Rheological Characteristics of Clays

Caractéristiques Rhéologiques des Argiles

H.SEKIGUCHI Disaster Prevention Research Institute, Kyoto University, Japan

SYNOPSIS This paper deals with time-dependent and elastoplastic properties of normally consolidated clay. It is shown that the new interpretation of a unified relation for isotropic compression, dilatancy and secondary compression leads to the concept of a viscoplastic potential. Constitutive relations are then derived. They permit a consistent description, compatible with hitherto made observations, of strain rate effects on undrained stress-strain response, of stress relaxation characteristics, and of creep rupture characteristics.

INTRODUCTION

There is an increasing demand for careful field observations in the rapid construction of embankments on deep deposits of clay. However, few criteria have been established which enable us to judge precisely that the loaded soil mass is undergoing a stable, continued deformation, or is undergoing local failure growing finally to overall failure.

The present study is aimed at providing a theoretical basis that could be used for a proper interpretation of observed rates of deformation and pore-water pressures in a loaded or an excavated clay. For this purpose constitutive relations will be developed below for normally consolidated clay, with particular emphasis on its rheological behavior.

CONSTITUTIVE RELATIONS

In order to obtain the stress-strain-time relation relevant to the process of one-dimensional consolidation of clay exhibiting creep under constant effective stress, Sekiguchi and Toriihara (1976) have derived the following relation for sustained loading.

$$v=\frac{\lambda}{1+e_o}\ln\left(\frac{p}{p_o}\right)+D\cdot\left(\frac{q}{p}-\eta_o\right)-\alpha\cdot\ln\left(\dot{v}/\dot{v}_o\right) \quad (1)$$

where v is the volumetric strain, λ the compression index, e_o the value of void ratio immediately before the change of loading, p the mean effective stress, D the coefficient of dilatancy, q the principal stress difference, α the secondary compression index, $\dot{v}$ the volumetric strain rate, and p_o, η_o and $\dot{v}_o$ denote the values of p, q/p, and $\dot{v}$ immediately before the change of loading, respectively. The physical meaning of each term on the right-hand side of Eq. (1) is simple. The first term is the contribution of isotropic compression, the second term is the contribution of dilatancy, and the third term represents the secondary compression effect. It may be noted that if no time effect exists, i.e. if α tends zero, the resulting relation reduces to that derived by Shibata (1963).

To solve Eq. (1), we assume that the volumetric strain v_i immediately after the change of loading takes the form:

$$v_i=\kappa\cdot\ln(p/p_o)/(1+e_o) \quad (2)$$

where κ is the swelling index. Then, we introduce the viscoplastic volumetric strain v^p by subtracting v_i from v after obtaining the solution to Eq. (1). Thus, after some manipulation, we obtain

$$F=\alpha\cdot\ln\{1+(\dot{v}_o t/\alpha)\cdot\exp(f/\alpha)\}=v^p \quad (3)$$

in which t is the elapsed time after the change of loading, and the function f is defined by

$$f=\frac{\lambda-\kappa}{1+e_o}\ln\left(\frac{p}{p_o}\right)+D\cdot\left(\frac{q}{p}-\eta_o\right) \quad (4)$$

Here, we redefine the stress parameters p and q as follows:

$$p=\sigma_{ii}'/3\ ;\ q=\sqrt{(3/2)s_{ij}\ s_{ij}} \quad (5)$$

where σ_{ij}' and s_{ij} are the effective and the deviatoric stress tensor, and the summation convention is used.

When the elapsed time t is kept constant in Eq. (3), this equation is considered to represent a surface in effective stress space. Also, v^p in Eq. (3) could be regarded as a strain-hardening parameter. Therefore, let us assume the function F to be a viscoplastic potential from which we obtain the viscoplastic strain rate tensor, $\dot{\varepsilon}_{ij}{}^p$, in the form

$$\dot{\varepsilon}_{ij}{}^p=\Lambda\cdot\partial F/\partial\sigma_{ij}' \quad (6)$$

where Λ is the proportional constant. To determine Λ, we introduce the condition for a continued viscoplastic flow, i.e. $\dot{F}=\dot{v}^p$. Thus, we obtain

$$\dot{\varepsilon}_{ij}{}^{p} = \{\frac{\partial F}{\partial \sigma_{mn}'} \dot{\sigma}_{mn}' + \dot{v}_o \cdot \exp\left(\frac{f-v^p}{\alpha}\right)\} \times \frac{\partial F/\partial \sigma_{ij}'}{\partial F/\partial p} \qquad (7)$$

where

$$\partial F/\partial \sigma_{ij}' = \{1 - \exp(-v^p/\alpha)\} \cdot \partial f/\partial \sigma_{ij}' \qquad (8)$$

And, let us introduce the elastic strain rate tensor, $\dot{\varepsilon}_{ij}{}^{e}$, in the form:

$$\dot{\varepsilon}_{ij}{}^{e} = \frac{\kappa \dot{p}}{3(1+e_o)p} \delta_{ij} + \frac{1}{2G} \dot{s}_{ij} \qquad (9)$$

where δ_{ij} is the unit tensor, and G is the modulus of rigidity.

Thus, the total strain rate tensor, $\dot{\varepsilon}_{ij}$, is given by

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}{}^{p} + \dot{\varepsilon}_{ij}{}^{e} \qquad (10)$$

In the subsequent sections, the validity of the constitutive relations just derived will be discussed.

STRAIN RATE EFFECTS

This section is devoted to discuss strain rate effects on undrained stress-strain response under conventinal isotropically consolidated triaxial conditions. A set of values for the five material constants of the proposed constitutive relations is listed in Table I, together with a value of $\dot{v}_o$. Those values are arbitrarily chosen to be typical of those for a young normally consolidated clay.

Table I Values of parameters in proposed constitutive relations

$\frac{\lambda}{1+e_o}$	$\frac{\kappa}{1+e_o}$	D	α	$\frac{G}{p_o}$	$\dot{v}_o$ (%/min.)
0.0921	0.0199	0.053	0.0029	25	10^{-5}

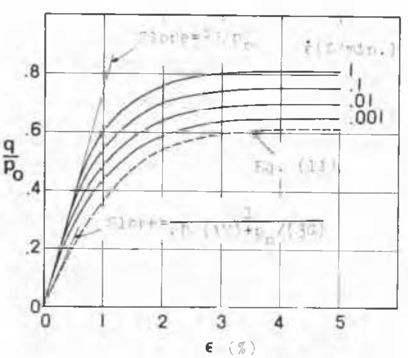

Fig. 1 Computed stress-strain curves

Figure 1 shows computed stress-strain curves at different rates of strain, together with the dotted curve computed using

$$\varepsilon = \frac{D\kappa}{\lambda} \ln\left(\frac{M}{M-q/p}\right) + \frac{q}{3G} \qquad (11)$$

where ε is the deviatoric strain defined by

$$\varepsilon = \sqrt{(2/3)(\varepsilon_{ij} - v/3\delta_{ij})(\varepsilon_{ij} - v/3\delta_{ij})}$$

and where $M=(\lambda-\kappa)/\{D\cdot(1+e_o)\}$.
If the second term on the right-hand side of Eq. (11) is ignored, the resulting relation reduces to that derived by Roscoe et al. (1963) and by Ohta and Hata (1971). Since Eq. (11) assumes an elastoplastic response, the dotted curve in Fig. 1 could be regarded as the stress-strain response at equilibrium. It is evident from Fig. 1 that the deviatoric stress, q, at any strain tends to increase almost proportionally to the logarithm of the deviatoric strain rate, $\dot{\varepsilon}$.

Relatively less attention has been paid to the strain rate dependency of the mean effective stress as compared with that of the deviatoric stress. Figure 2 shows computed effective stress paths at different rates of strain which are associated with the curves in Fig. 1. The dotted curve in Fig. 2 is computed using

$$q/p = \lambda/\{D\cdot(1+e_o)\}\cdot \ln(p_o/p) \qquad (12)$$

which has been obtained by Roscoe et al. (1963) and by Ohta and Hata (1971) with an assumption of no strain rate effect. It is seen from Fig. 2 that the mean effective stress at any deviatoric stress tends to be greater with increasing rate of strain.

Such predicted characteristics as shown in Figs. 1 and 2 are in qualitative agreement

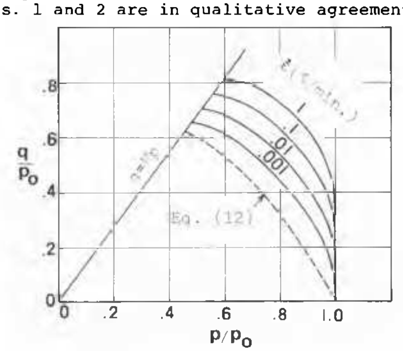

Fig. 2 Computed effective stress paths under undrained shear

with observations made, for example, by Richardson and Whitman (1963) and Akai et al. (1975).

STRESS RELAXATION CHARACTERISTICS

This section deals with characteristics of stress relaxation which occurs after the stoppage of undrained shear with a constant rate of strain. The merit of this type of stress relaxation test may be capable of testing at a wide range of strain levels.

Figure 3 indicates such stress relaxation curves at different strains, computed for conventional isotropically consolidated triaxial conditions. It is seen from Fig. 3 that after a finite value of elapsed time the deviatoric stress decreases in proportion to the logarithm of elapsed time. And the slope of the linear portion of each curve is found to be approximately independent of the magnitude of imposed strain. Such predicted properties appear to be confirmed by the observed ones at relatively large strains (Murayama et al., 1974 and Akai et al. 1975).

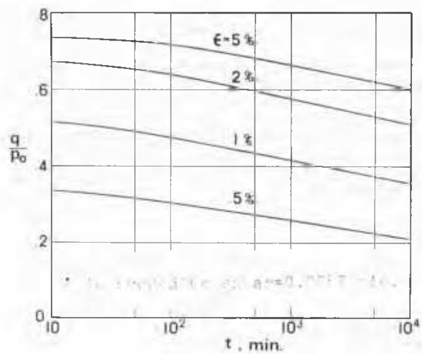

Fig. 3 Computed stress relaxation curves

Experimental evidence from Lacerda and Houston (1973), Murayama et al. (1974) and Akai et al. (1975) has suggested that during undrained stress relaxation the mean effective stress does decrease with time as well as the deviatoric stress. It may be appropriate here to supplement that results to support the just mentioned have also been obtained in the present computations.

CREEP RUPTURE CHARACTERISTICS

The present section is aimed at predicting creep rupture behavior of normally consolidated clay under conventional isotropically consolidated triaxial, K_o consolidated triaxial, and K_o consolidated plane strain conditions. Here K_o denotes the coefficient of earth pressure at rest.

Figure 4 shows computed changes of the mean effective stress during undrained creep at given deviatoric stresses for conventional isotropically consolidated triaxial conditions. It is suggested from Fig. 4 that the value of q/p_o at the point of intersection of the failure line (q=Mp) and the dotted curve corresponds to the upper yield value that has been originally proposed by Murayama and Shibata (1961). At lower stress levels than the upper yield value creep appears to stop or continue at an imperceptible rate after very large times from the initiation of creep (Fig. 5); and at greater stress levels than the upper yield value the initially decreasing creep rate may start to accelerate after a finite elapsed time and ultimately lead to creep rupture (Fig. 5).

Now let us consider the relationship between the time to failure and the minimum creep rate. Figure 6 indicates such relationships computed for conventional isotropically consolidated triaxial, K_o consolidated triaxial, and K_o consolidated plane strain conditions. Here the axial strain rate is used as a

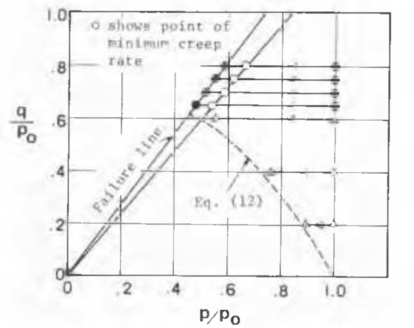

Fig. 4 Computed effective stress paths under undrained creep

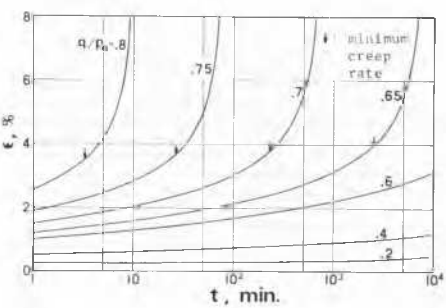

Fig. 5 Computed creep curves

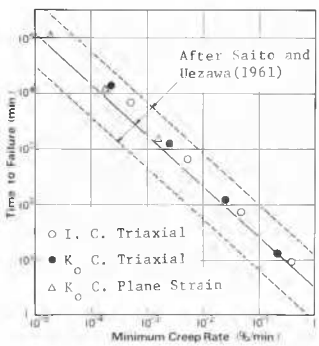

Fig. 6 Computed relationships between time to failure and minimum creep rate

measure of creep rate. It is clear from Fig. 6 that the time to failure is approximately in inverse proportion to the minimum creep rate for the three conditions as indicated. This prediction appears to be supported by the experimental results of Campanella and Vaid (1974). And, it is of interest to mention that those computed points on Fig. 6 fall into the bands that have been determined by Saito and Uezawa (1961) for a variety of soils.

CONCLUSIONS

Constitutive relations for normally consolidated clay have been derived based primarily on the concept of a viscoplastic potential. It has been shown that they permit a consistent description, compatible with hitherto made observations, of strain rate effects on undrained stress-strain response, of stress relaxation characteristics, and of creep rupture characteristics.

From a practical point of view the following are noteworthy:

1) they permit an evaluation of the upper yield value which corresponds to the so-called creep strength;
2) they are able to predict the relationship between the time to failure and the minimum creep rate that has been empirically proposed by Saito and Uezawa (1961); and
3) they are of rate type and are applicable to an analysis of a loaded or an excavated soil mass by using the incremental method of finite elements.

It is the subject for future study to apply them as a theoretical basis for a proper interpretation of observed rates of deformation and pore-water pressures in a loaded or an excavated clay.

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