Preconsolidation and Its Rheological Implications

Préconsolidation et Implications Rhéologiques

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SYNOPSIS "Internal state variables" have received much attention in recent years, but as it has happened with other developments of Continuum Mechanics, there has been a time lag between the formulation of this theory and its application to Soil Mechanics. In this paper, preconsolidation is defined as an internal state variable and a theory of stress-strain behavior of soils based on this definition is then developed, establishing its connection with elastoplastic behavior. As an application, the Cambridge theory of "wet" clays is discussed, demonstrating that it is inconsistent, because the assumed volumetric behavior implies non-vanishing elastic shear distortion which is independent of effective pressure. This is precisely the actual behavior observed in the laboratory and the results here presented prove that the introduction of a second yield locus is not necessary.

INTRODUCTION

In spite of some more recent investigations, the two most widely used theories incorporating the principles of plasticity theory are Rowe's stress dilatancy theory and the Cambridge theory of "wet" clays [5]. The first one of these theories does not consider elastic strains while the second one is unable to predict any elastic distortion. In previous work [1,2] one of the authors has proposed a method to develop constitutive relations for soils whose main characteristics are: rheological models based on clearly stated postulates; implications of the postulates must be explored as widely as possible; models must be constructed in succesive steps going from simpler postulates to those of greater complexity and development of artificial soils satisfying more closely the postulates. This approach has led him to revise the concept of preconsolidation [3], showing that it constitutes an "internal state variable". The concept of "internal state variable" developed within the realm of Continuum Mechanics, has received much attention in recent years [4], because it has demonstrated already its theoretical and practical value. Just as it has happened with other developments of Continuum Mechanics, there has been a time lag between the formulation of this theory and its application to Soil Mechanics, in spite of the fact that it can contribute much to a better understanding of many properties of soils. In this paper a new definition of preconsolidation as an internal state variable is given and it is shown that a proper understanding of the phenomenon of preconsolidation exhibited by many soils, requires treating preconsolidation as such a variable. A theory of stress-strain behavior of soils, based on this definition is then developed and applied to clays; its connection with elastoplastic behavior is explained. The model is applied to the Cambridge theory of "wet" clays and it is shown that the hypothesis of vanishing elastic shear distortion is inconsistent; indeed, it is shown that the assumed volumetric behavior implies non-vanishing elastic shear distortion which is independent of effective pressure. This is precisely the actual behavior observed in the laboratory and the results here presented prove that the introduction of a second yield locus is superfluous. The theory permits to determine easily the distortional response once the volumetric behavior is known. Previous experimental work is supplemented with tests on clays from the Valley of Mexico and some artificial soils, obtaining results which confirm the predictions of the theory.

2. THE BASIC CONSTITUTIVE EQUATIONS

For simplicity attention will be restricted to isotropic soils, subjected to axially symmetric stress states. The ideas presented are however, of more general applicability. The stress parameters used by Roscoe and Burland [5] are the deviatoric component q and the mean normal stress p, while deformation is specified by compressive volumetric strain v and deviatoric (shear distortional) strain ε. These parameters are enough to specify stress and strain states of the soil, because of the isotropy of the material and the assumed axial symmetry of the stresses. The relation between the volumetric strain and the voids ratio e is

$$\delta V = -\frac{\delta e}{1+e} \quad (1)$$

When a clay is subjected to stresses at levels occuring in engineering applications, neighbouring particles can get so close to each other that new internal forces develop. These forces are such that when stresses are removed they remain acting and therefore, the soil elastic properties are modified; this is the case, for instance, when a clay is isotropically consolidated and then expanded. The development of the internal forces just mentioned characterizes the phenomenon of preconsolidation. On the other hand, if a preconsolidated clay is subjected to varying stresses it behaves elastically as long as the current yield curve is not reached, but when the level of preconsolidation is changed the elastic properties are also changed. Thus, two samples of the same clay differing only on their level of preconsolidation constitute two different elastic materials.

Let ϕ, be a parameter specifying the level of preconsolidation; then, associated with every value of ϕ there will be an elastic material. Therefore, an approach similar to Hill and Rice [4], is appropiate, because this is precisely the type of situation described by them. For every ϕ, deformations are given by

$$V=F(p,q,\phi) \quad ; \quad \varepsilon=G(p,q,\phi) \tag{2}$$

Preconsolidation is frequently characterized by the maximum effective pressure ever supported by the soil; although such characterization is suitable when attention is restricted to isotropic paths of stress (i.e. when q=0), it is not so when anisotropic stresses are included (i.e. when $q\neq 0$). To overcome this difficulty, here it will be assumed that there is a function $\theta(p,q)$ such that

$$\phi(t)=\max_{\tau\leq t}\theta(p(\tau),q(\tau)) \tag{3}$$

where the shape of the function θ is to be determined experimentally. It will also be assumed that elastic deformation is derivable from a potential [4]; i.e.

$$F_q(p,q,\phi)=G_p(p,q,\phi) \tag{4}$$

where subscripts stand for partial derivatives with respect to corresponding arguments. It is straight forward to show that eqs. 1 and 2 imply that

$$e=H(p,q,\phi) \tag{5}$$

where H and F are functionally related.

3. RELATION WITH A PLASTIC MODEL

If elastic response is defined by

$$\delta V^E=\frac{\partial F}{\partial p}dp+\frac{\partial F}{\partial q}dq \quad ; \quad \delta\varepsilon^E=\frac{\partial G}{\partial p}dp+\frac{\partial G}{\partial q}dq \tag{6}$$

and plastic response by

$$\delta V^P=\frac{\partial F}{\partial \phi}d\phi \quad ; \quad \delta\varepsilon^P=\frac{\partial G}{\partial \phi}d\phi \tag{7}$$

then $dV=\delta V^E+\delta V^P$ and $d\varepsilon=\delta\varepsilon^E+\delta\varepsilon^P$. With these definitions the three basic postulates of a plastic model are satisfied [4]. An increment of stress is called "loading" if θ increases and "unloading" if θ decreases; the current yield locus is defined by

$$\theta(p,q) = \phi_o \tag{8}$$

where ϕ_o is the present value of ϕ.

4. THE STATE BOUNDARY SURFACES AND THEIR POTENTIAL

Let f,g and h be defined by

$$f(p,q)=F(p,q,\theta(p,q));\ g(p,q)=G(p,q,\theta(p,q));\ h(p,q)=H(p,q,\theta(p,q)) \tag{9}$$

Then, stress states on the current yield locus satisfy

$$V=f(p,q),\varepsilon=g(p,q),e=h(p,q) \tag{10}$$

The last of eqs. 10 defines a surface (fig 1) on the p-q-e space which separates those states which are accesible to a given clay from those which are not. Roscoe and Burland [5] have called it the "state boundary surface". Similar considerations apply to the surfaces defined by the other two equations; therefore, it is natural to extend this concept and say that each of the surfaces defined by eqs 10 constitute a state boundary surface in each of their respective spaces.

Plastic models are usually supplemented with Drucker's orthogonality condition. It has always been recognized that his condition is not a law of nature and that therefore, the class of materials satisfying it, is necessarily restricted. Drucker's condition will be incorporated in the constitutive equations here developed, because the class of materials satisfying it, is wide and because its application to clays has been satisfactory. Drucker's normality condition states that plastic deformations are orthogonal to the yield curve; using eqs 7 and 8 it is seen that this is equivalent to:

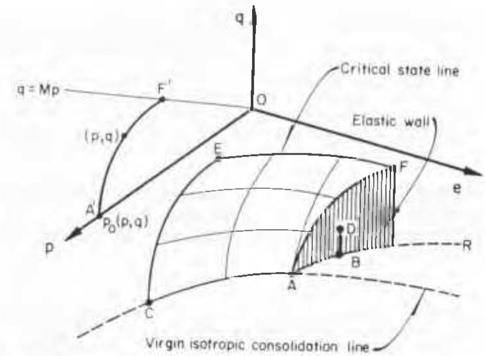

Fig. 1 Illustration of state boundary surface on p-q-e space.

$$\frac{\partial\theta}{\partial q}F_\phi = \frac{\partial\theta}{\partial p}G_\phi \tag{11}$$

whenever p,q satisfy 8. Now, from 9 it follows that:

$$f_q(p,q)=F_q(p,q,\theta(p,q))+F_\phi(p,q,\theta(p,q)\frac{\partial\theta}{\partial q}) \tag{12a}$$

$$g_p(p,q)=G_p(p,q,\theta(p,q))+G_\phi(p,q,\theta(p,q)\frac{\partial\theta}{\partial p}) \tag{12b}$$

Eqs 4, 11 and 12 together imply that

$$f_q = g_p \tag{13}$$

Eq 13 is the condition for the existence of a potential function $\Sigma(p,q)$ such that

$$\frac{\partial\Sigma}{\partial p} \equiv f \text{ and } \frac{\partial\Sigma}{\partial q} \equiv g \tag{14}$$

5. A MINIMAL SET OF EXPERIMENTAL DATA

Define the function $p_y(\phi,q)$ by the condition

$$\theta(p_y(\phi,q),q) = \phi \tag{15}$$

Thus, p_y is the pressure corresponding to the point on the current yield curve for which the deviatoric component of stress is q; it will be assumed that there is only one such point. By direct integration it is seen that for any p,q and ϕ

$$G(p,q,\phi)=g(p_y(\phi,q),q)+\int_{p_y(\phi,q)}^{p} G_p(s,q,\phi)ds \tag{16}$$

where s is a parameter of integration. In view of eq 4, eq. 16 becomes

$$G(p,q,\phi)=g(p_y(\phi,q),q)+\int_{p_y(\phi,q)}^{p} F_q(s,q,\phi)ds \tag{17}$$

On the other hand

$$g(p,q)=g(1,q)+\int_1^p g_p(s,q)ds \tag{18}$$

which can be transformed into

$$g(p,q)=\varepsilon_a(q)+\int_1^p f_q(s,q)ds \tag{19}$$

using eq 13 and writing ε_a for g(1,q). Therefore

$$G(p,q,\phi)=\varepsilon_a(q)+\int_1^{p_y(\phi,q)} f_q(s,q)ds+\int_{p_y(\phi,q)}^{p} F_q(s,q,\phi)ds \tag{20}$$

The function $p_y(\phi,q)$ is determined by the function θ, which in turn is determined by the yield curves. Thus, eq 20 shows that when the volumetric behavior of the clay is known, it is only necessary to determine $\varepsilon_a(q)$ in order to know the deviatoric behavior.

A manner of specifying the function H that will be used in the sequel is:

$$H(p,q,\phi)=h(1,0)+\int_1^{p_P(\phi)} h_p(s,0)ds+\int_{p_P(\phi)}^{p} H_p(s,0,\phi)ds+$$
$$+\int_0^q H_q(p,s,\phi)dq \tag{21}$$

where the function $p_P(\phi)$ is defined as the pressure corresponding to the point of intersection of the virgin isotropic line with the elastic surface determined by ϕ (point A in fig 1). Eq 21 can be obtained by integration along a path like CABD in fig 1.

It is worth noticing that the only condition that the function $\theta(p,q)$ has to satisfy is that it be constant on each yield curve. If the function $p_o(p,q)$ is defined by the condition that p_o be the pressure corresponding to the point where the yield locus passing through the point (p,q) meets the p-axis (fig 1), then the function $p_o(p,q)$ has the property of being constant on every one of the yield curves; thus, θ can be taken identically equal to p_o. When this is done the preconsolidation parameter ϕ, turns out to be identical with p_P. If attention is restricted to paths of isotropic stress, then $p_o \equiv p$ and p_P=max p; however, in general, p_o differs from p and it will be called "équivalent consolidating pressure". The parameter p_P will be called "equivalent preconsolidation pressure" and it is given by

$$p_P(t) = \max_{\tau\le t} p_o(t) \tag{22}$$

Notice that a clay is preconsolidated if the equivalent consolidating pressure p_o is smaller than the equivalent preconsolidation pressure p_P; otherwise, it is normally consolidated. Another fact worth noticing is that a clay can be preconsolidated even if the present value of the pressure p is the maximum that it has ever sustained.

6. APPLICATION TO THE CAMBRIDGE THEORY

The Cambridge theory of the stress-strain behavior of "wet" clays [5], as developed for triaxial (axisymmetric) test conditions, was later extended to include plastic shear distortion for states beneath the state boundary surface. This theory constitutes an excellent illustration of the kind of constitutive equations presented in this paper, because it incorporated a great amount of experimental data and because it has been discussed extensively. It is based on a set of hypotheses which were introduced in an ad hoc manner [5]. However, it will be shown that these hypotheses are inconsistent. A minimal set of hypotheses for the basic (i.e., before it was modified) Cambridge theory of stress-strain behavior of "wet" clays is:

i) Wet clays possess constitutive equations with preconsolidation as an internal state variable of the type that have been here described, which satisfy Drucker's normality condition.

ii) For every p

$$h_p(p,0) = -\frac{\lambda}{p} \tag{23}$$

where λ *is a constant.*

iii) For every p *and* ϕ

$$H_p(p,0,\phi) = -\frac{K}{p} \tag{24}$$

where K *is a constant.*

iv) Elastic surfaces are vertical in the p-q-e *space; i.e.*

$$H_q(p,q,\phi) \equiv 0 \tag{25}$$

v) The slope of each yield locus is a function of q/p *only; i.e.*

$$\frac{\partial\theta}{\partial p} - \psi(\eta)\frac{\partial\theta}{\partial q} = 0 \tag{26}$$

where η=q/p *and* ψ *is a function of* η *to be determined.*

Hypotheses i) to v) would be enough to determine the rheological model, because when eqs 23 to 26 are substituted into eq 21 it is obtained

$$H(p,q,p_P)=e_a+(K-\lambda)\log p_P-K\log p \tag{27}$$

where as in [5], e_a stands for h(1,0). Eq 26 can be integrated to obtain

$$\theta(p,q)\equiv p_o(p,q)=p\exp\left\{\int_0^{q/p}\frac{d\eta}{\psi(\eta)+\eta}\right\} \tag{28}$$

On the other hand at any time t,

$$\phi(t)\equiv p_P(t)=\max_{\tau\le t} p_o(p,q) \tag{29}$$

Consequently

$$h(p,q)\equiv H(p,q,p_o(p,q)=e_a-\lambda\log p+(K-\lambda)\int_0^{q/p}\frac{d\eta}{\psi(\eta)+\eta} \tag{30}$$

Eqs 1,2 and 5 imply

$$F_q=-\frac{H_q}{1+H} \quad \text{and} \quad f_q=-\frac{h_q}{1+h} \tag{31}$$

In view of eqs 28 and 29, it can be seen that the function $p_Y(q,p_P)$ is the solution of

$$\phi\equiv p_P=p_Y\exp\left\{\int_0^{q/p_Y}\frac{d\eta}{\psi(\eta)+\eta}\right\} \tag{32}$$

Substitution of eqs 27 and 30 to 32 into 20 determines $G(p,q,\phi)$ leaving $\varepsilon_a(q)$ as an arbitrary function.

However, the Cambridge stress-strain theory adds the hypothesis:

vi) Elastic shear distortion vanishes identically; i.e.

$$\frac{\partial G}{\partial p} \equiv \frac{\partial G}{\partial q} \equiv 0 \tag{33}$$

by virtue of eq 6.

It can be shown that hypothesis vi) is inconsistent with the other five. Indeed, due to eq 33, G is a function of ϕ only. But eqs 11 and 26 together, imply

$$\psi(\eta) = \frac{F_\phi(p,q,\theta(p,q))}{G_\phi(\theta(p,q))} \tag{34}$$

where $\eta = q/p$. In order for eq 34 to be satisfied, it is required that when p and q move along a path on which θ remains constant, $F_\phi(p,q,\theta)$, be the function ψ of q/p only. If this would happen, it would be very fortunate because $F_\phi(p,q,\theta)$ and $\psi(\eta)$ were chosen independently. By taking the derivative of eq 34 along a yield curve, it can be shown that for the Cambridge theory of "wet" clays it would be required that

$$1+e_q-\lambda \log p \equiv -\frac{\psi(\eta)}{\psi'(\eta)[\psi(\eta)+\eta]}+(\lambda-K)\int_0^\eta \frac{d\xi}{\psi(\xi)+\xi} \tag{35}$$

which is impossible unless λ vanishes, because p and η can be varied independently.

To remove the inconsistency, one has only to disregard hypothesis vi) and apply eq 20 instead. Doing so and using hypothesis iv), it is obtained

$$G(q,p_p)=\varepsilon_a(q)+\int_1^{p_Y(p_p,q)} f_q(s,q)\,ds \tag{36}$$

where the variable p was droped out from the left-hand side of this equation because the right hand side is independent of p.

The fact that the elastic deviatoric deformation given here by

$$d\varepsilon^E = G_q(q,p_p)\,dq \tag{37}$$

is a non-vanishing quantity independent of p, was found experimentally by the Cambridge group [5]. However, in order to account for it, they introduced a second yield surface which as it has been shown, is not required. The practical value of eq 36 stems from the fact that deviatoric behavior can be deduced from volumetric behavior with economy of experimental work required.

7. EXPERIMENTAL RESULTS

A series of laboratory tests were carried out on clays from the Valley of Mexico, as well as in some artificial ones. An example of the results is shown in Fig. 2.

It is clear from this figure that except for a noticeable hysteretic effect, distortional deformations are recoverable and therefore are non-plastic; this is not surprising because experimental results obtained by other authors also correspond to this kind of behavior. Indeed, Roscoe and Burland introduced a second yield locus to account for distortional deformation in spite of the fact that the clays they used in their own experimental work exhibit recoverable distortional deformation beneath the first yield curve (see for example fig 4 of reference [5]) and therefore contradicts the assumption of plastic behavior in this region.

Results obtained previously by other investigators [5] according to which distortional deformations are independent of effective pressure beneath the yield locus, have been confirmed in the clays tested in

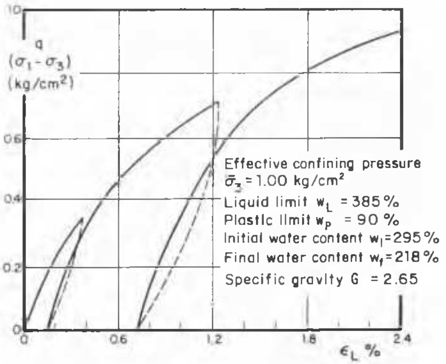

Fig 2 Load and unloading in a remolded sample from the Valley of Mexico. Triaxial isotropically consolidated test.

this work. Thus, the experimental results lead to the following conclusions:

i) Keeping the equivalent preconsolidation pressure fixed, distortional deformations are independent of effective pressure; and

ii) Beneath the yield locus, distortional deformations are recoverable and therefore, elastic.

This is precisely the behavior predicted by the theory here developed.

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