Stresses and Strain in Undrained Test

Tension et Déformation dans l'Essai Non Drainé

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SYNOPSIS The deformation, which occurs during the undrained shearing to the saturated clay sample isotropically consolidated in triaxial apparatus, has been studied mainly as "plastic yielding occurience". After the short "start stage" assumed elastic the sample yields thus, that the yield locus is in the first yield stage the exponent function. In the second yield stage, which ends with the failure of the sample, the yield locus is the ellipse. On the basis of both yield locus functions is arrived at a result that the relationship ε_s-q can be during the undrained shearing to approximate with the same simple hyberbola from the beginning of the shearing (stage) to the failure of the sample. In the paper dealt with the clay sample is a soft postglacial clay. The sample is on purpose overstressed isotropically before the undrained shearing.

THE BASIC ASSUMPTIONS

The sample is assumed to be and to remain isotropic and homogenous. It, further, is assumed that the volume change occurs according to the equation (1) (Fig. 1a).

$$\delta\varepsilon_v = \delta\varepsilon_v^e + \delta\varepsilon_v^p \quad \ldots\ldots\ldots\ldots (1a)$$

$$\delta\varepsilon_v^e = \frac{k}{1+e}\,\frac{\delta p}{p} = m_e\,\frac{\delta p}{p} \quad \ldots\ldots\ldots (1b)$$

$$\delta\varepsilon_v = \frac{\lambda}{1+e}\,\frac{\delta p}{p} = m\,\frac{\delta p}{p} \quad \ldots\ldots\ldots (1c)$$

$\delta\varepsilon_v$ is the total volume change. $\delta\varepsilon_v^e$ is the elastic one and $\delta\varepsilon_v^p$ is the plastic one; in the undrained shearing $\delta\varepsilon_v = 0$. It is assumed that $\delta\varepsilon_s^e > 0$ and $\delta\varepsilon_s^p = 0$ between the points A and B (Fig. 2) of the stress path, whereas between the points B and D of stress path $\delta\varepsilon_s^e = 0$.
ε_s^e is the elastic shear strain and ε_s^p is the plastic one. δp is the change in mean principal stress.

Provided the soil stable in the sense defined by Drucker (1959) then the strain-rate vector expressed in terms of only the plastic components of strain $\delta\varepsilon_s^p$ and $\delta\varepsilon_v^p$ must be normal to the yield locus. This "normality condition" gives:

$$\frac{d\varepsilon_v{}^p}{d\varepsilon_s{}^p} = -\frac{dq}{dp} \quad \ldots\ldots\ldots\ldots (2)$$

$q = \sigma'_1 - \sigma'_3$ is the deviator stress and p is the mean principal stress. Combining equations (1) and (2) and taking into account

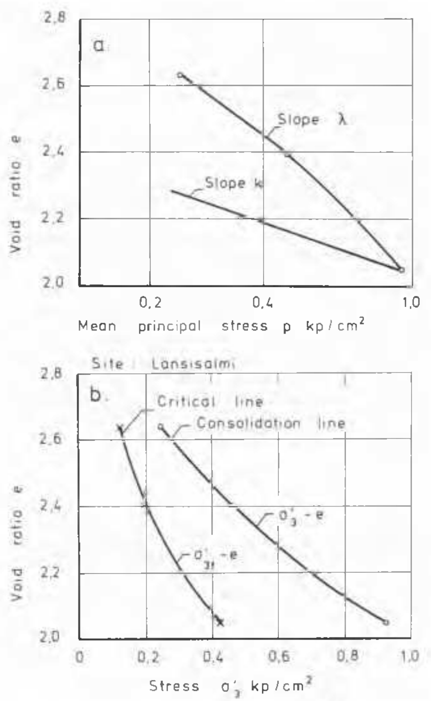

Fig. 1 Relationship between p, σ'_3 and void ratio

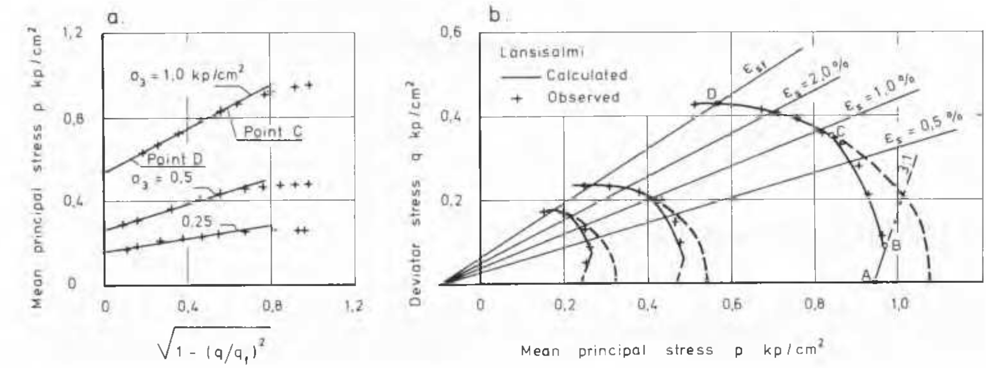

Fig. 2 Undrained isotropically consolidated triaxial test. Stress path and yield locus. The soil tested: postglacial clay, w_L = 78 %, I_P = 46 %

that in the undrained shearing $\delta\varepsilon_v = 0$, is obtained:

$$E = \frac{dq}{d\varepsilon_s} = m_e\, p \left[\frac{dq}{dp}\right]^2 \quad \ldots\ldots\ldots\ldots(3)$$

E is the tangent modulus on ε_s-q space

STRESS PATH, YIELD LOCUS AND DEFORMATION FUNCTIONS

In Fig. 2 have been shown three typical stress paths of the overconsolidated (or overstressed) clay in the undrained triaxial compression test. The stress path is devided into three parts AB, BC and CD. At the space AB the inclication of stress path is 3:1, the same in the undrained compression test. The behaviour of the sample at this stage can be to consider elastic. At the space BC it begins into the overstressed sample to develope the pore pressure and into its deformations also the plastic component obviously come along. At the space CD the sample can be to find yielding plastically. Next is examined the behaviour of the sample at the space BC of the stress path. According to notes of Fig. 3 is

$$p_o + \frac{1}{3}q = p + u_a + u_d$$
$$p_o = u_a + p \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

The component u_a of the pore pressure is assumed to develope proportionally to "the rest of the shear stress mobilization degree", that is to say:

$$\frac{dq}{du_a} = D\left[1 - \frac{q}{q_f}\right] = -\frac{dq}{dp} \quad \ldots\ldots\ldots(5)$$

From the equation (5) is obtained by integrating (if u_a=0, when q=0):

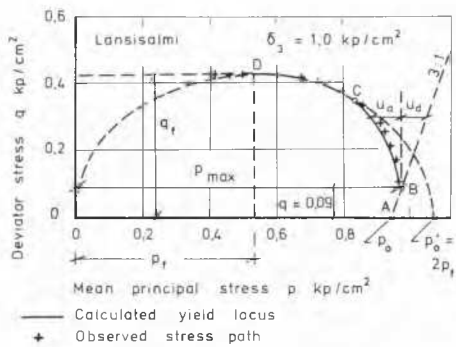

Fig. 3 Stress path ABCD has been devided into three spaces AB, BC and CD. At the space AB the sample behaves elastically. At the space BC the stress path joins the exponential yield locus and at the space CD it joins the elliptical yield locus.

$$q = q_f\left[1 - \exp -\left(\frac{u_a D}{q_f}\right)\right] \quad \ldots\ldots\ldots(6)$$

On the basis of Fig. 4 can be found that the curve of equation (6) joins the points of observations, when to q_f is used the theoretical value $q_{ft} > q_f$, when q_f is the "observed" deviator stress at failure.

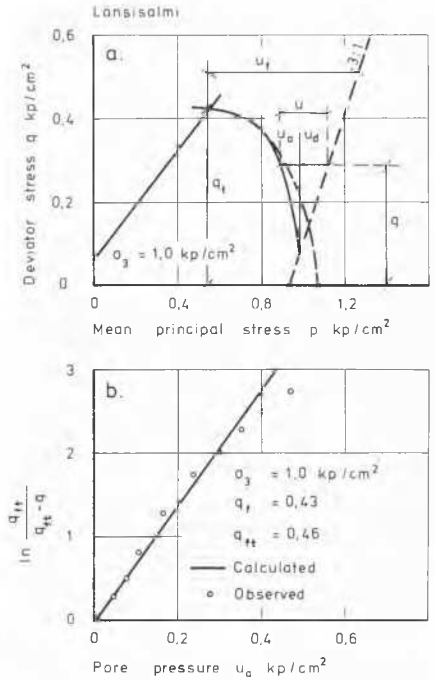

Fig. 4 Undrained triaxial test. Relationship between pore pressure (u_a) and the rest of the shear stress mobilization degree ($1-q/q_f$).

If it is assumed that the fully saturated sample is at the point B of stress path in the elastic condition, is obtained the approximate value of the coefficient D for equations (5) and (6), as follows:

$$\frac{\Delta u_a}{\Delta \sigma_3} = B \approx 1 \ ; \ \Delta\sigma_3 = \frac{\Delta q}{3}$$

$$\frac{\Delta u_a}{\Delta q} = \frac{du_a}{dq} = \frac{B}{3} \quad (\text{when } dq \to 0)$$

At the point B:

$$D = \frac{dq}{du_a} \approx \frac{3}{B} \approx 3$$

B is the pore pressure coefficient of the fully saturated sample (Skempton 1954).

If the stress path can be to interpret the yield locus between B and C, is got on the basis of the equation (3) and (5):

$$E = \frac{dq}{d\varepsilon_s} = m_e p \left[\frac{d\varepsilon_v{}^p}{d\varepsilon_s{}^p}\right]^2 = m_e p \left[\frac{dq}{dp}\right]^2 \text{..(3)}$$

$$E = \frac{dq}{d\varepsilon_s} = m_e p \left[D\left(1-\frac{q}{q_f}\right)\right]^2 = 9 m_e p \left[1-\frac{q}{q_f}\right]^2$$

$$E = E_o \left[1 - \frac{q}{q_f}\right]^2 \quad \text{........(8a)}$$

$$q = \frac{\varepsilon_s}{a+b\varepsilon_s} = \frac{\varepsilon_s}{\frac{1}{E_o} + \frac{\varepsilon_s}{q_f}} \quad \text{........(8b)}$$

$1/a = E_o$ is initial tangent modulus

$1/b = q_{ft} > q_f$ is the theoretical deviator stress at failure

The equation (8) presents the simple hyberbola, which has been in several connections stated to be suited to the approximating for the relationship ε_s-q [among others Botkin (1939), Kondner and Zelasko (1963), Korhonen (1972), Duncan and Chang (1970), Domaschuk and Wade (1969)]. On the basis of Fig. 5 can be stated that the equation (8) is also in this case approximately valid. It is true, in Fig. 5 the curve of the equation (8b) has been presented in "the linear shape".

As earlier stated the deformation of the sample at the space C and D of the stress path is plastic. The yield locus usually joins completely to the stress path (Fig. 2). The yield locus can be approximated with the ellipse (9), which Burland (1965) has derived on the basis of the energy balance for the sample.

$$\left[\frac{p-p_f}{p_f}\right]^2 + \left[\frac{q}{q_f}\right]^2 = 1 \quad \text{........(9a)}$$

$$\frac{dq}{dp} = -\frac{q_f}{p_f}\,\frac{(p-p_f)}{(2pp_f-p^2)^{1/2}} = -\frac{q_f}{p_f}\,\frac{\left\{1-\left[\frac{q}{q_f}\right]^2\right\}^{1/2}}{q/q_f} \quad \text{........(9b)}$$

q_f is the deviator stress at the failure and
p_f is the mean principal stress at the same moment

When into the equation (3) is set the value dq/dp from the equation (9b) is got:

$$E = \frac{dq}{d\varepsilon_s} = m_e p \left[\frac{q_f}{p_f}\right]^2 \frac{\left\{1-\left[\frac{q}{q_f}\right]^2\right\}}{(q/q_f)^2} \text{..(10)}$$

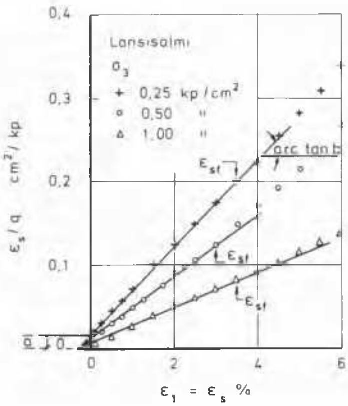

Fig. 5 Hyberbola $q = \varepsilon/a + b\varepsilon$. The line is straight according to the coordinate of figure.

Because it is in question the undrained compression test, where the total volume of the sample does not change (at least theoretically) during the shearing, the value m_e probably stays unchangeable in the equation (10). Thus is obtained:

$$\frac{E}{E_o} = \frac{m_e p}{9 m_e p_o} \left[\frac{q_f}{p_f}\right]^2 \frac{\left\{1 - \left[\frac{q}{q_f}\right]^2\right\}}{\left[q/q_f\right]^2} \quad ..(11)$$

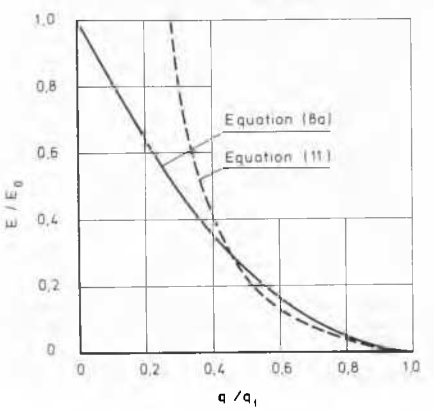

Fig. 6 The curve of constitutive equation (11), correspond to ellipse yield locus, joins the curve of the hyberbolic, constitutive equation (8a) at the space $q/q_f \simeq 0,5 - 1,0$.

In the Fig. 6 has been drawn the curve of the equation (11) with the values: $q_f/p_f = 0,80$ and $p_o/2p_f = 0,87$. On the basis of figure can be found that the curve of the equation (11) nearly joins the curve of the equation (8a), when $q/q_f > 0,5$. Thus can be found that the relationship ε_s-q can be approximated with the simple hyberbola (8b) from the start of the shearing stage in the undrained triaxial compression test up to the failure of the sample. It is also evident that the sample deforms at the space BCD (Fig. 2) of the stress path mainly plastically and that the stress path joins the yield locus at above-mentioned space. The yield locus can be to approximate with equations (6) and (9a). As before mentioned Burland (1965) has derived the equation (9a) on the basis of the energy balance for the sample during the shearing. It is possible that can be derived the energy balance equation for the yield locus (6).

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