Saturated Sands under Cyclic Loading

Sables Saturés soumis à la Charge Cyclique

W.S. WANG Ph. D., Senior Research Engineer, Deputy Head, Department of Earthquake Engineering, Water Conservancy and Hydro-Electric Power Research Institute, Beijing, China

SYNOPSIS The problem of strength, liquefaction and failure of saturated sands has always been greatly concerned in dealing with the safety of foundations of hydraulic structures and earth or earth-rock dams against earthquake action. The theory of limit equilibrium is adopted as a criterion in evaluation of the incipient failure state of saturated sands under cyclic loading, and the corresponding laboratory tests and analytical methods are suggested. The difference between liquefaction and incipient failure is clarified. Two types of approach are proposed for evaluation of pore water pressure changes induced by cyclic loading, in one of which their dispersion and dissipation behaviour is taken into account.

INTRODUCTION

How to evaluate the strength, liquefaction and failure of saturated sands under cyclic loading is still a not well solved problem in earthquake resistant design of hydraulic structures erected on sand foundations and earth or earth-rock dams composed of zones of saturated sands. This paper presents some of the writer's opinion and methodology about them.

STRENGTH

It has long been recognized that the strength parameter of the soil has to be compatible with the actual engineering condition and the method used in evaluation. The triaxial test technique is most popular in the determination of soil strength. The strength parameter of the soil is expressed either in terms of effective stress or in terms of total stress. It has been found that the angle of internal friction in terms of effective stress of the saturated sand under cyclic loading of durations as short as those of earthquakes is very close to the value determined by the ordinary static test. However, in terms of total stress the strength parameter is affected by the pore water pressure changes induced by cyclic loading. In the cyclic triaxial test the incipient failure state of the specimen usually can not be clearly identified on the stress-strain curve. It is suggested to define the incipient failure state of the specimen by the limit equilibrium condition which can be determined with the aid of following equations for two different cases as shown in Fig. 1:

Case I: Failure by extension (Fig. 1a)

$$\Delta\sigma_1 > \frac{\sigma_1-\sigma_3}{\sin\phi'} \quad (1)$$

$$\sin\phi' = \frac{-\sigma_1+\sigma_3+\Delta\sigma_1}{\sigma_1+\sigma_3-\Delta\sigma_1-2u_0-2\Delta u_{cr}} \quad (2)$$

$$\Delta u_{cr} = \frac{\sigma_1+\sigma_3}{2} - \frac{\sigma_1-\sigma_3}{2\sin\phi'} - \frac{\Delta\sigma_1(1+\sin\phi')}{2\sin\phi'} - u_0 \quad (3)$$

Case II: Failure by compression (Fig. 1b)

$$\Delta\sigma_1 \leqslant \frac{\sigma_1-\sigma_3}{\sin\phi'} \quad (4)$$

$$\sin\phi' = \frac{\sigma_1-\sigma_3+\Delta\sigma_1}{\sigma_1+\sigma_3+\Delta\sigma_1-2u_0-2\Delta u_{cr}} \quad (5)$$

$$\Delta u_{cr} = \frac{\sigma_1+\sigma_3}{2} - \frac{\sigma_1-\sigma_3}{2\sin\phi'} - \frac{\Delta\sigma_1(1-\sin\phi')}{2\sin\phi'} - u_0 \quad (6)$$

where σ_1 and σ_3 are the static axial and lateral pressures respectively. $\Delta\sigma_1$ is the amplitude of the axial cyclic stress; ϕ' is the angle of internal friction in terms of effective stress; u_0 is the initial pore water pressure; Δu_{cr} is the critical excess pore water pressure.

The shear stress τ_{fs}, that is the algebraic sum of the static shear stress τ_{f0} and the cyclic shear stress $\Delta\tau_f$ in the incipient failure plane of the specimen, is defined as

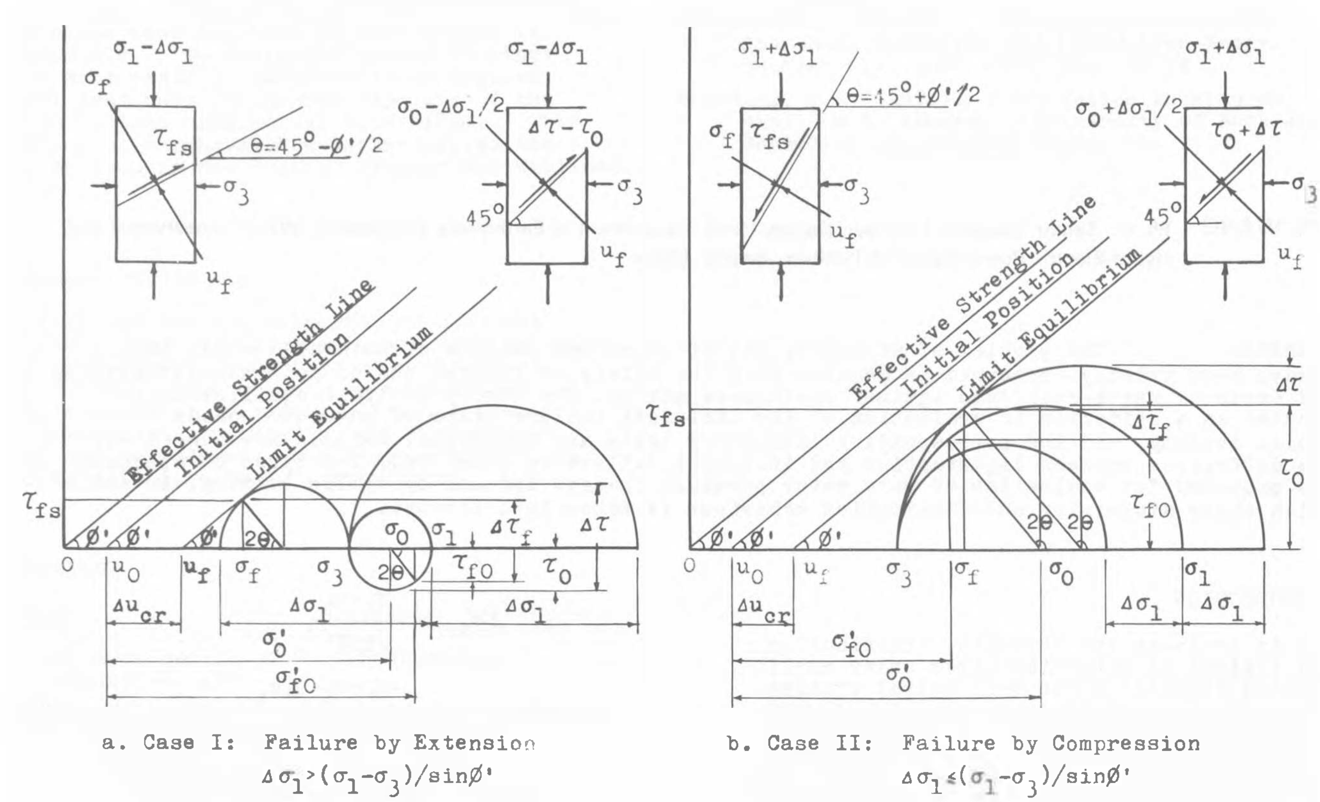

Fig. 1 Limit Equilibrium Condition of Saturated Sand in Cyclic Triaxial Test

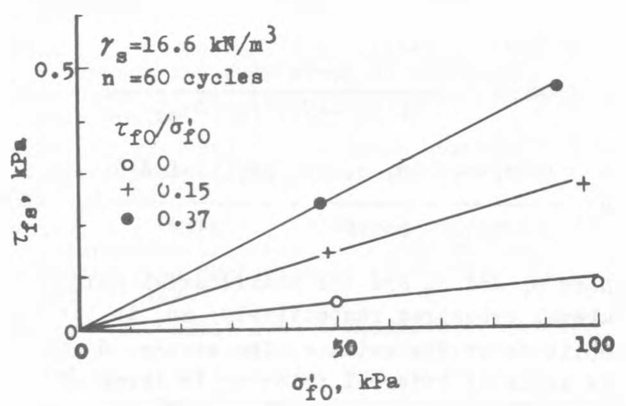

Fig.2 Shear Strength of a Saturated Sand under Cyclic Loading versus Initial Effective Normal Pressure with Different Initial Shear Stress Ratio

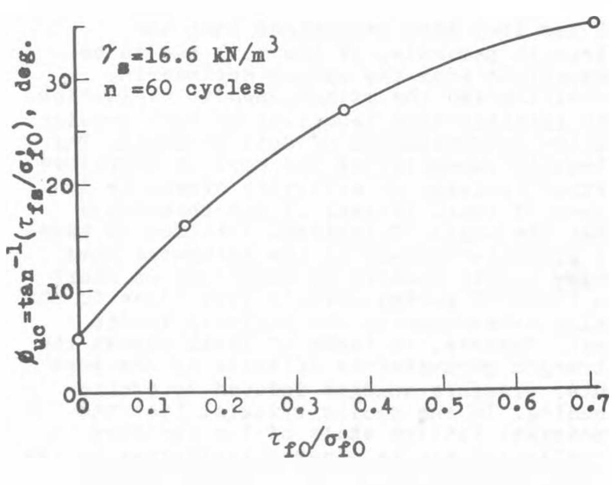

Fig. 3 Apparent Angle of Internal Friction in Terms of Total Stress of a Saturated Sand under Cyclic Loading versus Initial Shear Stress Ratio

the shear strength of the saturated sand when it reaches the limit equilibrium condition under cyclic loading. The value of the shear strength τ_{fs} is in correlation with the number of loading cycles n, the initial normal effective pressure σ'_{f0} and the initial shear stress ratio τ_{f0}/σ'_{f0}, where σ'_{f0} is the initial normal effective pressure acting on the incipient failure plane of the specimen. For any assigned number of loading cycles n we can get a set of τ_{fs} versus σ'_{f0} curves with different values of τ_{f0}/σ'_{f0} as a parameter as shown in Fig. 2. For most consolidated undrained cyclic triaxial tests these curves are rather straight and radiate from the origin of the coordinate system. The angle ϕ_{uo} between each of these straight lines and the horizontal axis σ'_{f0} may be called the apparent angle of internal friction in terms of total stress of the saturated sand under cyclic loading. Fig. 3 shows the relation between ϕ_{uo} and τ_{f0}/σ'_{f0}.

LIQUEFACTION AND FAILURE

Role of Pore Water Pressure

Under cyclic loading the skeleton structure of the sand usually tends to be further densified. If the water in pores of the saturated sand is unable to be squeezed out promptly, then the pore water pressure will increase and the shear resistance between sand grains will decrease consecutively. As soon as the pore water pressure rises to such a magnitude which causes the shear resistance to be reduced to the value of the shear stress in the saturated sand, the limit equilibrium condition is reached. Further increasing in pore water pressure will make it unstable. If the pore water pressure continues to increase and finally reaches an ultimate value at which the shear resistance between sand grains becomes zero, then the saturated sand is said to be liquefied.

Incipient Failure versus Liquefaction

The limit equilibrium condition is adopted by the writer to define the incipient failure state of the saturated sand under cyclic loading, at which the shear stress is just balanced by the shear resistance. On the other hand, at the state of liquefaction the shear resistance in the saturated sand should be zero. The difference between incipient failure and liquefaction can therefore be distinguished.

Evaluation of Pore Water Pressure Changes

Because the occurrence of liquefaction or failure of the saturated sand due to cyclic loading is essentially governed by the pore water pressure changes as having been mentioned above, therefore it is important at the first place to evaluate the pore water pressure changes and their distribution within the saturated sand mass after the commencement of cyclic loading. Two types of approach are proposed as given in the following.

(1) Close type

In the close type pore water pressure evaluation approach it is assumed that each of the elementary units of the saturated sand mass is kept closed with the neighbouring ones and is not permitted to drain during cyclic loading. Therefore the pore water pressure changes at any point in the saturated sand mass can be determined directly from the consolidated undrained cyclic test with the similar stress condition as in the field. This type of treatment was originally suggested by Huang (1961) and the cyclic triaxial test apparatus was first used for this purpose. After some refinements have been made by the writer in recent years, the test results can now be presented by the relationship between the excess pore water pressure ratio $(\Delta u/\sigma'_0)_n$ at the end of nth loading cycle, the cyclic shear stress amplitude ratio $\Delta\tau/\sigma'_0$ and the initial shear stress ratio τ_0/σ'_0 as shown in Fig. 4, where $\sigma'_0 = (\sigma_1 + \sigma_3)/2-u_0$, $\Delta\tau = \Delta\sigma_1/2$ and $\tau_0 = (\sigma_1-\sigma_3)/2$ (as shown in Fig. 1). If the values of σ'_0, τ_0 and $\Delta\tau$ in the saturated sand mass have been determined, then the values of $(\Delta u/\sigma'_0)_n$ can be located from Fig. 4 and the excess pore water pressure Δu after n cycles of loading can be computed by

$$\Delta u = (\Delta u/\sigma'_0)_n\sigma'_0 \qquad (7)$$

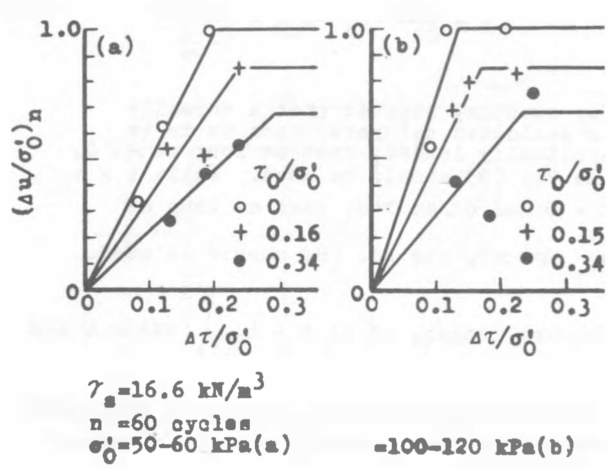

Fig. 4 Excess Pore Water Pressure Ratio at End of nth Cycle of Loading Versus Cyclic Shear Stress Amplitude Ratio with Different Initial Shear Stress Ratios

(ii) Open type

If the restriction of undrained condition of every elementary unit in the saturated sand mass is released, then the pore water pressure increments generated by cyclic loading will disperse and dissipate through unsteady seepage flow. Based on the compression, rebound and recompression characteristics of the sandy soil, the writer (Wang, 1962 and 1980) has derived following equations.

In case without residual rebound

$$\frac{1}{\gamma_w} \operatorname{div} (k \operatorname{grad} u_d) = \alpha \frac{\partial}{\partial t} (u_d - \sigma'_{m0} - u_d^*) \qquad (8)$$

In case with residual rebound

$$\frac{1}{\gamma_w} \operatorname{div} (k \operatorname{grad} u_d) = \beta \frac{\partial}{\partial t} (u_d - \sigma'_{m0}) \qquad (9)$$

where γ_w is the unit weight of water; k is the coefficient of permeability of sand; u_d is the excess pore water pressure due to cyclic loading for natural drainage condition; u_d^* is the fictitious excess pore water pressure for close type condition as having been mentioned in last section (i): α and β are the coefficients of volumetric compression and rebound or recompression respectively of the sand under drained condition ; $\sigma'_{m0} = \sigma_{m0} - u_0$, in which σ_{m0} and u_0 are the mean total pressure and initial pore water pressure respectively; t is time.

Whether the Eq. (8) or (9) should be used is governed by the following function

$$A = \frac{\partial}{\partial t} \left(u_d - \sigma'_{m0} - \frac{u_d^*}{1 - \frac{\beta}{\alpha}}\right) \qquad (10)$$

For example, suppose that a normally consolidated saturated sand is to be cyclically loaded, then as long as $A < 0$, the Eq. (8) should be used. While $t = t_1$, $A = 0$ and $dA/dt > 0$, then as long as $\int_{t_1}^{t} A dt > 0$, the Eq. (9) should be used.

Further longer, if at $t = t_2$, $\int_{t_1}^{t_2} A dt = 0$ and $A \leqslant 0$, then the Eq. (8) should be used again.

Another equation needed is the equation of equilibrium, namely

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = -[M]\,\ddot{U}_g(t) \qquad (11)$$

where [M], [C] and [K] are the matrices of mass, damping and rigidity respectively; $\{\ddot{X}\}$, $\{\dot{X}\}$ and $\{X\}$ are the vectors of acceleration, velocity and displacement respectively; $\ddot{U}_g(t)$ is the earthquake acceleration of the bed rock.

The writer has worked out two computation examples before (Wang, 1962), in which σ_{m0} was assumed to be maintained unaltered during and after the cyclic loading.

COMMENTS ON STABILITY ANALYSIS

If the pore water pressures in the saturated sand mass under cyclic loading can be predicated, then the effective stress method is preferred. On the other hand, if the total stress method is used, then the shear strength determined by the method described in the first part of this paper is recommended. However, the results of the two methods may not be identical.

CLOSING REMARKS

To take the limit equilibrium condition as the incipient failure state of saturated sands under cyclic loading is suggested. In the evaluation of pore water pressure changes produced by cyclic loading the open type approach is recommended, in which the dispersion and dissipation behaviour of the excess pore water pressures has been taken into consideration and the corresponding equations for calculation are proposed. These are only thought to be very preliminary according to the writer's own intuition. Further studies are indeed necessary for substantiating their practical value and possibly making further improvements.

REFERENCES

Huang, W. X. (1961). Investigation on stability of saturated Soil Foundations and Slopes against Liquefaction. Proc. 5th ICSMFE, (2),629-631,Paris.

Wang, W.S. (1962). Generation, Dispersion and Dissipation of Pore Water pressures of Saturated Sands during Vibration. Proc. 1st National Conf. on Soil Mech. and Found. Engg.,China Civil Engg. Society, 224-235, Beijing.

Wang, W.S. (1980). Strength, Liquefaction and Failure of Saturated Sands under Cyclic Loading. J. Hydraulic Engg.,1, 14-27, Beijing.