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# Application of a Stress-Strain-Time Relation

## Application d'une Rélation Tensions-Déformations-Temps

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SYNOPSIS Modern numerical methods and computer technique offer a possibility for a refinement in estimation of time-dependent problems. This report outlines the utilizing of a general constitutive equation for solving time-dependent strain and stability problems. An example shows the applicability.

INTRODUCTION

The today used methods for estimation of stability consider the limit state of stress. It is assumed, that the form of the slip surface is known and all points along the slip surface are in the limit state. The sliding body is taken as ideal plastic and does not deform during the sliding process. The parameters of failure criterion must be used as being constant. Measurements and observations show contradictions between the real mechanism of motion and the assumptions of the physical model.

The development of modern mathematical methods and large computers give the possibility to utilize a general stress-strain-time relation also in soil mechanics. Soils are very inhomogenous and anisotropically. The properties vary in space and time. The present used experimental methods do not allow to get them correctly. Therefore in practical estimations simple relations for the behaviour of the material must be taken. They are connected with prescribed stress paths and with special rules for the experimental determination of soil parameters.

CREEP MODEL FOR SATURATED FINE-GRAINED SOILS

Often it is possible to use a plane state of strain. Fig. 1 shows as an example the general structure for a slope. In a region $\Omega$ there are acting body forces $F_i$. At one part of the boundary $\Gamma_\sigma$ are prescribed stresses $\sigma_{ij}$, at another part $\Gamma_u$ displacements $u_i$. Large deformations are taken into account by using the deformation tensor of Green. Assumption in our visco-plastic relation are zero stress rate. This means, that all changes in loading and geometry take place momentary. This is the same for softening in the state of failure. The real stress-strain behaviour is idealized as shown in Fig. 2. The used stress-strain relation describes the behaviour in the first

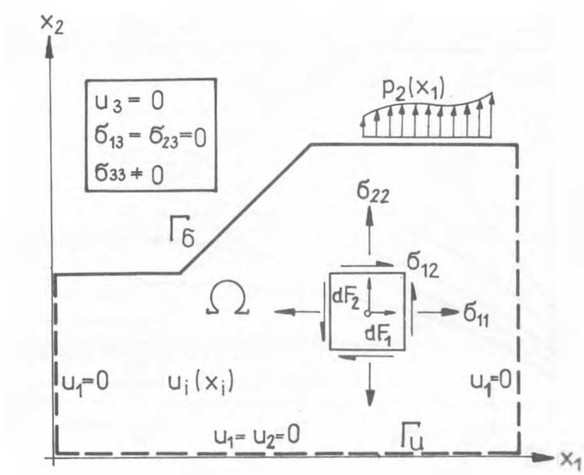


Fig. 1 Stresses and displacements in a state of plain deformation

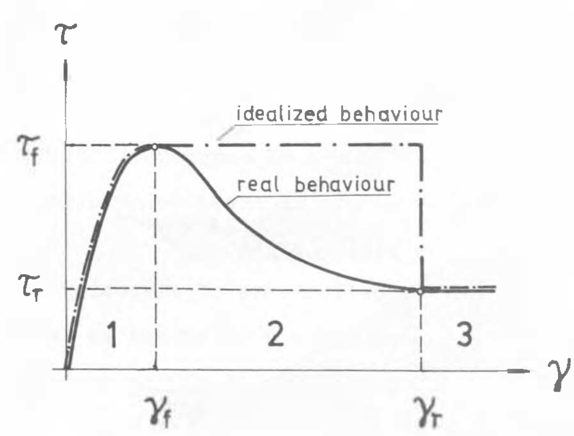


Fig. 2 Real and idealized stress-strain behaviour

and second section. Softening is realized by special conditions. The rheological model is that of Fedder (1973) as shown in Fig. 3. It involves all in fine-grained soils observed phenomena. Nonlinearities are approximated linearly in sections. Further assumptions are

(i) isotropy of the material

(ii) time-independent, linearly elastic behaviour of volume deformation

(iii) visco-plastic behaviour of distorsion

(iv) visco-elastic unloading

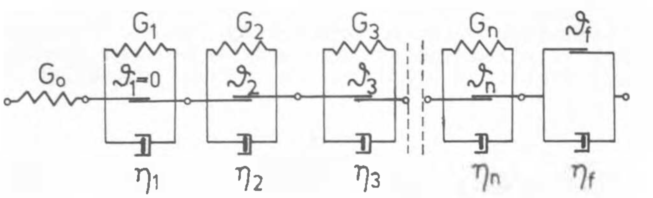


Fig. 3 Rheological model given by Fedder

So we have the following equations as relations between octahedral strains ($\varepsilon_0, \gamma_0$) and stresses ($\sigma_0, \tau_0$):

(i) for volume deformation

$$\varepsilon_0 = \frac{\sigma_0}{K} \tag{1}$$

(ii) for distorsion

$$\gamma_0 = \frac{\tau_0}{G_0} - \sum_{k=1}^{n}\left[\frac{\tau_0 + \vartheta_k \cdot \sigma_0}{G_k \cdot \sigma_0}\left(1 - e^{-\frac{G_k}{\eta_k}t}\right)\right] - \frac{\tau_0 + f_1 \cdot \sigma_0 - f_0}{\eta_f\, \sigma_0}\, t \tag{2}$$

valid if: $\sigma_0 < 0$ (3)

$$\gamma_0 \leq g_0 - g_1 \frac{\sigma_0}{\tau_0} \tag{4}$$

Parameters of material are

K - modulus of compressibility

$G_0, G_1 \ldots G_n$ - moduli of shear deformation

$\vartheta_2 \ldots \vartheta_n$ - yield limits

$\eta_1 \ldots \eta_n, \eta_f$ - coefficients of viscosity

$f_0, f_1$ - parameters of stress at failure

$g_0, g_1$ - parameters of strain at failure

Index f denotes the state of failure.

The behaviour of the material according to the above written equations shows Fig. 4. Fig. 4 a represents the relation between strains and stresses at different times. As shown in Fig. 4 b failure happens if two conditions are fulfilled:

(i) condition for the state of stress

$$\tau_0 \leq \tau_f = f_0 - f_1 \cdot \sigma_0 \tag{5}$$

a generalized Mohr-Coulomb criterion and

(ii) condition for the state of strain

$$\gamma_f = g_0 - g_1 \frac{\sigma_0}{\tau_0} \tag{6}$$

In Fig. 4 c are shown strain-time curves. For $\tau_0/\sigma_0$ = const. we have a fixed value of $\gamma_f$. The time of failure increases with decreasing $\tau_0$; $\lim_{\tau_0 \to \tau_f} t_f = \infty$ .

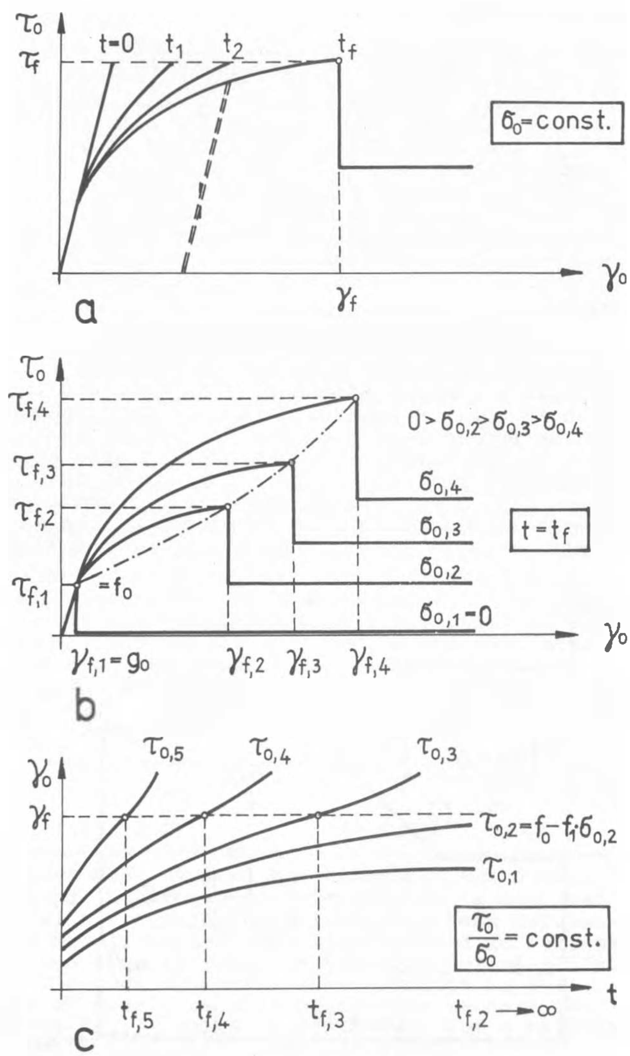


Fig. 4 Stress-strain-time relation

Creep tests in a triaxial apparatus were done. For estimation of failure parameters some of the tests were performed with stresses over long-terme strength. The results were analysed by utilizing a computer for determination of the above mentioned parameters.

EXAMPLE FOR APPLICATION

A developed computer programme (Georgi and Koehler 1979) using FEM is based on the described constitutive equation. The initial-boundary value problem was integrated over the time for an interval. The programme allows to solve problems with the following conditions:

(i) physically linear or nonlinear

(ii) geometrically linear or nonlinear

(iii) time-independent or time-dependent

The initial state of stress is prescribed or can be calculated by the computer. Changing the geometry of the structure and the loads in time can be done. The failure time of an element is computed and it is assumed this element can only be loaded by a state of stress consistent with the residual strength. The further stresses are supported by loadable elements (Malina 1969). In this way successive failure can be modeled.

We want to show the performance and results for the example of a slope. Fig. 5 shows the dividing of the continuum into finite elements. The initial state of stress was calculated under the condition of normal consolidation and then stored. The slope is formed by reduction of the initial mesh. The initial state of stress gives the nodel forces for the new structure. Then the stress-strain state is calculated on the base of the theory of linear elasticity. Before further computation it must be compared the state of stress in each element to decide between regions of loading and such of unloading to get the right constitutive equation for solving the nonlinear time-dependent problem. The condition of deciding

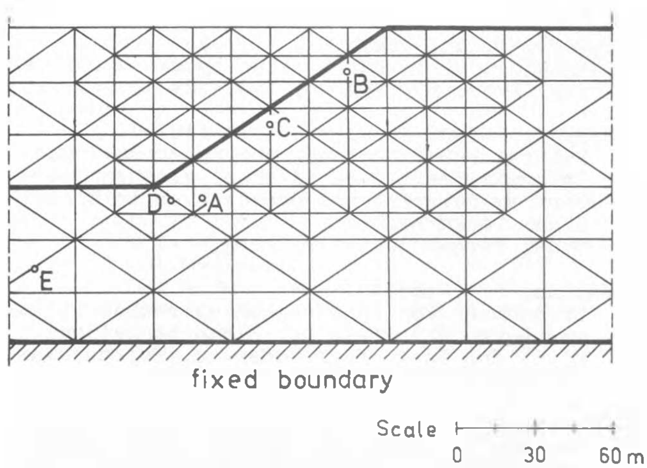


Fig. 5 Finite element mesh for the example

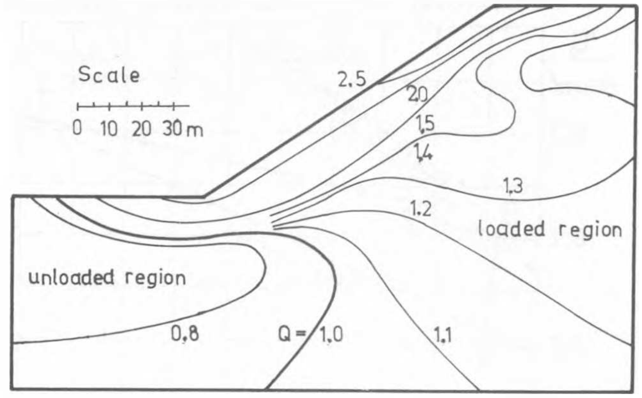


Fig. 6 Loaded and unloaded regions after forming of the slope

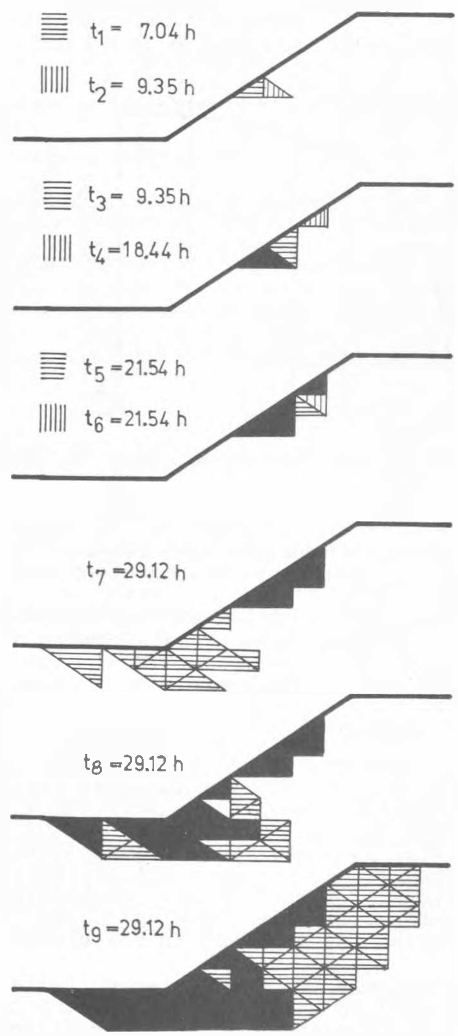


Fig. 7 Succession of failure

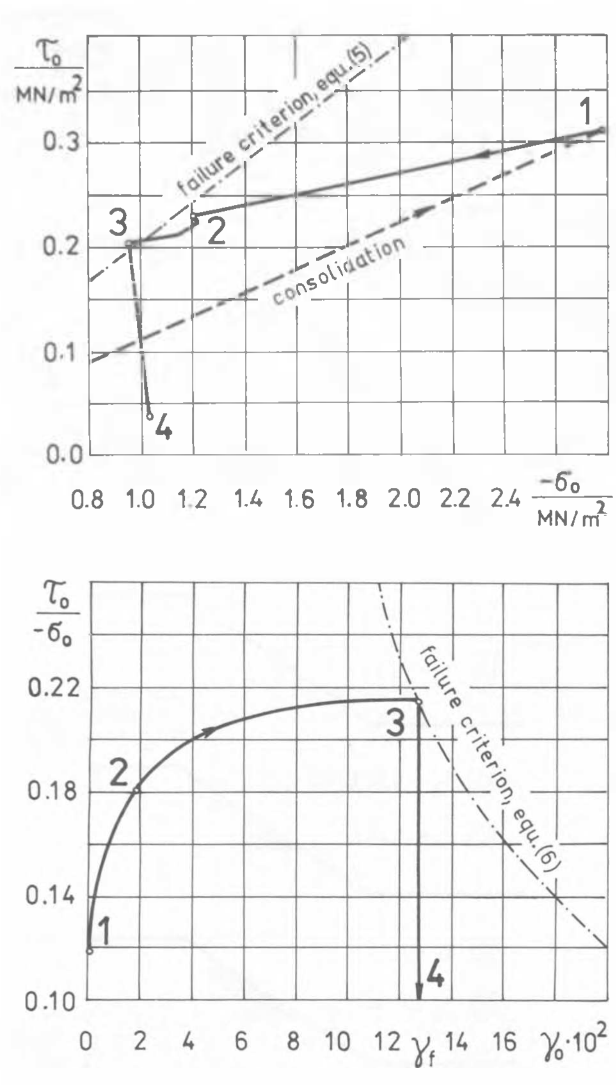


Fig. 8 Stress path and stress-strain behaviour of the element A

1 - initial state of stress
2 - stress after forming of slope
3 - failure state
4 - final state

is the quotient $Q = \lambda/\lambda_i$, where $\lambda = -\tau_0/\sigma_0$; index i denotes initial state. $Q < 1$ means unloading, $Q > 1$ means loading (see Fig. 6). The state of stress and strain provides a first approximation for failure time in the structure. With it a nonlinear calculation yields a corrected value for the start of failure. Further steps of iteration are necessary. The final state of strains comes up to the condition of equation (6) at least in one element.

In our example the first element failed after a time of 7.04 h. After further four time steps and nine steps of stress redistribution a general failure of the structure happened. The order of succession of failure in elements together with failure times is shown in Fig. 7.

It seems very interesting to represent stress paths for elements A, B, C, D, E (see Fig. 5). Fig. 8 illustrates the stress paths of element A and the stress-strain behaviour of the same element. Fig. 9 shows stress paths for the elements from B to E. Elements from A to D are always loaded, element E gets first loading and then unloading. The stress paths of elements from A to D are similar to an active compression test and the stress path of element E to an active extension test. For getting reliable soil parameters such tests must be performed.

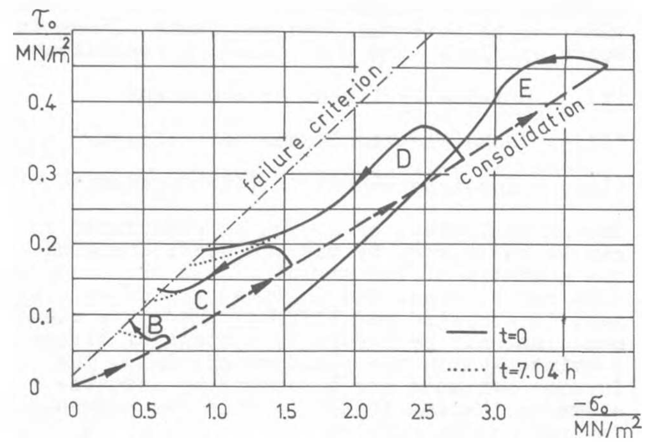


Fig. 9 Stress paths of elements B, C, D, E

## CONCLUSION

The described method provides a mean for computation of a stress-strain-time state in a continuum and allows to estimate the successive failure as a function of time. Provided that reliable soil parameters are available it is valuable for evaluation of long-term problems and long-term stability.

## REFERENCES


Fedder, D. and Breth, H. (1973).
Rheological investigations by a new apparatus. Proc. 8th ICSMFE, (1), 129-134, Moscow.

Georgi, P. and Koehler, H.-G. (1979).
Beitrag zur Berechnung von Spannungs- und Deformationsfeldern in rheologischen Koerpern am Beispiel von Boeschungen in bindigem Lockergestein.
Diss. Bergakademie Freiberg, 1-177.

Malina, H. (1969).
Berechnung von Spannungsumlagerungen in Fels und Boden mit Hilfe der Elementenmethode.
Mitt. des Inst. fuer Boden- u. Felsmechanik der Universitaet Karlsruhe, 40, 1-86.

Tschirner, N. (1974).
Der Einfluss der Zeit auf die Bruchfestigkeit eines dichten ungestoerten Tones beim einfachen Scheren.
Diss. Bergakademie Freiberg, 1-97.