From Hvorslev's Failure Criterion to Failure Condition

Du Critère de la Rupture par Hvorslev à la Condition de la Rupture

K.HASHIGUCHI and
M.UENO Faculty of Agriculture, Kyushu University, Fukuoka, Japan

SYNOPSIS M.J. Hvorslev (1960) has extended the Coulomb-Mohr's failure condition to express unifyingly the failure stresses of clays at various void ratios. However, the failure stresses can not be predicted by the Hvorslev's failure criterion itself. In this paper, at first, it is explained that a " failure state" corresponds to a "softening state" defined by the theory of plasticity and that the Hvorslev's criterion can not be regarded as a "failure condition" (softening yield condition) which expresses directly the failure stress, but can be regarded physically as a "constant volume condition" in the failure state. Then, this criterion is reformed to a failure condition which has a rather complex form. Moreover, two alternative failure conditions in the Coulomb-Mohr's form, which may be convenient for practical use, are provided.

INTRODUCTION

Hvorslev's failure criterion (1960) for clays is highly rated, as this criterion is an extended Coulomb-Mohr's failure condition to express unifyingly the failure stresses of materials at various void ratios. However, it is well-known that the failure stresses can not be predicted by this criterion itself. In this paper, at first, the definition of the term "failure" is given from the standpoint of the theory of plasticity and the mechanical meaning of the Hvorslev's criterion is discussed. Then, a failure condition based on this criterion, by which the failure stress can be predicted directly, is deduced. Further, two alternative failure conditions in a simple, linear form of the Coulomb-Mohr's equation are provided. Stresses, appear in this paper, mean the so-called effective stress.

FAILURE AND FAILURE CONDITION

Usually, it is thought that a failure occurs when the material becomes unable to bear the loads more than those applied at that time, and then the material breaks to two or more pieces. Accordingly, a failure stress is measured as the peak stress in a triaxial test, a direct shear test, etc. However, this interpretation of the failure is not a definite one based on mechanical reasons but is merely an intuitive one. Then, in the following, the failure is rationally defined from the standpoint of the theory of plasticity, since the failure state can be regarded as an extreme of the plastic state. Increasing the stress applied to the soil, microscopically, soil particles begin to slip each other remarkably at certain stress, and this results macroscopic, plastic deformation of the soil as an assemblage of the particles. Such stress, i.e., an yield stress is represented by an yield surface in stress space. It is idealized that any change of the stress within an yield surface generates not a plastic deformation but an elastic deformation. Further, as the plastic deformation proceeds, generally the yield stress varies, and a hardening or softening of the soil occurs. Now, it can be interpreted that the hardening or the softening of the soil occurs with an irreversible, i.e., plastic change of the void ratio or the volume, though it is not a change of the void ratio or the volume itself, since the elastic deformation does not generate any irreversible change of materials. Thereupon, a quantitative measure of the hardening or the softening can be represented by the specific volume $\bar{V}$ of the soil at a definite stress $\overline{\sigma}_{ij}$. Accordingly, the yield condition for the soil is represented by

$$f(\sigma_{ij}) - F(\bar{V}) = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

where f and F are functions of the stress σ_{ij} and $\bar{V}$ respectively.

Now, under a low pressure, the plastic deformation proceeds with a volume expansion opposing the pressure, and accordingly a lowering of strength, i.e., a softening occurs. On the other hand, under a high pressure, the plastic deformation proceeds with a volume contraction, and the hardening occurs. At a critical state, the plastic deformation proceeds with no plastic volume change, and neither the hardening nor the softening occurs. The failure occures when the deformation proceeds under a constant or rather decreasing stress. Therefore, the failure state can be defined as a softening state including the above-mentioned critical state . An yield condition related only to the failure state, excluding the hardening state, can be called a failure condition.

HVORSLEV'S FAILURE CRITERION AND AN EXPLICIT FAILURE CONDITION DERIVED FROM IT

M.J. Hvorslev (1960) has proposed that the following equation holds at the failure state

$$\mathscr{F} - P_{yn}\exp\left(\frac{V_{yn} - V}{\lambda}\right) = 0 \quad \ldots\ldots \quad (2)$$

where V_{yn} is a value of the specific volume V in an isotropic, normally consolidated state at which a mean stress $P = P_{yn}$ is applied to soils. λ is a compression index in the plastic (normally consolidated) state. $\mathscr{F}$ is denoted by

$$\mathscr{F} = \frac{1}{K_e\cos\phi_e}\left(\frac{\sigma_I - \sigma_{III}}{2} - \frac{\sigma_I + \sigma_{III}}{2}\sin\phi_e \right) \quad \ldots\ldots\ldots \quad (3)$$

where σ_I and σ_{III} are maximum and minimum principal stresses respectively. ϕ_e and K_e are the effective angle of friction and the coefficient of cohesion defined by Hvorslev, respectively.

Parameters related to the elastic deformation vary in the process of deformation up to the yield state, whereas those related only to the plastic deformation does not vary in this process. Since an yield condition includes only the latter parameter, it represents the yield stress directly. Although Hvorslev has called eq. (2) the "failure criterion", it includes the former parameter, and therefore this is not the failure condition defined before from the standpoint of the theory of plasticity, and then the failure stress can not be predicted by this equation.

The failure condition denotes the stress states at which the failure occurs in a certain degree of the hardening or the softening , for the fixed values of the plastic hysteresis parameters included in this condition, whereas the Hvorslev's criterion denotes the stress states at which the failure proceeds in a constant volume, for the fixed values of the parameter V included in this criterion. Accordingly, in a mechanical sense, we must regard the Hvorslev's criterion as a "constant volume condition" in the failure state, which is specifically termed a "failure-constant volume condition". On the basis of this mechanical interpretation ϕ_e and K_e can be easily determined by a single constant volume test in the failure state. Usual tedious tests are not required for determination of these values.

Now, eq. (2) can be reformed to the failure condition by modifying the parameter included in it to be independent from the elastic deformation and by introducing the relationship between the stress and the elastic volume change. Then, assume that the elastic volume change is not affected by a deviatric stress and that the following relation holds generally in an isotropic stress state.

$$V = V_0 - \kappa\ln(P/P_0) \quad \ldots\ldots\ldots \quad (4)$$

where κ is a compression index in the elastic (swelling) state, and P_0 is a value of P in the state at which $V = V_0$. Then, substituting the relation of eq. (4) to eq. (2) with replacements of $P_0 \longrightarrow P_{yn}$ and $V_0 \longrightarrow \bar{V}$ in eq. (4), the following failure criterion, that is a reformed Hvorslev's failure criterion, is obtained.

$$\left(P^{-\kappa}\mathscr{F}^{\lambda} \right)^{\frac{1}{\lambda - \kappa}} - P_{yn}\exp\left(\frac{V_{yn} - \bar{V}}{\lambda - \kappa}\right) = 0 \quad .(5)$$

As an example, constant volume lines and failure curves for a triaxial compression state prescribed by eqs. (2) and (5) are shown in Fig. 1, in which material constants $\lambda = 0.093$, $\kappa = 0.0346$, $\phi_e = 18°$ and $K_e = 0.047$ are derived from experimental data for

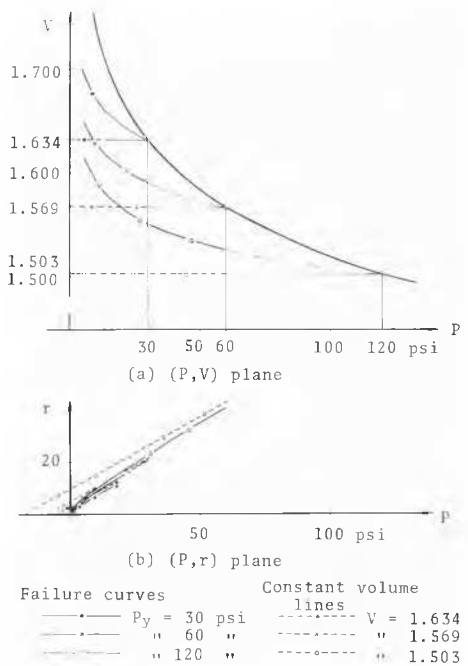
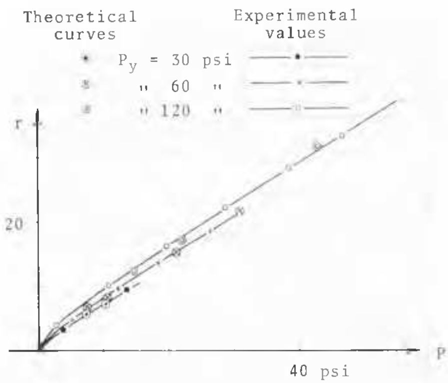
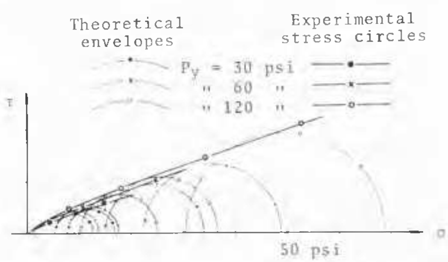


Fig. 1 Failure curves and constant volume lines based on Hvorslev's failure criterion



(a) (P,r) plane



(b) (σ,τ) plane

Fig. 2 Comparison of failure condition based on Hvorslev's failure criterion with experimental values

Weald clay reported by D.J. Henkel (1956) and R.H.G Parry (1960). Further, in Figs. 2 (a) and (b), comparisons between the theoretical failure curves and the experimental values in the (p,r) plane and the (σ,τ) plane are shown, where $r = \sqrt{2/3}(\sigma_I - \sigma_{III})$, and σ and τ are normal and shear stresses respectively. It seems that theoretical curves express the trends of experimental values fairly well.

ALTERNATIVE FAILURE CONDITIONS IN COULOMB-MOHR'S FORM

As shown by eq. (5), the failure condition deduced from the Hvorslev's failure criterion does not have a simple linear relation between components of the principal stress, which is a distinctive feature of Coulomb -Mohr's equation, and is rather complex. Hence, alternative failure conditions conforming to the Coulomb-Mohr's form are formulated.

(A) A failure condition based on a linear relation V − ln P

Adopting the linear relation of V to ln P, it holds that

$$\bar{V} = \bar{V}_0 - (\lambda - \kappa)\ln(P_y/P_{y0}) \quad \cdots\cdots\cdots \quad (6)$$

where P_y is P in the latest plastic state and P_{y0} is a value of P_y in the initial state. Then, choosing the size of hardening function F so as to satisfy $F = P_y$, and let $\bar{\sigma}_{ij}$ be isotropic stress state, by eq. (6) the yield condition (1) becomes that

$$f(\sigma_{ij}) - f_0\exp\left(\frac{\bar{V}_0 - \bar{V}}{\lambda - \kappa}\right) = 0 \quad \cdots\cdots \quad (7)$$

where f_0 is a value of f in the state at which $\bar{V} = \bar{V}_0$. Now, let the function f have the form of Coulomb-Mohr's equation itself. Hence, the following failure condition is obtained.

$$\frac{1}{K'\cos\phi'}\left(\frac{\sigma_I - \sigma_{III}}{2} - \frac{\sigma_I + \sigma_{III}}{2}\sin\phi'\right) - f_0\exp\left(\frac{\bar{V}_0 - \bar{V}}{\lambda - \kappa}\right) = 0 \quad \cdots\cdots \quad (8)$$

where φ' and K' are material constants.

(B) A failure condition based on modified characteristics of consolidation

In the linear relation of V − ln P, the relation between V − $\bar{V}$ and P is not affected by a plastic loading hysteresis and is always identical as shown in eq. (4). However, it may be natural that the smaller the $\bar{V}$, the smaller the V − $\bar{V}$ for a certain change of the pressure from one fixed value to another fixed one. Moreover, the pressure range in eq. (4) is limited to P >> 0 and therefore eq. (4) is not applicable to general boundary-value problems which include stress-free surfaces.

To supplement these fundamental shortcomings in the linear relation of V − ln P, K. Hashiguchi (1974) has proposed the following simple relations between specific volume and pressure in isotropic consolidation.

$$\bar{V}/\bar{V}_0 = -\alpha\ln(P_y/P_{y0}) \quad \cdots\cdots\cdots \quad (9)$$

$$\frac{V}{\bar{V}} = -\beta\ln\left(\frac{P - P_r}{P_0 - P_r}\right) \quad \cdots\cdots\cdots (10)$$

where α, β and Pr ($\leqq 0$) are material constants The left sides of eqs. (9) and (10) are the plastic and the elastic volumetric strains respectively which agree with the concept of a logarithmic strain.

By adopting eq. (9) and choosing the size of F so as to satisfy $F = P_y$, the yield condition is given as follows.

$$f(\sigma_{ij}) - f_0\exp[(\bar{V}_0/\bar{V})^{1/\alpha}] = 0 \quad \ldots (11)$$

Let the function f have the form of Coulomb -Mohr's equation itself, as did in (A). Then, the following failure condition is obtained.

$$\frac{1}{K\cos\phi}\left(\frac{\sigma_I - \sigma_{III}}{2} - \frac{\sigma_I + \sigma_{III}}{2}\sin\phi\right) - f_0\exp\left[\left(\frac{\bar{V}_0}{\bar{V}}\right)^{1/\alpha}\right) = 0 \quad \ldots (12)$$

where ϕ and K are the material constants.

Comparison between the theoretical failure curves expressed by eq. (8) or (12) and the experimental values is shown in Fig. 3 in a (σ,τ) plane with $K'=K=0.017$ and $\phi'=\phi=20.5°$.

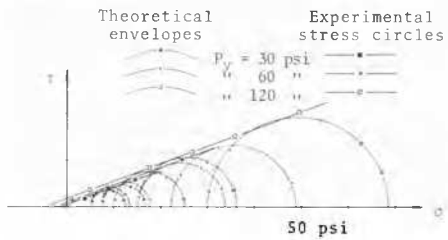


Fig. 3 Comparison of failure conditions in Coulomb-Mohr's form with experimental values

The Hvorslev's criterion provides flat constant volume surfaces and convex yield surfaces, whereas both of eqs. (8) and (12) provide concave constant volume surfaces and flat yield surfaces. Most of the experimental values show trends nearer to those indicated by the former. On the other hand, as shown in Fig. 3, it seems that failure stresses can be expressed fairly well even by eqs. (8) or (12). However, it should be considered that substancially eqs. (5) and (8) are not applicable to $P \leq 0$, according to the characteristics of the elastic, isotropic consolidation to which they are related.

Hvorslev has called ϕ_e "effective angle of friction" and C_e "effective cohesion" shown by $C_e = K_e P_{yn} \exp[(V_{yn} - V)/\lambda]$. However, C_e is not constant even in the case loaded from the same initial state, since C_e depends upon V which varies elastically within and along the yield surface. On the other hand, C' and C shown by $C' = K' f_0 \exp[(\bar{V}_0 - \bar{V})/(\lambda - \kappa)]$ and $C = K f_0 \exp[(\bar{V}_0/\bar{V})^{1/\alpha}]$ obtained from eqs. (8) and (12) are constant for fixed value of $\bar{V}$. These parameters C' and C may be suitable to be termed "failure cohesion", and further ϕ' and ϕ "failure angle of friction".

COMMENTS ON MORE GENERAL FAILURE CONDITION AND CONSTANT VOLUME CONDITION

The above-mentioned failure conditions are substantilly based on the Coulomb-Mohr's equation. However, in general, yield surfaces for soils are represented by closed surfaces. Also, failure surfaces i.e., softening parts of yield surfaces, curve nonlinearly with respect not only to deviatric stress but also to mean stress. Adopting the associated flow rule, the vectors of normal to these surfaces lose a component of mean stress at critical state, as shown in Fig.4. Further, regarding for the elastic volume change, it is understood that constant volume surfaces are also represented by closed surfaces, whereas the vectors of normal to these surfaces have a component of mean stress at critical state as shown in Fig.4. Softening parts of the surfaces are curved so remarkably that their vectors of normal have not only negative but also positive components of mean stress. Accordingly, neither a failure condition nor a failure-constant volume condition can be represented satisfactorily by a simple equation such as Coulomb-Mohr's. Hereafter, we should find a more general failure condition which is to be free from Coulomb-Mohr's equation.

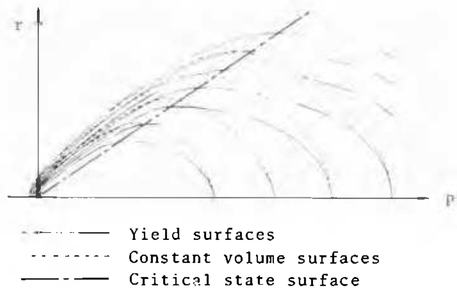


Fig. 4 General yield surfaces and constant volume surfaces

CONCLUSIONS

In this paper, a failure condition is derived from Hvorslev's failure criterion, and further two alternative failure conditions in Coulomb-Mohr's form are presented. The first condition seems to represent the trends of failure stress for soils most suitably, except for the state at which the stress is near to zero. But, this condition is rather complex. On the other hand, it seems that the failure condition which adopts the modified consolidation characteristics and the Coulomb-Mohr's form gives a proper approximation and is very convenient for practcal use.

However, general failure stresses for soils can not be represented by these conditions based on the Coulomb-Mohr's equation.
A more general failure condition for soils, independent of the Coulomb-Mohr's equation, should be found.

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