A Common Stress-Strain-Relationship for Soils

Une Relation Générale Contrainte-Déformation pour Sols

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SYNOPSIS The stress-strain-relationship for the determination of the soil compressibility for settlement prediction can be obtained from two different common tests: the triaxial compression test with σ_3 = const. and the constant stress ratio test with $K = \sigma_3/\sigma_1$ = const, including the special cases of $K = K_0$ which corresponds to the oedometer test and K = 1.0 which corresponds to isotropic consolidation.
The autors show that it is possible to get a general stress-strain-relationship for all these tests, based on the hyperbolic function according to Duncan/Chang, assuming that the stress path is negligible.
This means that the difficult constant stress ratio test can be substituted by the more simple triaxial compression test or, if the angle of internal friction is known or estimated, even by the oedometer test.

INTRODUCTION

For many years it has been proposed to improve settlement calculations by using triaxial tests (Lambe 1964 and 1967, Brinch Hansen 1966, Kérisel/Quatre 1966 and 1968, Davis/Poulos 1968) instead of oedometer tests. For this purpose the following tests can be used:

(i) the triaxial compression test with a constant lateral pressure σ_3 (p.e. ODB in fig. 1),

(ii) the constant stress ratio test with $K = \sigma_3/\sigma_1$ (p.e. OB in fig. 1) including the special cases of $K = K_0$ which corresponds to the oedometer test and K = 1.0 which corresponds to the isotropic consolidation,

(iii) the special test in which the applied stresses are changed according to the stress path of the subsoil for different loading conditions (OAB in fig.1)

The last tests are the most difficult to execute and they are only valid for their special case. Also the constant stress ratio tests are more sophisticated than the normal triaxial compression tests which are often used to determine Mohr's shear parameters, so that more empirical knowledge is at hand. In order to combine and compare these tests the relationship between stress and strain has to be described by a theoretical equation. The often applied exponential function of Ohde (1939) and Janbu (1967/68) cannot be applied directly to the triaxial compression test with a constant lateral pressure, but it must be generalized. Therefore a hyperbolic relationship according to Duncan/Chang (1970) is used, containing the Mohr failure criterium.

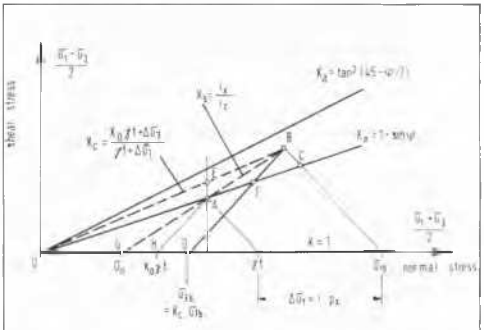

Fig. 1 Stress paths under foundation and during laboratory tests.

On assuming the material law to be valid for all states of stresses it must be taken into account that, according to Lambe (1967), that exact stress path of the test has more or less no big influence on the results. The stress path is therefore neglected in most of the known calculation methods (Amann et al. 1975). The ensuing faults can be stated more exact by appropriate comparing tests.

STRESS PATHS (fig. 2)

Nevertheless the various stress paths should be considered. If we consider an axial element in a depth of about 1/3 to 1/4 (Davis/Poulos) of the compressible layer it will have been consolidated anisotropically to Point A. The stress due to foundation loading which will act upon this representative element is going to have the ratio

$$\sigma_3/\sigma_1 = K_b = \text{const.}$$

as not only the vertical but also the horizontal stress increase linearly with the contact pressure p. The stress path AB is given by:

$$\frac{K_0 \cdot \gamma \cdot t + K_b \cdot \Delta\sigma_1}{\gamma \cdot t + \Delta\sigma_1} \quad \ldots\ldots\ldots\ldots \quad (1)$$

For the calculation of the settlement we need the displacement of the sample due to stress state A and B. The differential displacement multiplied with the height of the compressible layer will give the total settlement. We can reach these stress states by different triaxial tests (fig. 1 + 2):

(i) two triaxial compression tests, one including Point A with the lateral pressure of $K_0 \cdot \gamma \cdot t$ and one including point B with $\sigma_3 = K_c \sigma_{1b}$

(ii) two constant stress ratio test, one with $K = K_0$ and one with $K = K_c$

(iii) one single test with anisotropical consolidation ($K = K_0$) to Point A and shearing with the ratio of equ. (1) to Point B.

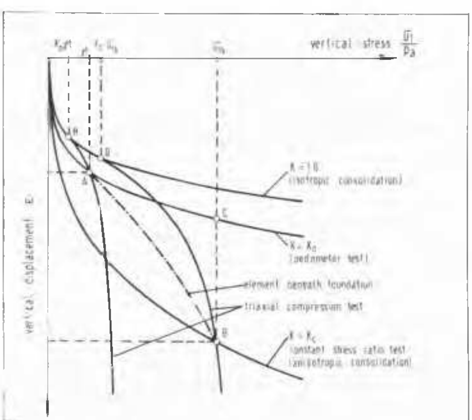

Fig. 2 Stress-strain-curves for different stress paths

As it is impossible to reach point B in the oedometer test, point C will be taken for the calculation having the same vertical but different horizontal stresses.

It must be noticed that in the common triaxial compression test the displacement of the sample during isotropical consolidation (OH and OD in fig. 2) will not be measured. An initial tangent modulus for this case must be determined separately.

STRESS-STRAIN-RELATIONSHIP

To get the transition from stress to strain we need a fitting material law. The following functions can be used:

(i) curves without asymptote, suitable for constant stress ratio tests, expecially for oedometer tests (fig. 5)

(ii) curves with asymptote, suitable for triaxial compression tests and more or less for plate loading tests

In both cases the parameters to describe the shape of the curve can be obtained by transformation into a logarithmic or hyperbolic plot.

For all these curves the equation according to Duncan/Chang (1970) is applicable. It has been set up for the most general case of the triaxial compression test with σ_3 = const. (curve i)

The tangent modulus depending on the lateral stress is as follows:

$$E_d = \left[1 - \frac{R_f(1-\sin\varphi)(\sigma_1-\sigma_3)}{2c\cdot\cos\varphi + 2\sigma_3\sin\varphi}\right]^2 m \cdot p_a \left(\frac{\sigma_3}{p_a}\right)^n$$

$$= \left[1 - R_f \cdot \psi\right]^2 m \cdot p_a \left(\frac{\sigma_3}{p_a}\right)^n \quad \ldots\ldots\ldots\ldots \quad (2)$$

with

p_a = atmospheric pressure = unit pressure
ψ = stress-level $= (\sigma_1-\sigma_3)/(\sigma_1-\sigma_3)_f$
c, φ = Mohr's shear parameters
f = failure
m,n = compression indices
R_f = relation between failure in test and asymptote of the theoretical curve

The 5 parameters which must be determined, can be obtained by 3 triaxial compression tests.

In order to apply this relationship to the constant stress ratio test we must get a relationship between ψ and K. Assuming that the angle of internal friction φ does not change with the applied stresses we get (Schultze/Schmidt-Schleicher 1973, S. 58)

$$\psi = \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_f} = \frac{K_a(1-K)}{K(1-K_a)} \quad \ldots\ldots\ldots (3)$$

with

$$K_a = (\sigma_3 / \sigma_1)_f = \tan^2(45 - \varphi/2) \ldots\ldots (3a)$$

= coefficient of earth pressure at failure

This means that in the plot with E_d as dependant and ψ as independent variable straight vertical lines for K = const. would be obtained (fig. 3) (Schultze 1976).

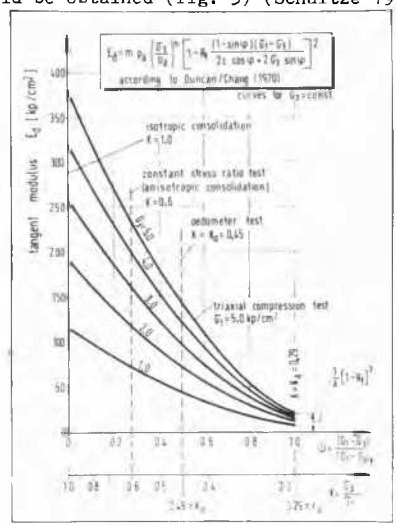

Fig. 3 Derived tangent modulus from triaxial compression test after the law of Duncan/Chang.

Inserting equ. (3) in equ. (2) and using the relationship of $\sigma_3 = K \cdot \sigma_1$ the function for this modulus is obtained:

$$E_d = \left[1 - R_f \frac{K_a(1-K)}{K(1-K_a)}\right]^2 m\, p_a\, K^n \left(\frac{\sigma_1}{p_a}\right)^n \quad \ldots (4)$$

If the constant parameters are combined and this factor is named

$$v_k = m \cdot K^n \left[1 - R_f \frac{K_a(1-K)}{K(1-K_a)}\right]^2 \cdot p_a \quad \ldots\ldots\ldots (5)$$

$$w_k = n$$

equ. (4) can be written as

$$E_d = v_k \left(\frac{\sigma_1}{p_a}\right)^{w_k} \quad \ldots\ldots\ldots (6)$$

and for K = K_0 (oedometer test)

$$E_s = v \left(\frac{\sigma_1}{p_a}\right)^w \quad \ldots\ldots\ldots (7)$$

The last leads to the equation of Ohde (1939) and Janbu (1967/68).

The curves are plotted in fig. 4.

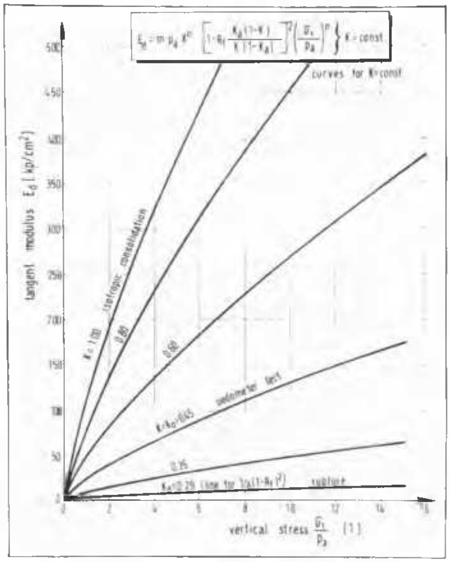

Fig. 4 Tangent modulus of constant stress ratio tests interpolated from triaxial compression test.

By integration the stress-strain-curves can be obtained:

$$\varepsilon_1 = q_k \left(\frac{\sigma_1}{p_a}\right)^{k_k} \quad \ldots\ldots\ldots (8)$$

with

$$q_k = 1 / \left[m K^n \left(1 - R_f \frac{K_a(1-K)}{K(1-K_a)}\right)^2 (1-n)\right] \quad \ldots\ldots (9)$$

$$k_k = 1 - n$$

The curves for constant stress ratio tests, interpolated from the normal triaxial compression tests, are plotted in fig. 5.

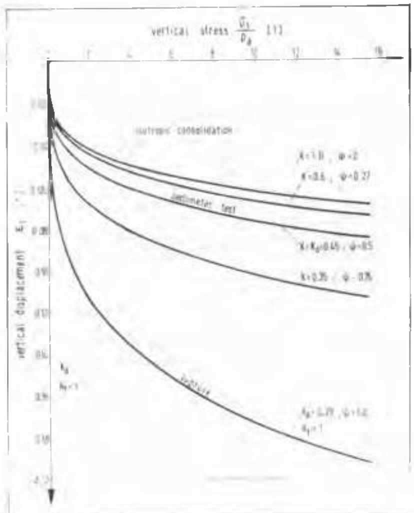

Fig. 5 Stress-strain-curves of constant stress ratio tests with different K-values, interpolated from triaxial compression tests.

CONCLUSIONS

For settlement calculations based on triaxial tests, the best way would be to perform tests with a stress path very close to the stress state of the element beneath foundation. As special tests are too expensive and too sophisticated it is preferable to calculate the displacement by means of triaxial compression tests, which have often been performed to determine Mohr's parameter φ and c. The above derived formulas allow the determination of the vertical compression of the soil considering the influence of the lateral pressure with quite a good accuracy for practial purposes.

If only oedometer tests are at hand and an estimation of the amount of settlement is wanted, this could only be done, if the angle of internal friction φ is known or sufficiently exact estimated. This leads to the calculation of K_a according to equ. (3 a), which makes it possible to determine v_μ according to equ. (5). The last required parameter R_f has to be estimated by experience and often lies between 0.75 and 1.0.

If a soil reacts very sensitivly to the applied stress path it would be letter to approximate it stepwise according to Lambe (1967) during test performance.

Furthermore it should be considered that these interpolations are only valid for soils without preconsolidation, i.e. for curves with only one branch.

This report is part of a research work on these relations for silt that is going on at the Technische Hochschule Aachen. The final report will be published in 1978 (see references).

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