Spatial Calculation of Slope Stability under Definite Surcharges

Calcul Spatial de la Stabilité de Talus sous des Charges Surfaciques Limitées

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SYNOPSIS Proceeding from the fact that too low safety factors contradictory to reality are established by means of procedures covering the plane state for calculating the slope stability in spatially limites failure patterns, a calculation method has been developed which approximately represents the spatial state. The final effects increasing stability are approximately represented by cone sections attached to a circular cylinder section.

The procedure is preferably suited for calculating the stability of homogeneous unseeped slopes that are loaded with a limited area on which surcharge is applied (e. g. vehicles, construction machinery or hoisting units). In this contribution, the fundamental statements and assumptions are discussed. Furthermore, diagrams and tables are presented which are summarized in a regulation of the national building supervisory board as a result of comprehensive variational computations by means of a computer programme for the abovementioned procedure as an aid for designing.

INTRODUCTION

In the practice of civil engineering, increasingly higher loads occur in the form of surcharges behind the slope edges of cuts and service trenches. This is due to the use of larger and thus heavier elements which are typical of civil engineering (e. g. precast concrete members for collectors, concrete pipes with large diameters and lengths), to the employment of larger and heavier construction machines and vehicles and to the limited space in development areas in town centres and factories under the conditions of reconstruction.

The specific national standard TGL 11 482/07 (1978), which is valid in the GDR, however, contains restrictive demands concerning the permissible surcharge behind the slope edge and its distance. In dependence on the soil type, a minimum distance from the toe of slope of 1.2 • H (noncohesive soils) or 1.5 • H (cohesive soils) is demanded for a uniformly distributed surcharge ≤ 10 kN/m^2. Due to their limited areas on which surcharge is applied, most of the construction machines and vehicles (excavators, cranes, lorries, bulldozers and others) considerably exceed the above-mentioned permissible surcharge (Bilz, 1979 a; 1979 b) and for meeting the demand made by the standard they should maintain a distance from the slope edge which, in many cases, is not possible for technological reasons. Especially for depths of cuts and trenches of > 3 m, this makes necessary hoisting units and excavators having larger working radii, which often do not belong to the standard equipment of smaller building firms.

SPATIAL CALCULATION OF THE SLOPE STABILITY

Only the plane (two-dimensional) case is covered by the methods for calculating the slope stability which are embodied in national and international standards or specifications (TGL 11 482/07, WAPRO 4.10, DIN 4084/02). The natural state in the case of a slope failure, however, is clearly characterized by a three--dimensional elongation which is finite also in the longitudinal direction of the slope. It is known from comparative analyses (Schubert, 1966; Baligh and Azzous, 1975) that for failure body lengths which are shorter than the 2.5fold slope height also in unloaded slopes the calculations based on the plane problem will yield too low safety values. For the case of a slope which is subjected to (concentrated) surcharges the deviations are still larger (Bilz, 1979 b; Bilz et al., 1980).

The analysis of the respective literature has shown us proposals for spatial (three-dimensional) slope calculations in some publications (Baligh and Azzous, 1975; Baligh et al., 1977; Hovland, 1977; Lefebre and Duncan, 1973; Schapiro, 1979; Vanmarke, 1977), but in only one case it has made possible the direct determination of line loads (Baligh et al., 1977). By this, the problem of the limited surcharges, which is of primary interest to us, could not be solved. The statements by Hovland can obviously be extended to cover the influence of surcharge; the procedure itself, however, is connected with a very high (semi--graphical) expenditure, which seems to be justified for the purpose proper - the investigation on the stability of dams in relatively narrow, V-shaped valleys.

Comperative investigations by using the representations given by Baligh and Azzous (1975) for unloaded slopes with the slope parameters and soil characteristics according to TGL 11 482/07 have shown an increase of safety of 20 to 25 % for the spatial case as compared with the two-dimensional calculation. By this, apparently a suitable basis was given for the derivation of a spatial calculation method for taking into account limited (definite) surcharges by using the fundamental statements by Baligh and Azzous (1975).

FUNDAMENTALS AND BOUNDARY CONDITIONS OF THE CALCULATION PROCEDURE

The calculation of the slope stability with the aid of a three-dimensional model according to Baligh and Azzous (1975) proceeds from a failure body constituting a segment from a cylinder having a finite length with cones attached **to it on both sides.** (Figure 1)

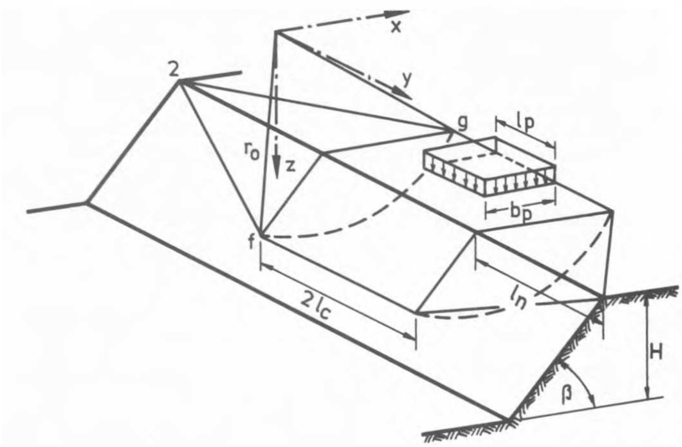


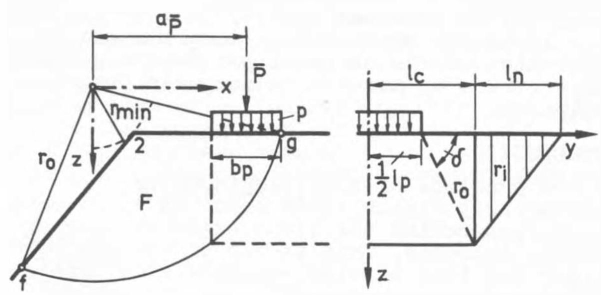
Figure 1. Geomet**ry and boundary** conditions

For such a failure body with an assumed cylinder radius (slip circle radius), all sorts of plane sections perpendicular to the slope edge are considered, and in them the slip-promoting and the slip-resisting moments are calculated (Bilz et al., 1980). Their determination is based on the mass force, on the one hand, and on the integral over the shear forces from friction and cohesion acting along the slip circle, on the other hand. If the plane section will cut the area of the application of surcharge, there have to be taken into account moments which are due to surcharge. To determine the resisting moment produced by surcharge, a distribution angle of $\delta = 60^o$ is assumed for the spreading of surcharge up to the slip circle (Bilz et al., 1980). This definition relies on data given by Tschebotarioff (1973) ($\delta = 60^o$), Kögler and Scheidig ($\delta = 55^o$), quoted in Tschebotarioff (1973), as well as according to TGL 33 342/02, and it is approximately equivalent to an angle of $45^o + \Phi/2$, the relevant friction angle Φ being taken into account.

To determine the safety which corresponds to a certain slip circle radius, integration is carried out over the slip-promoting and slip--resisting moments (having been calculated for each plane section) perpendicularly to the section plane. Here is to be taken into account for the resisting moment that integration proceeds along the slip plane, in which the moment originates. Safety is expressed by the quotient of the two integrals:

$$\eta \approx \frac{r_o l_c (T_1 + T_2) + r_o \bar{P} \frac{l_p}{2} \tan \Phi' + I_r}{W a \, l_c + \bar{P} a_{\bar{p}} \frac{l_p}{2} + I_d}$$

where $T_1 = \gamma \tan \Phi' \left\{ \left[\frac{1}{2} \left(x \sqrt{r^2 - x^2} + r^2 \text{ arc sin} \frac{x}{r} \right) \right]_{x_f}^{x_g} - \left[\frac{m}{2} x^2 + nx \right]_{x_f}^{x_2} - \left[z_2 x \right]_{x_2}^{x_g} \right\}$



$$T_2 = \left[c' r \text{ arc sin } \frac{x}{r} \right]_{x_f}^{x_g}$$

$z = m_x + n$ - equation of the slope area

$\bar{P} = p \cdot b_p$

$W = F \cdot \gamma$

$$I_r = \frac{1}{k} \sqrt{(r_o - r_{min})^2 + l_n^2} \sum_{i=1}^{k} M_r(y_i)$$

- **approximatively calculated integral of the resisting moments over the end cone**

$$I_d = \frac{l_n}{k} \sum_{i=1}^{k} M_d(y_i)$$

- **approximatively** calculated integral of the promoting moments over the end cone with r_o - the slip circle radius relevant to the specific case

r_{min} - radius of the (theoretical) slip circle, which is tangent to the slope line

F - area of the slope section on the slip surface

a - pertinent moment arm ∥ x-axis

$a_{\bar{F}}$ - moment arm of $\bar{F}$

(for the other designations see Fig. 1)

The length of the cylindrical part of the failure body was defind in such a way that with the above-mentioned assumed surcharge distribution at an angle of 60° the part of the slip surface influenced by it will lie within the cylinder. The height of the attached cones was assumed as half the slope height, since it had been found by comparative calculations that for surcharged slopes in most cases the safety minimum resulted in the range of $0.25\,H \leq l_n \leq 0.75\,H$, or $\eta \approx$ const. would occur. Furthermore, from these comparative calculations it is found that the variation of the slope angle as well as of the load distance from the slope edge experts relatively little influence on the position of the safety minimum, but that an increase of the slope height results in a shift of the position of η_{min} from $l_n \approx 0.25\,H$ to $l_n \approx 0.75\,H$.
An example is shown in Figure 2.

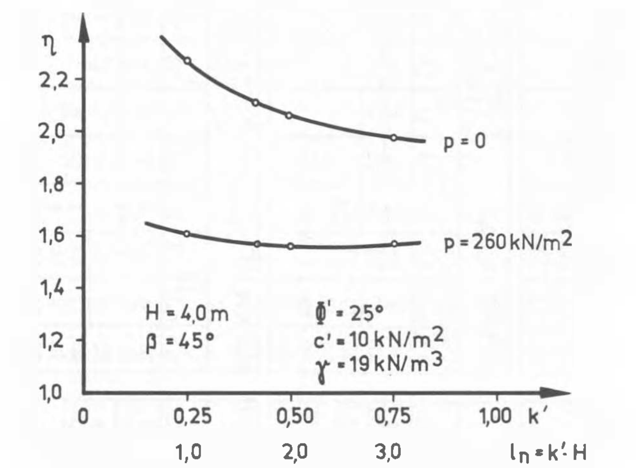


Figure 2. An example for the estimation of the influence of the length of the end cones upon safety

The safety calculation described proceeds from a previously defind slip circle radius. For determining the stability of a slope under surcharge, the slip circle radius is varied within specific bounds, and thus the minimum safety is calculated.

PRACTICAL APPLICATION

According to the demands made by practice, with the aid of the SLOPE 3-computer-programme extensive variational calculations were performed for typical parameters of slops for cuts and service trenches ($2.5\ m \leq H \leq 10\ m$; $30° \leq \beta \leq 80°$) as well as representative groups of surcharges (Bilz, 1979 a; 1979 b). Since due to the larger number of variables given the expenditure of evaluation and, mainly, of representation for the results of computations would have been increased many times in the case an additional free variation of the parameters for the soil characteristics (Φ', c', γ), in accordance with commonly used combinations of soil characteristics (Striegler and Werner, 1969, TGL 11 482/07, DIN 1055/02, TRL 2.4.6 - 1) the shear strength parameters Φ' and c' or c_u (as the capillary cohesion in sands and gravels) were selected according to Figure 3 for nine soils which are typical in the GDR.

The results are summarized in a number of tables within the regulation 84/79 of the national building supervisory board, which is already valid for the design practice. Table I shows an example of the surcharge groups $q_1 = 230\ kN/m^2$ and $q_2 = 900\ kN/m^2$, which, are equivalent to, (e. g.) a mobile crane MDK 204 being stabilized on slabs with $(1.0 \times 1.0)m^2$ or on slabs with $(0.3 \times 0.3)m^2$. All design tables are constructed according to the same pattern. For the safety factors which are in common use for cuts and service trenches or which are fixed in other regulations (Bilz et al., 1980) of $\eta = 1.1$ and 1.3, the respective required surcharge distance from the slope edge can be read from the sub-tables in dependence on the slope parameters and the soil characteristics. In each case, the unloaded strip required from the viewpoint of labour safety is taken as a minimum for this distance. On the other hand, the tables allow us to determine the permissible slope parameters also in the case of a technologically specified surcharge distance from the slope edge.

Necessarily, in the tables design bounds are represented (as horizontal lines), at which the limit values for the unloaded slope are reached. Here, a distinction is made between slopes with a theoretically infinite length with $L > 2\,H$ (full lines and short slopes with $L \leq 2\,H$ (broken lines).

Analogously to the large number of diagrams already published in literature concerning the two-dimensional case of unloaded slopes and being in common use, especially for short cuts with $L \leq 2\,H$ (L - slope length at the upper edge) the diagram represented in Figure 4 was developed. In the regulation 84/79 it is specified that the application of the tables and diagrams is permissible to:

- single slopes on cuts and service trenches
- systems of slopes which are interrupted by benches and can be approximately substituted by a mean slope an almost homogeneous soil in the range of the potential failure body
- unseeped or unflooded slopes or potential failure bodies.

TABLE I Estimation of necessary distance from the surcharge to the top of slope (example)

$q_1 = 230\,kN/m^2$ $\quad$ $q_2 = 900\,kN/m^2$

$b_{q1} = 1{,}00\,m$ $\quad$ $l_{q1} = 1{,}00\,m$

$b_{q2} = 0{,}30\,m$ $\quad$ $l_{q2} = 0{,}30\,m$

Validity limit for L > 2H and L ≤ 2H

T $\Phi' = 15°$ $\gamma = 20\,kN/m^3$ $c' = 25\,kN/m^2$

β \ H	2,5 η=1,1	2,5 η=1,3	5,0 1,1	5,0 1,3	7,5 1,1	7,5 1,3	10,0 1,1	10,0 1,3
30	e	e	e	e	e	e	e	e
40								
50								e
60								
70					e	e	e	
80	e	e	e	e	1,3			

UT $\Phi' = 20°$ $\gamma = 18{,}5\,kN/m^3$ $c' = 10\,kN/m^2$

β \ H	2,5 η=1,1	2,5 η=1,3	5,0 1,1	5,0 1,3	7,5 1,1	7,5 1,3	10,0 1,1	10,0 1,3
30	e	e	e	e	e	e	e	e
40		e				e	e	2,6
50		0,7		e	e	2,8	1,7	
60	e	1,3	e	1,3	1,6			
70	0,9	1,6	1,4	3,0				
80	1,3	1,9	2,4					

SU, ST, KT $\Phi' = 30°$ $\gamma = 20\,kN/m^3$ $c' = 10\,kN/m^2$

β \ H	2,5 η=1,1	2,5 η=1,3	5,0 1,1	5,0 1,3	7,5 1,1	7,5 1,3	10,0 1,1	10,0 1,3
30	e	e	e	e	e	e	e	e
40								
50								e
60						e	e	2,3
70		e		e	e	2,2	1,4	
80	e	0,9	e	1,7	1,5			

SN, SE, SF $(J_D > 0{,}4)$ $\Phi' = 32°$ $\gamma = 17\,kN/m^3$ $c_u = 3{,}5\,kN/m^2$

β \ H	2,5 η=1,1	2,5 η=1,3	5,0 1,1	5,0 1,3	7,5 1,1	7,5 1,3	10,0 1,1	10,0 1,3
30	e	e	e	e	e	e	e	e
40						e	e	1,3
50				e	e	1,5	1,2	2,0
60		e	e	1,3	1,2			
70		0,9	1,1					
80	e	1,2	1,5					

SA, SN, SE $(J_D > 0{,}4)$ $\Phi' = 35°$ $\gamma = 18{,}5\,kN/m^2$ $c_u = 6\,kN/m^2$

β \ H	2,5 η=1,1	2,5 η=1,3	5,0 1,1	5,0 1,3	7,5 1,1	7,5 1,3	10,0 1,1	10,0 1,3
30	e	e	e	e	e	e	e	e
40								
50								e
60						e		1,9
70		e		e	e	2,1	e	
80	e	1,0	c	1,9	1,7			

TU $\Phi' = 17{,}5°$ $\gamma = 20\,kN/m^3$ $c' = 20\,kN/m^2$

β \ H	2,5 η=1,1	2,5 η=1,3	5,0 1,1	5,0 1,3	7,5 1,1	7,5 1,3	10,0 1,1	10,0 1,3
30	e	e	e	e	e	e	e	e
40								
50								e
60						e	e	
70				e	e	1,7	1,9	
80	e	e	e	1,2	2,1			

U SF, SE, SN $(J_D < 0{,}4)$ $\Phi' = 25°$ $\gamma = 17\,kN/m^3$ c' bzw. $c_u = 7\,kN/m^2$

β \ H	2,5 η=1,1	2,5 η=1,3	5,0 1,1	5,0 1,3	7,5 1,1	7,5 1,3	10,0 1,1	10,0 1,3
30	e	e	e	e	e	e	e	e
40		e		e		e		1,3
50		0,9		0,9	e	2,0	e	
60	e	1,3	e	2,1	1,3			
70	0,9	1,7	1,4					
80	1,4	2,0	2,1					

KA, KE $(J_D < 0{,}4)$ $\Phi' = 32°$ $\gamma = 18{,}5\,kN/m^3$ $c_u = 1\,kN/m^2$

β \ H	2,5 η=1,1	2,5 η=1,3	5,0 1,1	5,0 1,3	7,5 1,1	7,5 1,3	10,0 1,1	10,0 1,3
30	e	e	e	e	e	e	e	e
40	0,7	1,3	e	1,5	e		e	
50	1,1	1,7	1,2					
60	1,4							
70	1,6							
80	1,9							

KA, KN, KF $(J_D > 0{,}4)$ $\Phi' = 37°$ $\gamma = 18{,}5\,kN/m^3$ $c_u = 2\,kN/m^2$

β \ H	2,5 η=1,1	2,5 η=1,3	5,0 1,1	5,0 1,3	7,5 1,1	7,5 1,3	10,0 1,1	10,0 1,3
30	e	e	e	e	e	e	e	e
40						e		e
50		e		e		1,1		1,7
60		1.1		1,5	e		e	
70	e	1,4	e		1,7			
80	1,0	1,6	1,5					

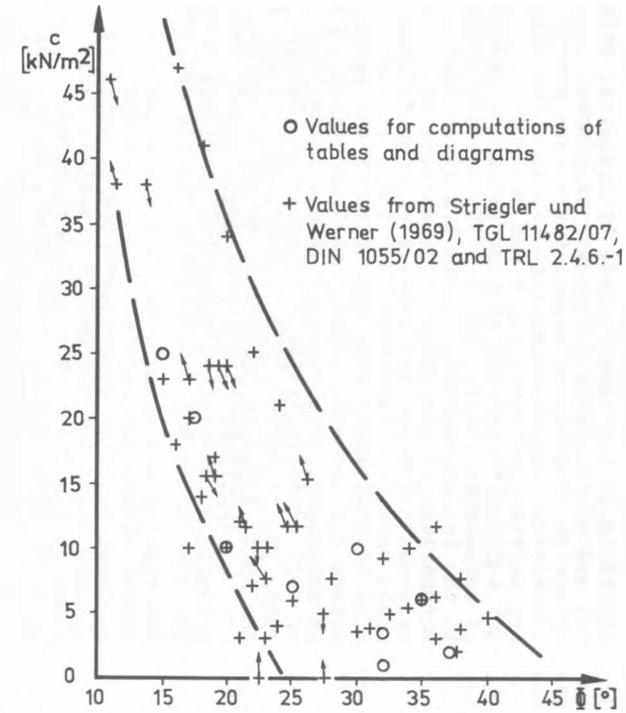


Fig. 3 Representative shear strength parameters for GDR-typical soils

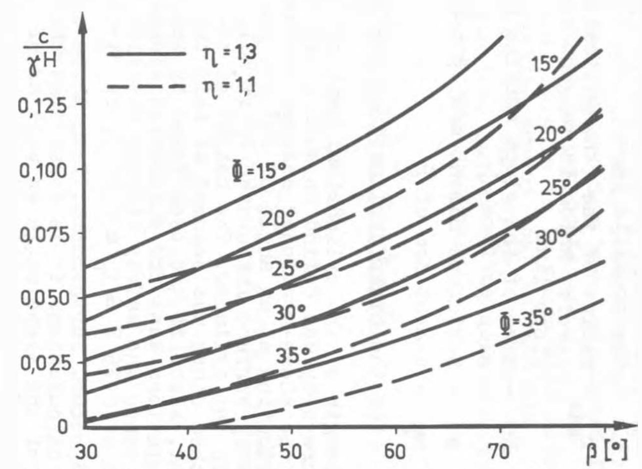


Fig. 4 Design diagram for excavations of limited length (L ≤ 2H)

If the slope-forming soils are layered, then one can calculate with mean soil characteristics, if the following criteria are observed:

$$\frac{\max \tan \Phi'}{\min \tan \Phi'} \leq 1.6 \qquad \frac{\max c}{\min c} \leq 3.0 \qquad \frac{\max \gamma}{\min \gamma} \leq 1.3$$

If one of the criteria mentioned before is exceeded, separate verifications have to be furnished for the total slope with the soil characteristics of the individual layers; the most unfavourable parameters resulting from this are decisive.

Table II Increase in safety due to the three--dimensional stability calculation

$\frac{c}{\gamma \cdot H}$	0.50	0.30	0.20	0.10	0.08	0.06	0.04	0.02
$\frac{\eta_2}{\eta_3}$	1.20	1.18	1.15	1.11	1.09	1.07	1.05	1.02

SUMMARY AND CONCLUSIONS

1. A spatial (three-dimensional) method for calculating the stability of slopes of cuts and trenches is represented with its fundamentals, boundary conditions and application limits. The procedure is based on a model by Baligh and Azzous and takes into account cone-shaped slip surfaces at the slope ends.
2. This procedure is especially suited for proving stability under the influence of limited (defined) surcharges behind the slope edge. Point and line loads can be determined, too. For slopes with a limited length ($L \leq 2$ H) which are unloaded or loaded by uniformly distributed, practically unlimited surcharges, the safety-improving effect can be utilized, too.
3. This procedure is applicable to the general case, i. e. to soils with friction and cohesion.
4. The safety-increasing effect of the spatial calculation is clearly dependent on the factor $\frac{c}{\gamma H}$ and only slightly dependent on the angle of slope and the angle of internal friction. In the range analysed, the average increase in safety amounted up to 20 % (cp. Table II).
5. The results of **many variational** calculations are summarized in design tables and diagrams, which are available to designers in the form of an national regulation. A selected example is represented. The regulation contains ten tables of this kind for representative groups of surcharges (excavators, cranes, bulldozers, lorries).

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