Failure Conditions in Sand and Remolded Clay

Conditions à la Rupture des Sables et Argiles Remaniés

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SYNOPSIS Triaxial compression and extension tests and tests with three unequal principal stresses performed on sand and on normally consolidated, remolded clay form the basis for an evaluation of the failure conditions in these soil types. It is shown that the three-dimensional failure surfaces are similar for sand and normally consolidated, remolded clay when the test results are interpreted in terms of effective stresses. The plastic potential surfaces are studied on the basis of the directions of the strain increments at failure and it is shown that they are similar for these two soils. These observations indicate that the basic framework for stress-strain models developed from concepts of plasticity theory should be similar for sand and normally consolidated, remolded clay.

INTRODUCTION

The stress-strain and strength characteristics of sand and clay are similar in many respects, especially when the soil behavior is interpreted in terms of effective stresses. The extent to which this similarity is found in sand and clay under three-dimensional loading conditions has great bearing on the similarity in the theoretical development of the basic framework for stress-strain models for these soil types. Of prime interest in this respect are comparisons of the failure surfaces and the directions of the strain increment vectors at failure. These two aspects of soil behavior are determining for the yield criteria and the plastic potentials used in stress-strain models based on plasticity theory. The conditions prevailing at failure in sand and in normally consolidated, remolded clay have been studied on the basis of results of triaxial compression and extension tests and tests with three unequal principal stresses. The influence of the intermediate principal stress on the strengths and on the relative magnitudes of the strain increments are compared, and the similarities and differences in the behavior of sand and normally consolidated, remolded clay are discussed.

THREE-DIMENSTIONAL TESTS

All tests performed for investigation of the influence of the intermediate principal stress on the behavior of sand and remolded clay were conducted in a cubical triaxial apparatus. A description of the design and performance of this apparatus was presented by Lade and Duncan (1973). The cubical specimens had side lengths of 7.6 cm, and lubricated end plates were used in all tests. The tests on both soils were conducted with constant values of the ratio $b = (\sigma_2-\sigma_3)/(\sigma_1-\sigma_3)$. The effect of stress-path on the effective friction angle for sand is negligible (Lade and Duncan, 1976), and it has often been observed that the failure surface corresponding to the maximum effective stress ratio for normally consolidated clay is for practical purposes independent of the stress-path leading to failure (Henkel, 1960; Olson, 1962). The fact that all tests on sand and remolded clay were performed with constant values of b has consequently no influence on the failure conditions in these soils. The conclusions presented below are therefore not limited to stress-paths with constant b-values.

Tests on Sand. - Drained tests were performed on specimens of Monterey No. 0 Sand with void ratios of $e = 0.78$ ($D_r = 27\%$) and $e = 0.57$ ($D_r = 98\%$). The minor principal stress, σ_3, was held constant at 0.30, 0.60, and 1.20 kg/cm^2 in the triaxial compression tests in which $\sigma_1 > \sigma_2 = \sigma_3$, and it was 0.60 kg/cm^2 in the tests with $\sigma_1 \geq \sigma_2 > \sigma_3$. The results of these tests have been reported previously by Lade and Duncan (1973), and the pertinent data are presented here for comparison with the results of the tests on remolded clay.

Tests on Remolded Clay. - Consolidated-undrained tests with pore pressure measurements were performed on saturated specimens of Grundite Clay with the following characteristics: Liquid limit = 54.8, plastic limit = 24.7, and activity = 0.58. The specimens were prepared from a clay slurry consolidated in a double draining consolidometer at a vertical pressure which was 0.1 kg/cm^2 smaller than the final isotropic consolidation pressure to be used in the tests. After thoroughly remolding the clay, cubical specimens were trimmed and consolidated isotropically at 1.00, 1.50, and 2.00 kg/cm^2 corresponding to water contents of 34.4%, 31.9%, and 30.1%, respectively. The normally consolidated, remolded specimens were then loaded at a constant vertical strain rate of 0.04%/min chosen according to criteria set forth by Barden and McDermott (1965). Additional

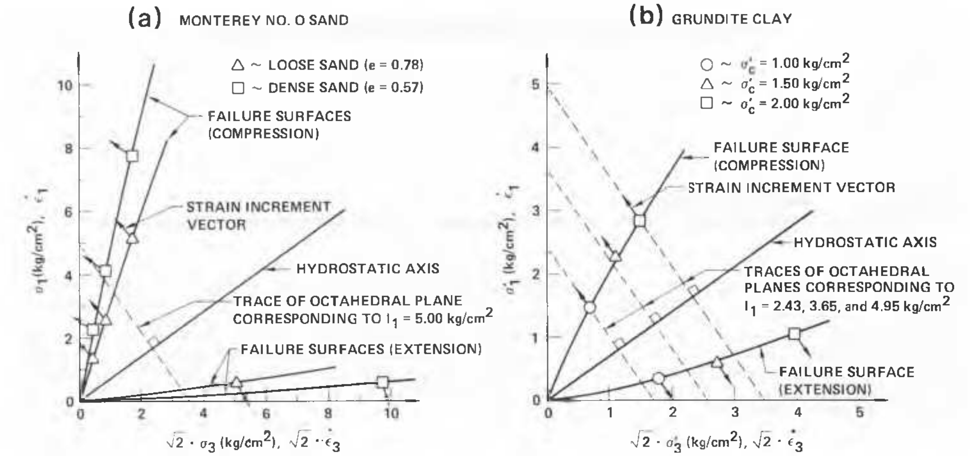

Fig. 1 Failure Surfaces and Strain Increment Vectors in Triaxial Plane for Tests on (a) Loose and Dense Monterey No. 0 Sand and (b) Grundite Clay.

tests at lower strain rates confirmed that the soil and pore pressure behavior were not significantly affected by the strain rate when lubricated ends were employed. The test results were corrected for the loads taken by the filter paper and the membrane.

FAILURE SURFACES

The effective strengths and the corresponding failure surfaces obtained from the tests on sand and remolded clay are shown and their shapes are discussed in view of their traces in triaxial and octahedral planes of the principal stress space. The experimentally obtained failure surfaces are compared to the Mohr-Coulomb failure surfaces and to failure surfaces corresponding to a recently developed failure criterion for cohesionless soil.

Triaxial Plane. - Fig. 1 shows the traces of the failure surfaces in the triaxial plane containing the σ_1-axis for loose and dense Monterey No. 0 Sand and for Grundite Clay. Comparison of the strengths in the triaxial plane indicates that the failure surfaces for sand and remolded clay are very similar, when the test results are interpreted in terms of effective stresses. The failure surfaces are curved in this plane, and they intersect the origin of the principal stress space. Curved failure surfaces have often been observed for sand as well as for clay soils (Bishop, 1966; Lee and Seed, 1967; Olson, 1974).

The strength characteristics of normally consolidated clay differ from those of sand in one respect. A sand can be deposited with different void ratios and these change only little with consolidation pressure. Different failure surfaces are obtained for different initial void ratios, as indicated on Fig. 1a for Monterey No. 0 Sand. In contrast, a normally consolidated clay can only exist in equilibrium at a void ratio which depends on the consolidation pressure. Therefore, only one failure surface is possible for a normally consolidated clay, as seen in Fig. 1b for Grundite Clay.

Octahedral Plane. - The strength results from the triaxial compression and extension tests and the tests with three unequal principal stresses on the Monterey No. 0 Sand and the Grundite Clay are shown on the octahedral planes in Fig. 2. The location of the octahedral planes are indicated by the values of the first stress invariant, I_1, and by their traces in the triaxial plane shown in Fig. 1. Note that the traces of the failure surfaces for the loose and dense sand are shown in one octahedral plane, whereas the single failure surface for the remolded clay is shown by traces in three different octahedral planes. Only the shapes (and not the sizes) of the failure surfaces are important for the considerations presented below. The specimens of Monterey No. 0 Sand were shown to be essentially isotropic (Lade and Duncan, 1973), and the tests on remolded Grundite Clay indicated that the specimens were isotropic. The failure surfaces are therefore symmetric around the three principal axes in the octahedral planes. The failure points shown in Fig. 2 have been adjusted for the curvature of the failure surfaces in planes containing the hydrostatic axis such as the triaxial planes.

The experimentally obtained failure surfaces are compared to the Mohr-Coulomb failure surfaces and to the surfaces corresponding to a failure criterion expressed in terms of the first and the third stress invariants, I_1 and I_3 (Lade and Duncan, 1975):

$$I_1^3/I_3 = \kappa_1 \tag{1}$$

where

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 = \sigma_x + \sigma_y + \sigma_z \tag{2}$$

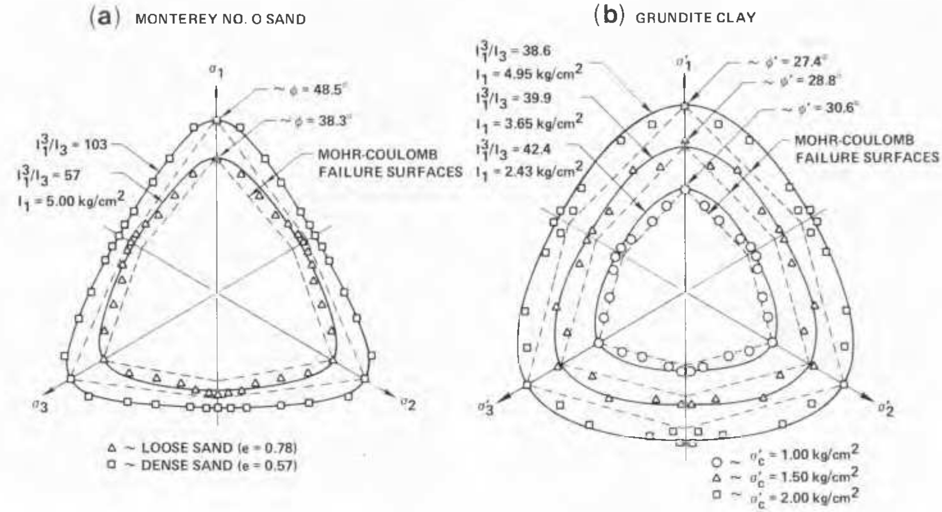

Fig. 2 Failure Surfaces in Octahedral Planes for Tests on (a) Loose and Dense Monterey No. 0 Sand and (b) Grundite Clay.

and

$$I_3 = \sigma_1 \cdot \sigma_2 \cdot \sigma_3 \tag{3}$$
$$= \sigma_x \cdot \sigma_y \cdot \sigma_z + 2 \cdot \tau_{xy} \cdot \tau_{yz} \cdot \tau_{zx} - \left(\sigma_x \cdot \tau_{yz}^2 + \sigma_y \cdot \tau_{zx}^2 + \sigma_z \cdot \tau_{xy}^2\right)$$

and κ_1 is a constant which may be determined from the results of triaxial compression tests only. The value of the ratio I_1^3/I_3 is 27 for isotropic stress conditions ($\sigma_1=\sigma_2=\sigma_3$), and it increases up to failure where κ_1 expresses the strength of the soil. The value of κ_1 increases with increasing friction angle in triaxial compression. In principal stress space the shape of the failure surface defined by Eq. 1 is conical, with the apex of the cone at the origin of the stress space, and with cross-sections of the types shown by solid lines in Fig. 2. The cross-sections are symmetrical around the principal axes in the octahedral planes, and the shape is essentially circular for values of κ_1 close to 27 and it becomes increasingly triangular with increasing values of κ_1. This may be seen by comparing the cross-sections in Fig. 2.

The irregular hexagons shown by dashed lines in Fig. 2 are described by the Mohr-Coulomb criterion corresponding to the friction angles in triaxial compression. The experimental failure points are all located outside the respective hexagons, thus indicating that the friction angles from tests with $\sigma_2 > \sigma_3$ are higher than those obtained in triaxial compression tests. Thus, the Mohr-Coulomb failure criterion underestimates the strength of both sand and remolded clay for all but triaxial compression conditions.

The failure surfaces corresponding to Eq. 1 with values of κ_1 determined from the triaxial compression tests appear to model the failure conditions for both sand and remolded clay quite well. Since the failure surfaces are curved as shown in Fig. 1, different values of κ_1 correspond to the cross-sections in the three octahedral planes shown in Fig. 2b. The differences between the measured strengths and those suggested by Eq. 1 are most pronounced for the tests on Grundite Clay performed with low to intermediate values of b and with a consolidation pressure of σ'_c = 1.00 kg/cm^2, as shown in Fig. 2b. This consolidation pressure produced specimens with rather soft consistency, and the relatively low strengths are, at least partly, due to difficulty in handling the soft specimens. The results of the tests with σ'_c = 1.50 and 2.00 kg/cm^2 are considered to be more reliable, because the specimens were stiffer and therefore easier to handle. On the basis of these considerations it is suggested that the failure criterion in Eq. 1 with the respective values of κ_1 models the three-dimensional strength of both Monterey No. 0 Sand and Grundite Clay with good accuracy.

STRAIN INCREMENT DIRECTIONS

Because it may be possible to model the soil behavior by plasticity theory, it is of interest to study the directions of the strain increment vectors relative to the yield or failure surfaces in the principal stress space. For this purpose the principal strain increment axes are superimposed on the stress axes in the principal stress space.

Plastic Potential Surface. - The plastic potential is a function from which the ratios of the components of the plastic strain increments can be determined by differentiating with respect to stresses. This function describes a surface in the stress space which is called the plastic potential surface, and the plastic strain increment directions are perpendicular to this surface when the principal strain increments are plotted in the same space as the principal stresses. According to the assumed normality condition used in classical plasticity theory, the plastic strain increment vector for a perfectly plastic material should be normal to the yield surface. If the normality condition is fulfilled, the yield surface and the plastic potential surface coincide. This condition simplifies the development of plastic stress-strain relations considerably. It is in fact often assumed a priori that this condition is satisfied for soils, and plastic potential surfaces determined from the plastic strain increment directions are employed as yield surfaces for the plastic stress-strain relations.

Based on the results from the tests on Monterey No. 0 Sand and Grundite Clay, the directions of the strain increment vectors relative to the failure surfaces are studied below. If the normality condition is fulfilled, the pertinent strain increment vectors are perpendicular to the traces of the failure surfaces in a triaxial plane, and the projections of the strain increment vectors on an octahedral plane are perpendicular to the trace of the failure surface in this plane. The magnitudes of the strain increment vectors are immaterial to the question of normality. Only the directions of the strain increments at failure will be considered, because normality concerns only plastic strains, and by definition all strain increments are plastic at failure.

Triaxial Plane. - The directions of the strain increment vectors measured at failure for loose and dense Monterey No. 0 Sand and for Grundite Clay are shown initiating from the respective failure points on the triaxial plane in Fig. 1. Since $\varepsilon_2 = \varepsilon_3$ in compression and $\varepsilon_2 = \varepsilon_1$ in extension, the corresponding strain increment vectors are contained in the triaxial plane. Because the volumetric strain is zero in undrained tests, the strain increment vectors are perpendicular to the hydrostatic axis and they are therefore contained in the octahedral planes corresponding to the respective failure points, as shown for the Grundite Clay on Fig. 1b. The strain increment vectors are not perpendicular to the failure surfaces obtained for sand and for clay in compression and extension. The normality condition is therefore not satisfied when the soil behavior is interpreted in terms of effective stresses.

Interpretation of the undrained behavior of saturated soil at a given void ratio in terms of total stresses leads to failure surfaces whose traces in a triaxial plane are parallel to the hydrostatic axis. The strain increment vectors would be perpendicular to these traces, and the normality condition would be fulfilled when the total stress failure surface is used. There are definite advantages in retaining the normality condition for the purpose of simplifying the analysis procedures for soil structures (Prévost and Höeg, 1975). However, the strains (direction and magnitude) are controlled by effective stresses rather than total stresses, and yield criteria and plastic potential functions should therefore be expressed in terms of effective stresses. In addition, only one failure surface is obtained for e.g. normally consolidated clay under undrained conditions when the behavior is interpreted in terms of effective stresses. Different total stress failure surfaces would have to be employed for the same soil, because the undrained strength increases with increasing consolidation pressure.

Octahedral Plane. - The projections of the strain increment vectors on octahedral planes are shown in Fig. 3. Because of symmetry around the principal stress axes, only one sixth of the planes are shown. Since the volumetric strain is zero in undrained tests, the strain increment vectors for the Grundite Clay shown in Fig. 3b are contained in the respective octahedral planes. The projections of the strain increment vectors are very nearly perpendicular to the traces of the failure surfaces in the octahedral planes for loose and dense sand and for remolded clay. Only a few of the vectors at intermediate values of b deviate slightly from perpendicularity. The magnitudes of these deviations are so small that they could even be due to experimental inaccuracies.

It is clear from these results that the condition of normality is satisfied or very nearly satisfied in the octahedral planes, but not in the triaxial plane, and it is therefore not fulfilled in general. According to the directions of the strain increment vectors shown in Figs. 1 and 3, the plastic potential surfaces should have cross-sections similar to those of the yield surfaces and they should be inclined to the hydrostatic axis at angles smaller than those between the yield surfaces and the hydrostatic axis. A plastic potential function whose shape in stress space has these properties was suggested by Lade and Duncan (1975) for cohesionless soil. It is interesting to note that sand and normally consolidated, remolded clay are characterized by very similar behavior patterns at failure, and that the above conclusions apply equally well to both soil types. Thus, it is likely that the basic framework for stress-strain models developed from concepts of plasticity theory should be similar for sand and for normally consolidated, remolded clay.

COMPARISON WITH PREVIOUS STUDIES

Investigations of the influence of the intermediate principal stress on the failure conditions in sand and clay have been performed previously. The results of these studies are briefly reviewed on the basis of the concepts of the failure criterion and the plastic potential as used here.

Sand. - The behavior of sand under three-dimensional loading conditions has been reviewed previously (Lade and Duncan, 1973, 1975) and it will not be repeated here. However, it should be mentioned that tests performed by Poorooshasb et al. (1966) and Ko and Scott (1968) showed that the strain increment directions were not perpendicular to the failure surface in the triaxial plane. Goldscheider and Gudehus (1973) demonstrated that the projection of the strain increment directions on octahedral planes were perpendicular to the traces of the failure surfaces in these planes. These results are in agreement with those presented here.

Clay. - Studies of failure in normally consolidated clays have resulted in different conclusions. Tests performed by Wu et al. (1963), Pearce (1971), and Yong and McKyes (1971) indicated that the Mohr-Coulomb criterion adequately characterizes failure in clays. Shibaba and Karube (1965) and Yong and McKyes (1967) found that the effective friction angles in triaxial compression (b=0) and extension (b=1) were similar in magnitude, but friction angles for intermediate values of b were higher than those for b=0 and b=1. Thus, the traces of the experimental failure surfaces in

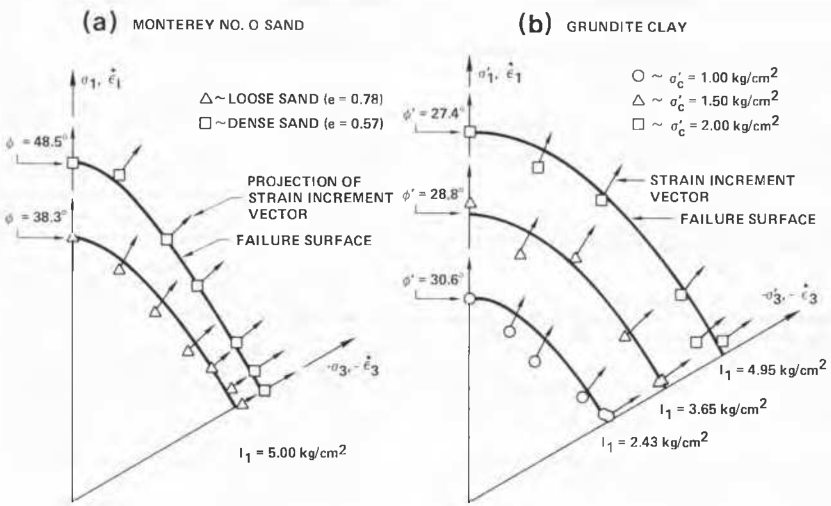

Fig. 3 Directions of Projections of Strain Increment Vectors on Octahedral Plane for (a) Loose and Dense Monterey No. 0 Sand and (b) Grundite Clay.

octahedral planes were curved and circumscribed the Mohr-Coulomb failure surfaces. Vaid and Campanella (1974) presented effective friction angles from undrained tests in plane strain and triaxial extension which were higher than those obtained from triaxial compression. Only the strength results from the latter study can be described by the failure criterion in Eq. 1.

The strain increment directions from the investigations by Yong and McKyes (1971) and Vaid and Campanella (1974) were perpendicular to the traces of the failure surfaces in octahedral planes. However, different failure surfaces were found in these studies.

Because some ambiguity existed regarding the behavior of normally consolidated clays at failure, the study of Grundite Clay presented above was undertaken. The results of this study tend to support the results presented by Vaid and Campanella (1974).

CONCLUSIONS

The conditions prevailing at failure in sand and in normally consolidated, remolded clay have been studied on the basis of results of triaxial compression and extension tests and tests with three unequal principal stresses. The influence of the intermediate principal stress on the effective strengths of both soils is modeled quite well by a failure criterion expressed in terms of the first and the third stress invariants, and the behavior characteristics at failure are shown to be very similar for the two soil types. However, different failure surfaces are obtained for sand with different initial void ratios, whereas only one failure surface is possible for normally consolidated clay. The test results from both soils indicate that the yield criterion and the plastic potential should be different when the soil behavior is interpreted in terms of effective stresses. The basic framework for stress-strain models developed from concepts of plasticity theory should be similar for sand and for normally consolidated, remolded clay.

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