Simplified Search for Noncircular Slip Surfaces

Récherche des Surfaces Critiques de Glissement Non Circulaires

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SYNOPSIS The accuracy of slope stability analyses can be improved by studying noncircular as well as circular slip surfaces. The automated procedure described in this paper makes it simple and practical to search thoroughly for the critical noncircular surface, and thus to determine more reliably the minimum factor of safety.

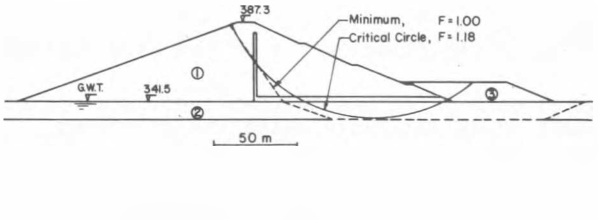
INTRODUCTION

Many methods of slope stability analysis have been developed, such as the Ordinary Method of Slices, Bishop's Modified, Morgenstern and Price's, Janbu's, and Spencer's methods. Each of these methods provides a means for calculating the factor of safety for given slip surfaces. They do not include techniques for locating the slip surface with the minimum factor of safety. This important aspect of the analyses is accomplished by repeated trial--by analyzing many possible slip surfaces to locate the critical one.

When circular slip surfaces are studied the repeated trials can be done methodically, and a large number of computer programs have been developed which search automatically for the critical circular slip surface. The same is not true for noncircular slip surfaces. Because of the greater difficulties involved in automating the search for noncircular surfaces, most available slope stability computer programs that can be used to analyze noncircular slip surfaces leave it to the user of the program to locate the most critical slip surface, using judgment and repeated trials, outside the computer. This adds considerably to the time and cost of the analysis, and the results are less reliable: Such a search often fails to locate the most critical noncircular slip surface, and thus fails to evaluate correctly the minimum factor of safety.

Because of the great difficulties involved in locating critical noncircular slip surfaces, in most cases only circular slip surfaces are analyzed. This simplifies the analysis, especially when a computer program is used which performs the search automatically. In some cases, however, the most critical slip surface may not be approximated accurately by a circular arc. In these cases the apriori assumption that the slip surface is circular will result in an error of unknown magnitude: The smallest value of safety factor found by studying circular arcs will be higher than the correct minimum value, and the results will thus overestimate the actual stability of the slope.

An example of a case where analyses of circular slip surfaces gives a misleading estimate of the factor of safety is the Agua Vermelha Dam embankment, shown in Fig. 1. The upper 10 m of the foundation are weak highly weathered rock with a low residual angle of shearing resistance. Circular arc analyses, (performed using Bishop's Modified Method) resulted in location of the critical circle shown in Fig. 1, which has a factor of safety equal to 1.18. Analyses of noncircular surfaces (performed using Spencer's Method) resulted in location of the noncircular surface shown in Fig. 1, which has a factor of safety about 15% lower than the critical circle. This difference is not due to the use of two methods of analysis. Both give the same factor of safety for the critical circle. Thus, in the case of Agua Vermelha Dam embankment the most thorough possible slope stability analysis performed using only circular slip surfaces would have resulted in a calculation error of 15% on the unconservative side, and an unconservative evaluation of the safety of the embankment. This is not an extreme example. In some cases the error due to analyzing only circular slip surfaces will be greater, and in other cases it will be less than 15%.



Material	γ (t/m^3)	c (t/m^2)	ϕ (°)
1 - Compacted Fill	2.0	0.5	29
2 - Highly Weathered Basalt (residual soil)	2.0	0	10*
3 - Dumped Berm	1.8	0	0

* Residual angle of shearing resistance

Fig. 1 Analyses of Agua Vermelha Dam using Residual Friction Angle in Foundation

The search procedure described in the next section provides a simple methodology which can be adapted for use with any slope stability analysis program capable of analyzing noncircular slip surfaces. Using this search method brings the power of the computer to bear on both aspects of slope stability analyses with noncircular slip surfaces--calculating the factor of safety and locating the critical slip surface.

The theoretical basis of the search procedure is described in the following section, and subsequent sections describe its application to various types of problems.

THE SEARCH PROCEDURE

A simple numerical method, applicable to practical problems involving layered soils and other complexities, is used to search for the critical slip surface. An alternative is the use of analytical procedures (Garber, 1973; Chen and Snitbham, 1975; Biernatowski, 1976; Revilla and Castillo, 1977; Baker and Garber, 1978).

To search for the noncircular surface with the lowest factor of safety, a number of repeated trials are performed. Each trial begins by shifting each of the points defining the slip surface to two new positions, and calculating the factor of safety. As this is done, all of the other points are kept at their original estimated positions, as shown in Fig. 2. The first and second shifts are always equal and in the same direction.

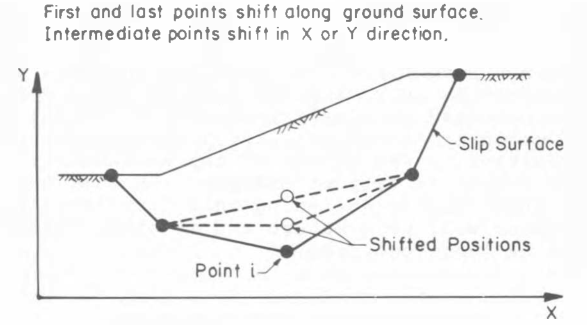


Fig. 2 Shifting Points on the Slip Surface

After each point has been shifted, its optimum position is estimated using the following equations:

$$x_i^* = x_i^o + \frac{\Delta x}{2} + \Delta x \frac{F_o - F_1}{F_o - 2F_1 + F_2} \qquad (1a)$$

$$y_i^* = y_i^o + \frac{\Delta y}{2} + \Delta y \frac{F_o - F_1}{F_o - 2F_1 + F_2} \qquad (1b)$$

where x_i^*, y_i^* = estimated optimum coordinates of point i; x_i^o, y_i^o, = original coordinates of point i; Δx, Δy = shift increments; F_o = factor of safety (F) before shifting; F_1 = F after the first shift of point i; F_2 = F after the second shift of point i.

These equations are derived according to a minimizing scheme which assumes that for each iteration: (1) the optimum position for a point can be estimated based on the change in F caused by the movement of this point only, and (2) the partial minimum of F with respect to a single variable x_i or y_i can be estimated using a local approximation of the function F versus x_i or y_i by a parabola of second degree. It is clear, however, that these assumptions are used only for piecewise local approximations of F. Because the process is repeated for a number of iterations, it is effective for funtions that do not obey these assumptions globally.

This method is described in the literature of nonlinear programming among others for minimization of general multivariable functions. Burley (1974), for example, calls the scheme described by assumption (1) the "alternating variable method." Assumption (2) is called "quadratic fit"; it is widely used for minimizing general functions along particular directions. It is also described by Burley (1974).

To check whether the values of x_i^* and y_i^* estimated using equations (1a) and (1b) are reasonable, the factor of safety of the slope is calculated with point i moved to x_i^*, y_i^* and all other points in their original positions. If this value of F (called F*) is smaller than F_o, the values of the coordinates x_i^* and y_i^* are retained for use in the next step of the iterative process. If not, then the point is returned to the position which corresponds to the lowest value among F_o, F_1 and F_2.

This process is repeated for each point in turn for a number of iterations. To check whether or not a minimum was reached after an iteration has been completed, the partial derivatives of F with respect to all x_i and y_i are calculated. If all are smaller than a specified value, the search is terminated. If not, it is continued. When the search has been completed using a given shift increment, another may be performed using a smaller shift increment.

SHIFT INCREMENTS AND ITERATIONS

In principle, the points defining the slip surface can be shifted in any direction. In practice it has been found to be effective to shift the end points along the ground surface, and to shift interior points vertically or horizontally.

If the shift increment used is too large, the slip surface will assume an awkward shape. When this happens the calculations to determine the factor of safety may encounter numerical problems, and it may then be necessary to begin the process again using a smaller shift increment. Thus, although large shift increments often result in faster convergence to the final shape of the slip surface, they may cause numerical problems which lead to abortion of a computer run.

Usually three shift increment values, each smaller than the one before, are used in a computer run. Experience has shown that a reasonable value for the initial shift increment is about 10% of the least distance between neighboring points defining the slip surface. Subsequent values are typically about one half and one quarter of the initial value. For each value of the shift increment, as many as five or six iterations may be required.

Because the acceptable size of the shift increment is related to the spacing of points along the slip surface, using a large number of points to define the slip surface requires the use of smaller shift increments and slows convergence. The writers have found that it is usually acceptable to define the slip surface using four to six points. Between these points more than one slice is used if needed to accommodate changes in soil properties in different zones within the slope, and these are added automatically by the computer program.

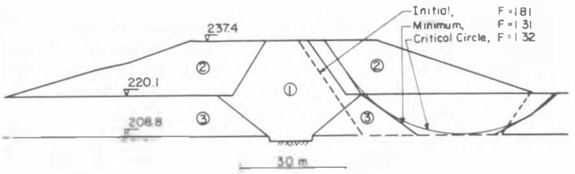
EXAMPLES

To illustrate the use and results of this procedure, two examples are described in the following paragraphs.

Birch Dam

The purpose of this example is to show that, in the absence of conditions which cause the slip surface to deviate from a circular shape, the critical surface located using the search procedure will be close to the critical circle.

Birch Dam, shown in Fig. 2, is a Corps of Engineers dam in Oklahoma. Founded on soft alluvium, it was built in two stages to increase its short-term stability. The original stability analyses, performed during design, were done using Bishop's Modified Method. These analyses located the critical circle shown in Fig. 3, which has F = 1.32.



Material	γ (t/m^3)	c (t/m^2)	ϕ (°)
1	2.0	7.8	18
2	2.0	4.9	31
3	2.0	4.9	0

Fig. 3 Critical Circular and Noncircular Slip Surfaces for Birch Dam, End of First Stage Construction

Recently additional analyses of Birch Dam were performed using a computer program for Spencer's Method with the noncircular search procedure, and a slip surface defined by six points. The initial trial slip surface shown in Fig. 3 was purposely chosen quite different from the critical circle. The search involved 5 iterations with a shift increment of 2.5 m, 5 with a shift increment of 1.2 m, and 4 with a shift increment of 0.4 m. It may be seen that the final slip surface is very close to the critical circle, and the factor of safety is essentially equal to the value for the critical circle.

The difference in the factors of safety for Birch Dam which were calculated using circular and noncircular surfaces is not significant. Thus it may be seen that for conditions where the critical slip surface can be accurately approximated by a circular arc, the new search procedure gives virtually identical results, and it does so using very few points to define the slip surface.

Reinius' Model Tests

Reinius (1961) performed a series of model tests to study the mechanism of failure for triangular-shaped sand fills like those shown in Fig. 4, which simulated the downstream shell of zoned embankment dams with weak central cores. Sliding within the shell was induced by increasing the fluid pressure on the vertical face of the wedge.

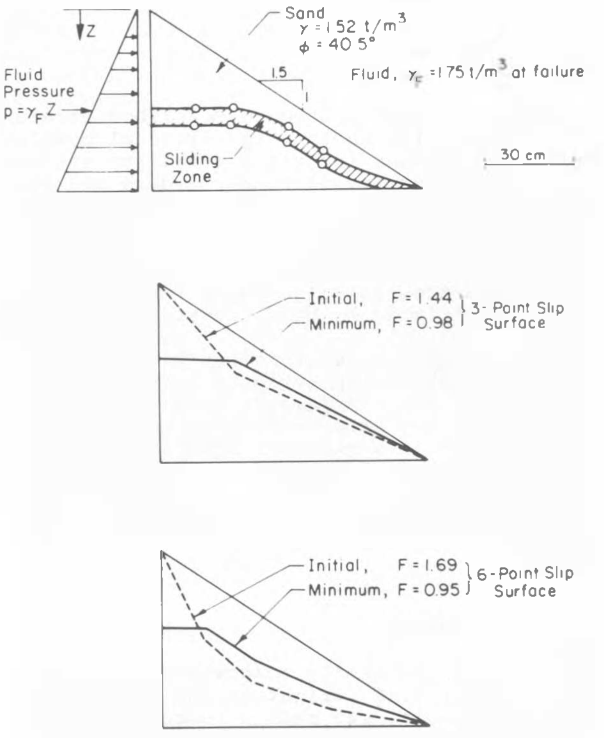


Fig. 4 Analyses of Model Tests Performed by Reinius (1961)

The shapes of the slip surfaces in the models were determined using vertical strips of colored sand. The slip surface (or band) observed in one of the models is shown in Fig. 4. It may be seen that this slip surface has an unusual shape; the upper portion is nearly horizontal, and the sloping portion includes a reversal of curvature.

The purpose of this example is to illustrate that the search procedure is capable of locating slip surfaces of such unusual shapes, which are believed to result from the pressure boundary conditions rather than nonhomogeneity or anisotropy of the soil.

Results for two analyses are shown in Fig. 4. The first, shown in the center part of the figure, was conducted using a slip surface defined by only three points. The initial slip surface, purposely chosen quite different in shape from that found experimentally, had F = 1.44. The critical surface, located after a total of 4 iterations with a shift increment of 6 cm, had F = 0.98. The second analysis, conducted using a slip surface defined by six points, is shown in the lower part of Fig. 4. After a total of 13 iterations, using shift increments of 3, 2, and 1 cm, the slip surface shown by the solid line was located. This slip surface has a calculated value of F = 0.95.

The critical slip surfaces located in both analyses correspond approximately to the shape observed in the laboratory tests, which illustrates the effectiveness of the search procedure for these unusual conditions, where the shape of the slip surface could not be closely approximated by an arc of a circle.

CONCLUSION

The simple search procedure described herein has proven effective in locating critical noncircular slip surfaces for slope stability analyses. A subroutine employing the procedure can be fairly easily adapted to any computer program for analysis of noncircular surfaces. (An example of such a subroutine is available without cost from the writers.) Use of this procedure offers great advantage over searching for critical noncircular surfaces by hand, outside the computer.

The ability to search efficiently and effectively for noncircular slip surfaces can improve the accuracy of slope stability studies in those cases where the slip surface is not accurately approximated as a circular arc. Such cases include conditions where the shape of the slip surface is constrained by a thin layer of weak material, anisotropic shear strength, or unusual boundary conditions.

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