No.			Ref. Test No.		
								16	25	34	16	25	34
2	34.6	1718	0.01020	13.3	10.4	314	366	25	28	25	7	55	56
9	34.6	2302	0.00510	11.7	3.9	408	342	27	26	26	30	51	52
16	34.6	1554	0.03935	13.3	39.6	297	407	0	20	10	0	61	60
25	52.2	2192	0.00510	23.6	5.8	417	341	11	0	12	28	0	70
34	52.2	1378	0.01020	27.3	12.7	357	272	22	16	0	20	65	0
46	51.8	2530	0.01020	21.7	13.7	394	365	9	2	14	35	72	72
52	69	5820	0.03935	17.0	14.5	277	246	25	0	12	39	46	47

TABLE - II

TEST DATA, VALUES OF k_{sh} AND ERRORS IN PERCENTAGE IN PREDICTION OF RESONANT FREQUENCY AND PEAK AMPLITUDE - SLIDING MOTION.

Test No.	Equivalent radius in cm.	Mass m, in kg.sec^2/cm.	Eccentric moment $m_e l_c$, in kg.sec^2.	Observed f_{nx}, in c.p.s.	Observed $A_{x(max)}$ x 10^{-4} in cm.	Average k_{sh} from elastic half-space kg/cm^2.	Average k_{sh} from model kg/cm^2.	Errorsin prediction of f_{nx}			Errors in prediction of $A_{x(max)}$		
								Ref. Test No.			Ref. Test No.		
								16	25	34	16	25	34
2	33.8	0.70	0.01020	20.3	194.0	-	251	25	0	12	125	21	17
9	33.8	0.86	0.00510	17.5	89.5	-	214	28	4	8	117	23	18
16	33.8	0.83	0.03935	19.3	635.0	-	214	0	9	19	0	17	14
25	50.8	1.23	0.00510	20.0	89.0	343	227	11	0	12	1	0	64
34	50.8	0.89	0.01020	20.7	192.0	265	209	22	17	0	13	58	0
46	50.8	1.19	0.01020	22.5	190.2	413	363	9	3	15	8	66	68
52	67.7	2.28	0.03935	16.0	127.5	-	240	24	28	27	58	32	36

Note:- - represents cases where k_{sh} could not be determined from elastic half-space theory.

REFERENCES

Rama Sastri, K. (1975), 'Prediction of Response of Footings subjected to Horizontal Vibrations', Thesis presented to the I.I.T., Madras, India, in fulfilment of the requirements for the Degree of Doctor of Philosophy.

Sreekantiah, H.R. (1978), 'Prediction of Coupled Vibrations of Footings', Thesis presented to the I.I.T., Madras, India, in fulfilment of the requirements for the Degree of Doctor of Philosophy.

Whitman, R.V., and Richart, F.E., Jr. (1967), 'Design Procedures for Dynamically Loaded Foundations', Journal of the Soil Mechanics and Foundations Division, Proce. A.S.C.E., Vol.93, pp.1969-1993.