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Foundation Rocking under Earthquake Loading

Le Mouvement des Semelles dans les Tremblements de Terre

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SYNOPSIS A simple theory is developed for the moment-rotation behaviour of a rigid footing under constant vertical load on a Winkler-type foundation, extended to include uplift and yield. Laboratory tests on model footings (0.5 m x 0.25 m) on clay and on sand are reported which support the general theoretical predictions. The moment-rotation relationship is found to be non-linear as a result of the lifting of one edge of the footing, even when the foundation material behaves elastically. At rocking amplitudes large enough to cause yielding beneath one edge, a strongly non-linear moment rotation relationship is formed, with hysteretic damping.

The research shows that rotational yield may occur under spread footings when earthquake loadings are applied, without serious detriment to vertical load-bearing capacity, and that, under suitable conditions, vertical displacement, even after several cycles of rocking, can be small. Thus, spread footings may be intentionally designed to yield in high-intensity earthquakes and this may be preferable to the yielding of columns at base level in reinforced concrete frame structures.

INTRODUCTION

It is well-known that the compliance of foundations has a considerable effect on the response of structures to earthquakes. This effect may be taken into account by assuming that spread footings act as rigid plates attached to the surface of a semi-infinite elastic medium. (See, for example Parmalee, 1967.) Another effect which has been considered is that one edge of a footing may rise off the supporting soil. Housner (1963) considered the response of a rigid block on a rigid base. The effect was termed "tipping" by Meek (1975) who examined its effect on a single degree-of-freedom structure.

Here, one aspect of soil-structure interaction is considered using both theoretical and experimental methods. The moment-rotation relationship for a rigid rectangular footing on the surface of a soil is investigated. Experimental results for footings on sand and on clay are correlated with the simple theory proposed.

In addition to giving some insight into the compliance of such foundations for use in estimating the response of structures to earthquake loading, the work may be of value in the design of foundations for earthquake-resistant structures.

THEORETICAL BACKGROUND

Elastic Rocking

Using static conditions, a spread footing can normally be assumed to be in contact with the supporting subsoil over the whole of its area.

If a small overturning moment is applied, it will remain in full contact and elastic deformation of the soil will occur. Elastic theory can then be applied, the rotational stiffness being a function of the soil properties and the footing dimensions. This is the method usually adopted in the analysis of vibration of foundations for machines with out-of-balance forces. (Richart, Hall and Woods, 1970) Usually these forces are small in comparison with the weight of the machine and its foundation, and generate motions of small amplitude. Under these conditions, the assumptions that the soil behaves elastically and that the footings remain in full contact with the ground are reasonable.

The Winkler Model

Because of its mathematical simplicity, the Winkler model for soil behaviour has been extensively used for a variety of interaction problems (Biot, 1943; Merritt and Housner, 1954; Donald, 1965; Weissman, 1972). The main criticism of the model is that it fails to account for continuity in the supporting medium. Nevertheless, it provides solutions which may be regarded as useful engineering approximations. With suitable modification to include the effect of partial separation of the footing from the soil and the effect of plastic deformations, the Winkler model forms the basis of the theory presented here. The subsoil is assumed to act as a uniform bed of springs. Over the contact area, the ratio of contact stress to vertical displacement is assumed to be constant. This ratio defines the coefficient of subgrade reaction, k_s .

If the vertical load V is kept constant, while a moment M is applied, this is equivalent to an

eccentricity of loading $e = M/V$. As this moment is increased, the contact stress distribution remains linear and the stress is compressive over the whole contact area, provided $e < B/6$ where B is the breadth of the contact area.

Under these circumstances, the rotational stiffness is constant:

$$k_r = \frac{M}{\theta} = k_s I_o \quad (1)$$

where θ is the angle of rotation and I_o is the second moment of the footing area about the axis of rotation.

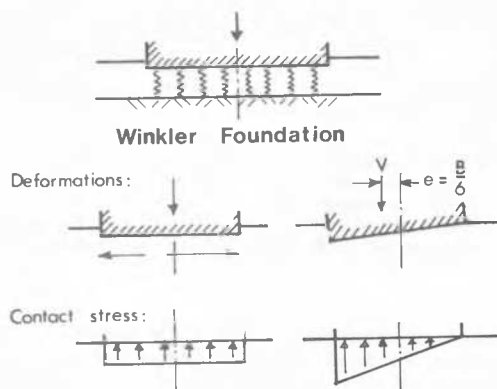


Fig. 1 Contact Stress under Axial and Eccentric Loading

As shown in Fig. 1, the contact stress becomes zero along one edge when $e = B/6$.

Separation of Footing and Subsoil

If it is now assumed that the 'springs' of the Winkler foundation cannot sustain tension, then for an applied moment $M > BV/6$ the footing will partially separate from the supporting medium. For these conditions it can easily be shown that the tangent rotational stiffness,

$$k_r' = \frac{dM}{d\theta} = k_s I \quad (2)$$

where I is the second moment of the area in contact about its neutral axis.

As the applied moment is increased, I decreases, resulting in a nonlinear (strain-softening) moment-rotation relationship. The nonlinearity arises from the changing geometry. The system is still completely elastic and deformation is fully recoverable, Fig. 2.

With partial separation, rotation of the footing occurs about the mid-point of the contact area, which is not coincident with the mid-point of the footing. As the footing rotates, its mid-point rises. Expressed in other terms, part of the energy supplied in applying the overturning moment is stored as potential

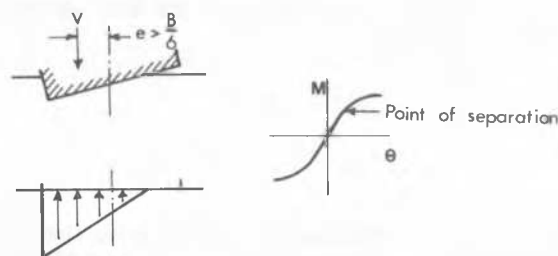


Fig. 2 Contact Stress and Moment-Rotation Relationship with Partial Separation

energy by raising the load, V , while the remainder is stored in the 'springs'.

Yield of Cohesive Subsoil ($\phi = 0$)

The Winkler model can be further extended to include yield, that is, the springs representing the subsoil assumed to behave elastically until the yield stress q_u is reached, after which they continue to deform plastically. This is equivalent to including a Coulomb slider element in series with each spring. As an approximate representation of a saturated clay subsoil, this yield stress can be assumed to be constant over the footing area. As footing rotation is increased, an area beneath which plastic deformation is occurring will form at one edge, with uniform contact stress as shown in Fig. 3.

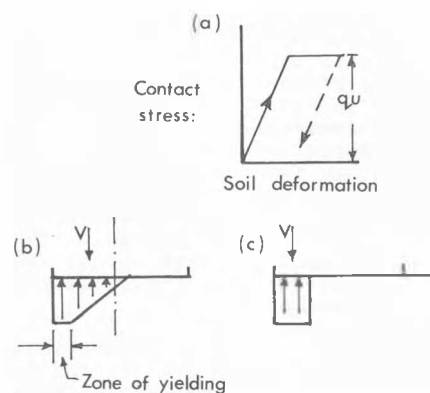


Fig. 3 Rotation with Yield
(a) Assumed Stress-Deformation Relationship
(b) Stress Distribution
(c) Ultimate Stress Distribution

Under these conditions, the tangent rotational stiffness,

$$k_r' = \frac{dM}{d\theta} = k_s I_e \quad (3)$$

where I_e is the second moment of area of that part of the contact area which remains elastic. (Bartlett, 1976)

The moment-rotation relationship, with yielding, is strongly nonlinear. With increasing rotation the contact stress distribution tends to

the ultimate condition where the contact area is fully plastic and the tangent rotational stiffness tends to zero.

Cyclic Rocking - Cohesive Subsoil

Based on this extended Winkler model, the response of a strip footing to a cyclic rocking disturbance (of constant angular amplitude) was computed. Two distinct types of behaviour are found depending on the factor of safety with respect to vertical loading only, F_v .

$$F_v = \frac{q_u A}{V} \quad (4)$$

where A is the area of the footing.

For values of $F_v > 2$, the separation of one edge of the footing occurs at a smaller angle of rotation than that required to cause yield at the other edge. Yielding occurs in the first cycle of loading only, accompanied by some permanent vertical deformation, and further rocking cycles at the same amplitude occur without further energy dissipation, and without further permanent vertical deformation (according to this simple model). After cyclic rocking with yield, the contact stress is reduced within the zones which have yielded, or, if amplitudes are great enough, may become zero over part of the area, as shown in Fig. 4.

It should be noted, however, that, provided the yield stress of the supporting medium has not decreased the factor of safety against failure under vertical load has not been reduced.

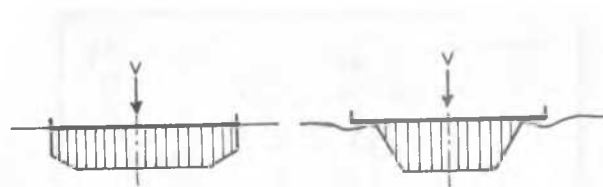


Fig. 4 Stress Distributions after Cyclic Rocking

The calculated response for $F_v = 3$ is shown in Fig. 5 in which rotation θ and moment M are normalised with respect to the values (θ_F and M_F respectively) at which full contact ceases (point F). Plastic yield occurs initially at point P. Also shown is the vertical displacement y at the centre-line of the footing in terms of the static vertical displacement y_0 under the axial load V .

For values of $F_v < 2$, yielding occurs before separation. Repeated cycling causes dissipation of energy in each cycle, with an increase in permanent vertical deformation in each cycle. Figure 6 shows the computed response for $F_v = 1.5$ for two cycles. Energy dissipation is represented by the area enclosed within the hysteresis loop of the moment-rotation relationship.

Yield of Cohesionless Subsoil

The model may be modified for cohesionless subsoil. In this case, the stress q_u at which

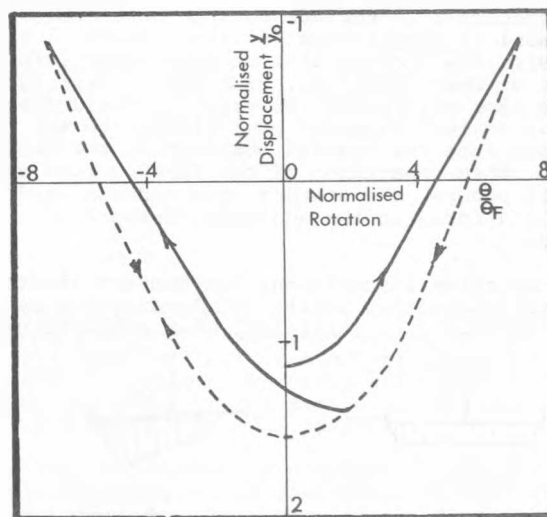
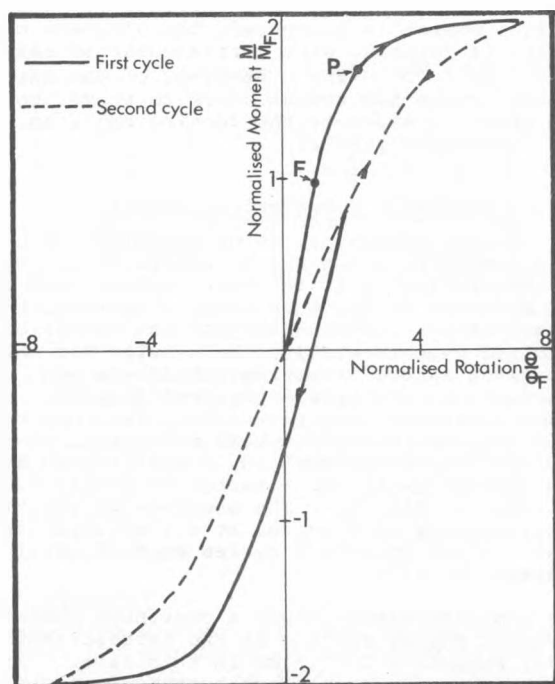


Fig. 5 Footing on Clay - Theoretical Response $F_v = 3$

yield occurs is not constant. If a footing on the surface of a cohesionless subsoil is loaded axially, the contact stress at ultimate load is known to increase from a minimum value

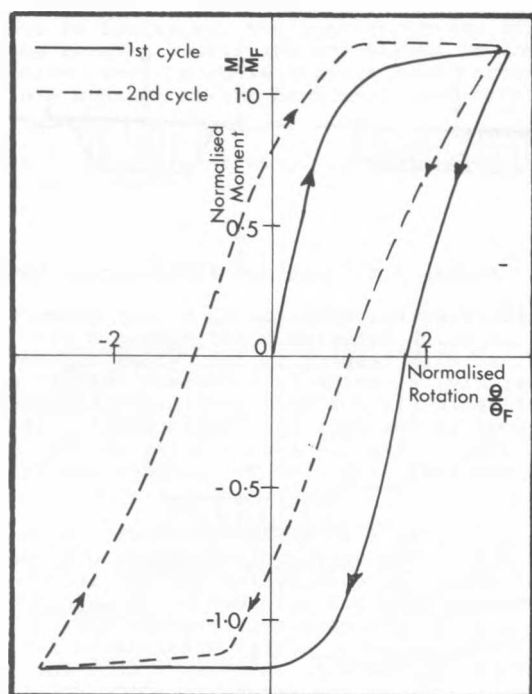
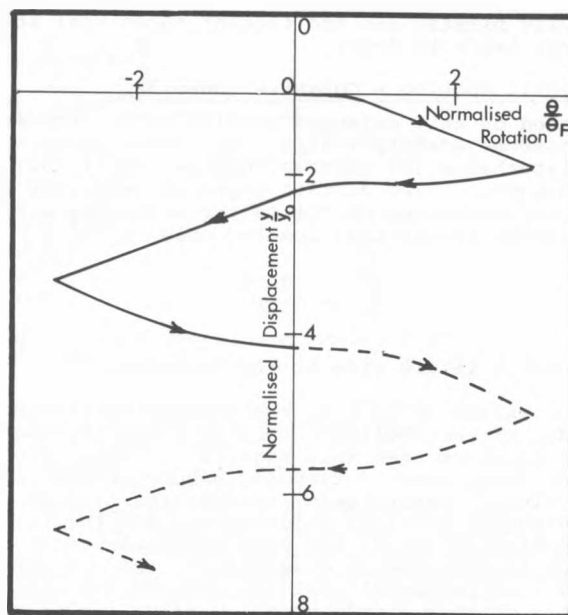


Fig. 6 Footing on Clay - Theoretical Response
 $F_v = 1.5$



at the edge to a maximum at the centre-line of the loaded area. The distribution assumed here (after Meyerhof, 1951) has a linear variation, increasing from zero at the edges. This is consistent with the fact that the ultimate bearing capacity of a surface footing on a cohesionless soil is proportional to its breadth. For analysis, the footing area is subdivided into a number of small sections, each with a "spring" of the same stiffness, but with the yield force (of the Coulomb element) proportional to the distance from the nearest boundary of the contact area. When separation of the footing from the subsoil occurs, the contact area reduces, and the yield force in the adjacent elements is reduced.

Based on these assumptions, the contact stress distributions under rotational loading are as shown in Fig. 7. Initially, when concentrically

loaded, contact stress is uniform except near the edges where it reduces to zero. Any applied moment, however small, must cause yield on one side (as it is at its yield stress already) and separation at the other (as the stress is zero at the edge initially). As the applied moment is increased, the ultimate condition is reached, with a triangular stress block under one edge. Reversal of the applied moment causes the stress block to travel to the opposite side, as the footing rolls on its supporting subsoil.

Cyclic Rocking - Cohesionless Subsoil

For several reasons it is not possible to compute moment-rotation relationships for a footing on cohesionless soil in dimensionless form as was done for footings on clay. Instead, the relationships computed are for the conditions examined experimentally, the results for which are given later. Vertical stiffness and bearing capacity were determined from vertical loading tests on the test rig. The computed behaviour of the 0.50×0.25 m footing, on the surface of a dense sand ($D_r = 93\%$) rocked about its shorter axis, for a factor of safety $F_v = 5$ is shown in Fig. 8. The sequence of rotational displacements is 5 cycles at ± 1 milliradian, 5 at ± 5 and finally 5 cycles at ± 20 milliradians ($\pm 1.1^\circ$).

The computed result shows a continued downward movement during cycling at the largest amplitude, amounting to 3.5 mm in this case. At the same time a continuous hysteretic energy loss is predicted.

Fig. 7 Rocking on Cohesionless Soil - Contact Stress State

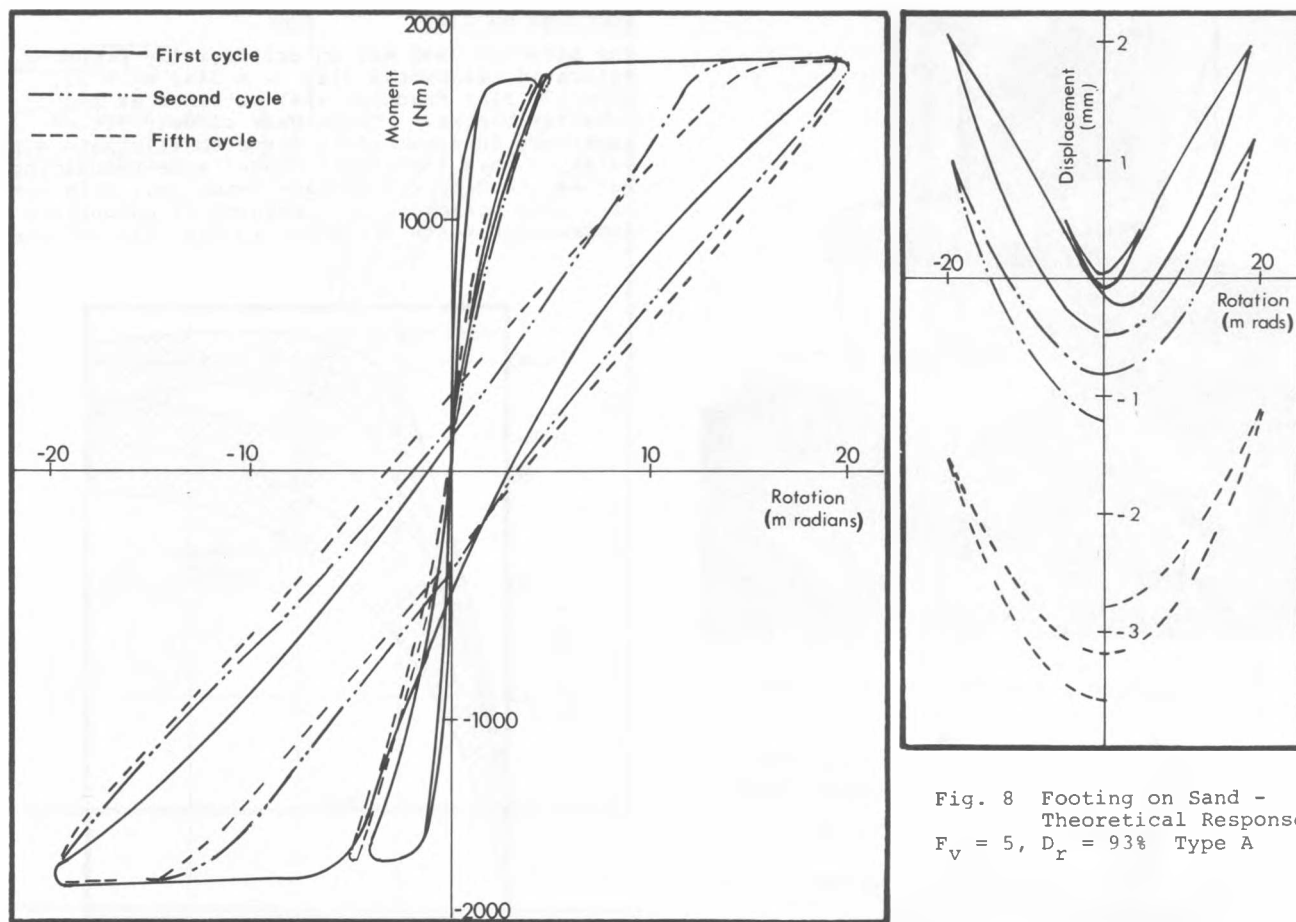
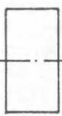
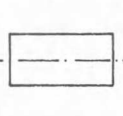


Fig. 8 Footing on Sand -
Theoretical Response
 $F_v = 5$, $D_r = 93\%$ Type A

TABLE I

Rocking on Sands - Computed Displacements

 Type A	 Type B	Type	F_v	Total vertical Displacement (mm)
		A	10	0.1
		A	5	3.5
		B	5	12.3
		A	2	25.0

This result is included in Table I, which gives computed vertical displacements after the same sequence of rotational displacements for rocking about the longer axis, and for other safety factors. It is seen that rocking about the longer axis produces greater vertical displacement, and that these displacements increase rapidly with reduction in safety factor.

EXPERIMENTAL STUDY

Basic Apparatus

The apparatus was made as large as was consistent with the avoidance of serious materials handling problems. As shown in the photograph, Fig. 9, the container for the soil (sand or clay) was constructed of heavy steel channel sections. It is 1.3 m square and 0.6 m deep. For the

footing size adopted (0.50 x 0.25 m) a rigid boundary of such dimensions enclosing an elastic medium should not affect rotational stiffness, although vertical stiffness would be expected to be greater than on a semi-infinite elastic medium (Sovinc, 1969).

The soil container was mounted on the reinforced concrete floor of the Test Hall, beneath which is a basement. This enabled the vertical load on the footing (from lead weights in the basement) to be transferred by hanger rods passing through holes in the floor as shown in Fig. 10. A hydraulic jack is used to apply the load, by lifting the weights off the basement floor. To allow rotation of the footing plate, the load is transferred to it through a low friction bearing. The footing plate is constrained against lateral movement by a pair of tie rods pivotted at their ends.

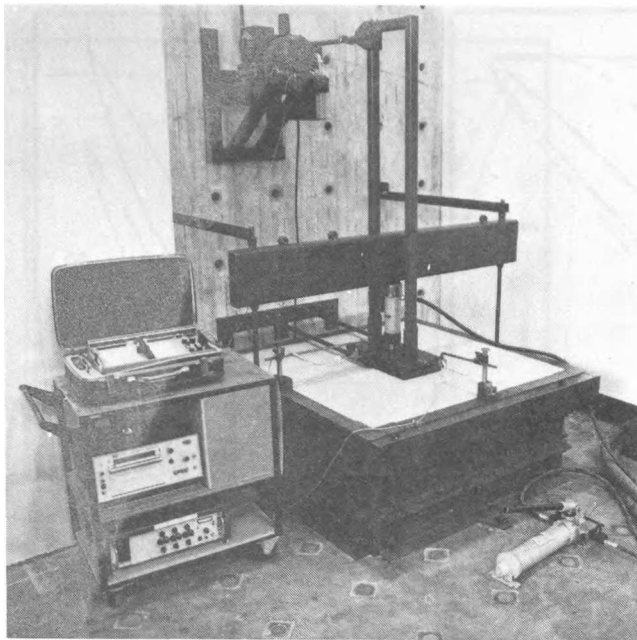


Fig. 9 Apparatus for Model Footing Tests

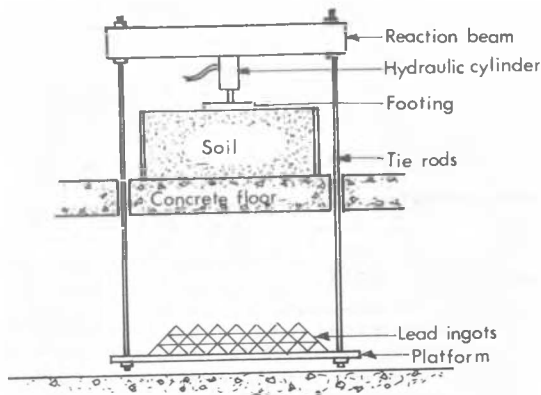


Fig. 10 Vertical Loading System

To apply cyclic rocking displacements, a vertical frame fixed to the footing is moved by a loading arm attached to its upper end. This arm is connected to an adjustable crank on the drive unit, mounted on the reaction wall, as can be seen in the photograph. In this way, harmonic rocking motion of adjustable amplitude can be applied to the footing at a frequency of 0.5 Hz.

Instrumentation

A load cell in the loading arm enables the applied moment to be determined. Two displacement transducers (LVDT's) one at each end of the footing, measure vertical displacements. The average of these readings is the central displacement, while the difference (divided by transducer spacing) determines the rotation. Throughout each test run, a continuous record was obtained on a photographic recorder while the moment-rotation relationship was obtained on an X-Y recorder.

Footings on Clay

The material used was an artificially prepared, saturated brickworks clay ($w = 34\%$, $w_p = 31$, $w_L = 67$, clay fraction 48%) obtained as rectangular blocks. These were closely fitted together, then rammed in place to eliminate air voids. This inevitably caused some remoulding but as the clay was already remoulded, this was of little consequence. Results of unconfined compression tests are shown in Fig. 11, for the

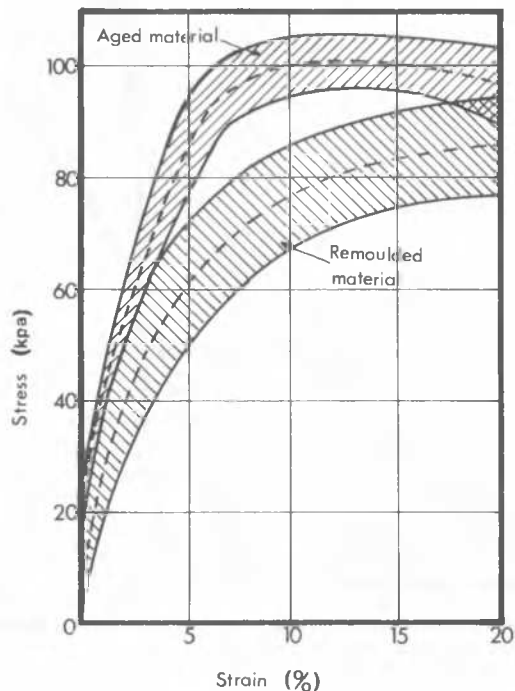


Fig. 11 Unconfined Compression Test Results

freshly remoulded clay, and after 'aging' for four weeks, when test runs are made. The remoulded soil had an undrained shear strength of 44 kPa initially, which increased to 49 kPa after aging. The clay was kept sealed at all times with heavy polythene sheets to prevent evaporation.

Four test runs were made, each comprising five loading cycles at each of three (increasing) amplitudes. The test conditions are listed in Table II, where the vertical displacements, measured in the five cycles at maximum amplitude, are also given.

Vertical displacements and moment-rotation relationships for tests 3 and 4 are shown in Figs. 12 and 13 respectively.

Footings on Sand

The same basic equipment was used for a series of test runs for surface footings on dry sand. A fairly uniform material of medium grain-size was used ($D_{10} = 0.18$ mm, uniformity coefficient 1.7). Predominantly a quartz sand, the solid density was 2663 kg/m^3 . Maximum and minimum dry densities, determined to ASTM D2049-69, were 1689 and 1421 kg/m^3 respectively.

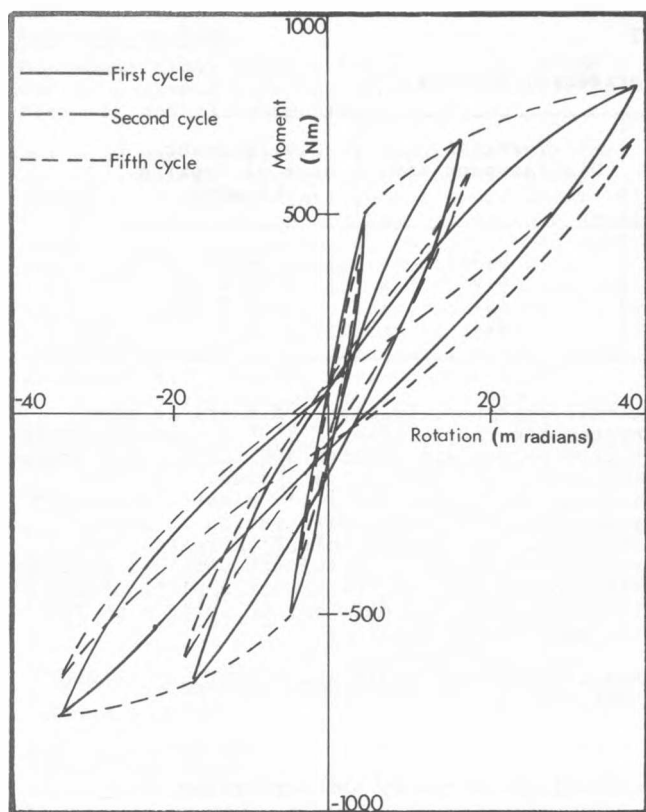


Fig. 12 Footing on Clay - Experimental Result - $F_v = 3$ Type B

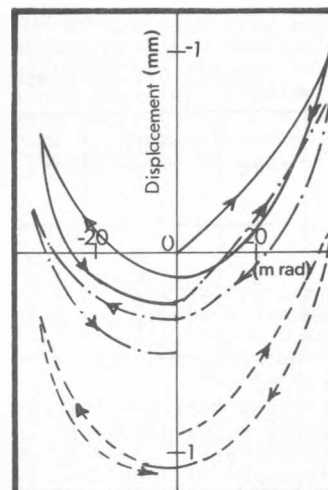


Fig. 13 Footing on Clay - Experimental Result - $F_v = 1.5$ Type B

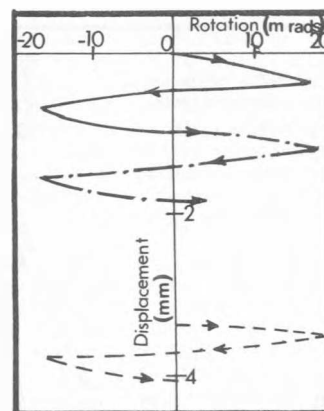
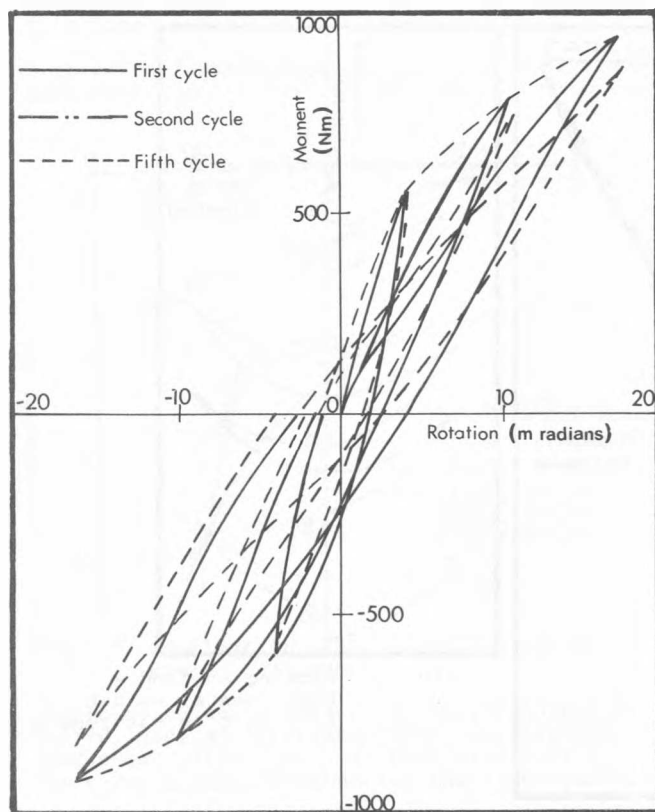


TABLE II
Rocking on Clay - Experimental Results

Test	Footing Type	F_v	Maximum Amplitude (m. rad)	Vertical Displacement (mm)	Displacement as % of footing breadth
1	B	8.0	37	0.1	0.04
2	A	3.0	21	0.8	0.2
3	B	3.0	37	1.1	0.4
4	B	1.5	17	4.1	1.6

Placement was effected by aerial deposition, the final density being dependent on the flow rate and height of fall. Considerable care was taken to ensure that the density throughout was as uniform as practicable. The sand was allowed to flow from an elevated hopper, through a flexible tube and then passed through two screens suspended at a constant height above the sand surface. Tests were conducted at relative densities of zero, $58 \pm 2\%$ and $93 \pm 2\%$. In the latter case, a low flow rate was used and filling took 5 hours. Sand at zero relative density was placed rapidly with the flexible hose flowing full, with zero height of fall.

Vertical displacements and moment-rotation relationships for the footing, type A (rocked about its shorter axis) are shown in Fig. 14. The sequence of applied rotations was 5 cycles at ± 1 , ± 5 and ± 20 milliradians each. It is seen that, at the lowest amplitude, rota-

tional stiffness is high and there is some hysteretic energy loss. At 5 m. rad amplitude, energy losses are greater and the maximum moment increases with successive cycles. At the maximum amplitude, the fully plastic moment is approached and energy dissipation is high. Here also the maximum moment increases with successive cycles. Permanent vertical deformation occurs even at the lowest amplitude and the total vertical deformation is 10.6 mm for the sequence of loadings.

Results of other tests are summarized in the next section.

COMPARISON OF THEORY AND EXPERIMENT

Considering the many gross simplifications made in formulating the model the correlation between theory and experiment is, in general, reasonably close.

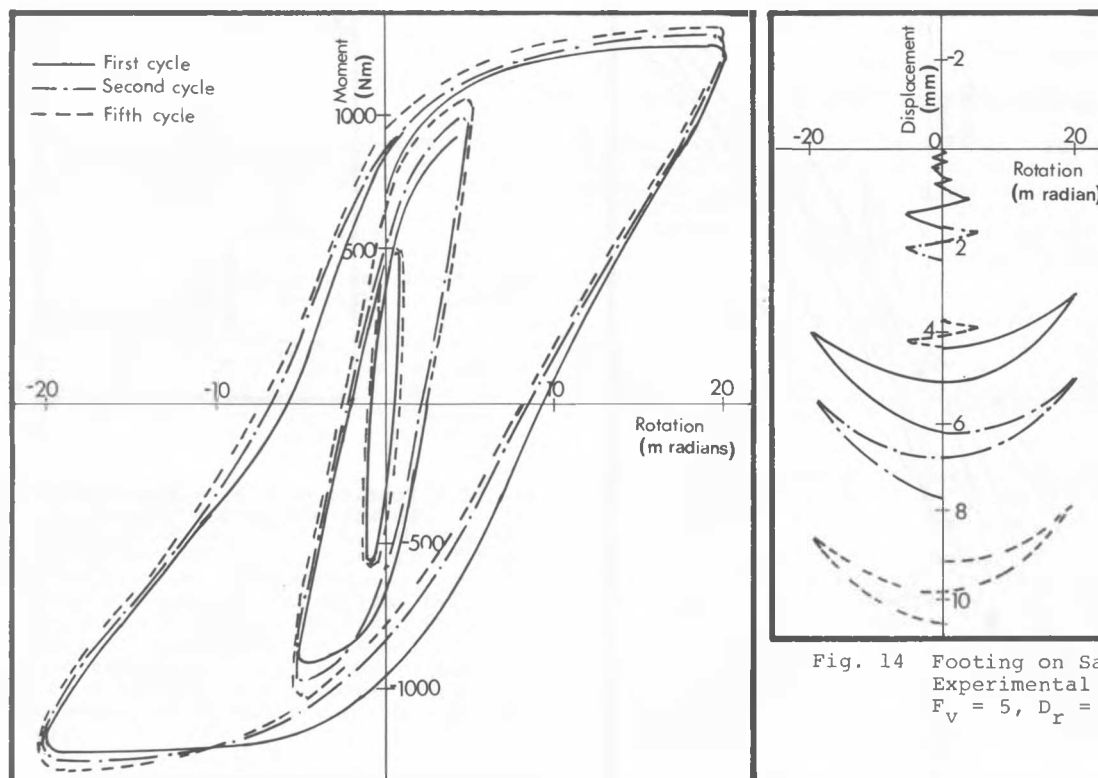


Fig. 14 Footing on Sand -
Experimental Result
 $F_v = 5$, $D_r = 93\%$ Type A

Footings on Clay

The theoretical results shown in Figs. 5 and 6 can be compared with the experimental results of Figs. 12 and 13 respectively.

While it was theoretically predicted that for $F_V = 3$, plastic deformation, accompanied by permanent vertical displacement would occur only in the first cycle of loading (at any one amplitude) (Fig. 5) it is found, in fact, that some small vertical displacement continues with repeated cycling (Fig. 12) and this is accompanied by continued energy loss, not predicted by the model. Secant stiffness is shown to decrease markedly with increase in amplitude, as predicted. At the maximum amplitude (37 m. rad) the maximum moment was 76% of the calculated fully plastic moment (970 Nm). Repeated cycling caused a reduction in the maximum moment, as a result of degradation of the clay.

For $F_V = 1.5$, the prediction (Fig. 6) that continued vertical displacements would occur was clearly shown (Fig. 13) and hysteretic loss was found to be greater than for $F_V = 3$. The shape of the hysteresis loop, however, is found to differ markedly from that predicted. In the test, although at lower amplitude than for $F_V = 3$, the maximum moment was 95% of the calculated maximum.

Footings on Sand

The theoretical response, shown in Fig. 8, for $F_V = 5$ may be compared directly with the experimental results given in Fig. 14. While the general features are very similar, the energy loss and vertical deformation are greater than predicted, and the actual moment-rotation relationship has a more rounded profile.

Vertical deformations observed in several tests are summarised in Fig. 15 and compared with the

In the preparation of the sand, the surface was finally screeded, probably causing a reduction in density within the uppermost few millimetres. Also, imperfect bedding of the footing on the sand surface may have been a contributing factor. Then, of course, there are the obvious deficiencies of the model.

CONCLUSIONS

It has been shown that when an overturning moment of sufficient magnitude to cause either uplift or soil yield is applied to a spread footing, the moment-rotation relationship becomes highly non-linear. The 'softening-spring' characteristic reduces the rotational stiffness at high amplitudes, and tends to increase the natural period of vibration of the structure it supports. In terms of earthquake response of structures, this is usually beneficial.

The extrapolation of model footing tests on sand to practical dimensions is inherently difficult, much more so than for footings on clay. The vertical deformation resulting from cyclic rocking, for example, is dependent on the size and shape of the footing, on the factor of safety, F_V , on the sand density and on the number and amplitude of rocking cycles. Based on conservative assumptions, it was estimated from the test results (Wiessing, 1979) that for a surface footing, 6 m x 2 m in plan, with a vertical load of 5 MN on sand with properties such that $F_V = 6$, the vertical deformation resulting from rocking, about its long axis, with five cycles each at ± 1 , ± 2 and ± 5 m. rad, would be 30 mm.

The tests on sands were conducted on surface footings, whereas in practice foundations on sands are invariably embedded. As is well known, the bearing capacity of sand increases markedly with embedment depth. It is considered that the vertical deformation resulting from cyclic rocking would be considerably reduced as a result of embedment. There is a clear need for further research on this aspect.

Both theoretical and experimental results for footings on clay suggest that, where the factor of safety against failure under vertical loading only is not less than 3 (a typical value used in design) several cycles of rotational displacement of as much as 2° can be sustained without causing vertical displacement of more than $\frac{1}{4}\%$ of footing breadth. It is true that the tests were carried out on clay of low sensitivity, whereas soils encountered in practice may have much higher values of sensitivity. It has been shown, however, (Seed, 1960) that more sensitive clays can undergo cyclic deformation with only minor loss of strength.

In the design of ductile, moment-resisting framed structures, the New Zealand Code requires "capacity" design methods to be applied. In addition to being designed to resist the "design earthquake" elastically, the structure must be able to undergo much larger deformations without serious damage. Then, the structure will behave as a mechanism, in

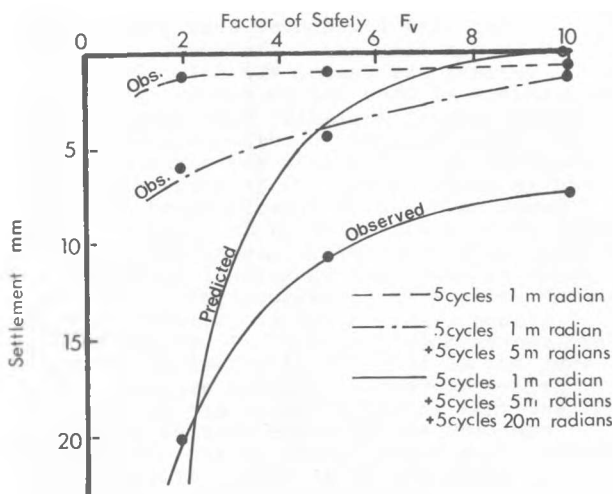


Fig. 15 Footings on Sand - Observed and Predicted Settlements

predicted values. After the full sequence of cyclic loading, deformations are greater than predicted, particularly at high values of F_V . There are several reasons for the discrepancy.

which yielding occurs at predictable locations. Generally, design is such that ductile hinges form in the beams, rather than the columns. With a fixed foundation, however, the formation of a hinge near the base of each column is unavoidable. By designing spread footings which will yield in rotation at an applied moment less than the moment capacity of the column, column hinging can be avoided. This approach to earthquake-resistant design has been applied in New Zealand (Taylor and Williams, 1979).

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ACKNOWLEDGEMENTS

The work on footings supported on clay was carried out by P.E. Bartlett, and that on sands by P.R. Wiessing, under the supervision of P.W. Taylor at the University of Auckland (Bartlett, 1976; Wiessing, 1979). The work was generously supported by the Ministry of Works and Development, New Zealand whose contribution is gratefully acknowledged.

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