Three-Dimensional Deformation Laws for a Sand-Clay

Lois de Déformation Tridimensionnelle pour une Argile Sableuse

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SYNOPSIS The paper begins by briefly summarising the theoretical and experimental criteria for evaluating consummate constitutive relationships for soils. It is demonstrated that such relationships necessarily include parameters reflecting the influence of the intermediate principal stresses and strains. A newly developed true triaxial apparatus is then described. The remainder of the paper is concerned with an experimental investigation of the response of a sand-clay to repeated true triaxial loading with particular emphasis on the role of the intermediate principal stress expressed in terms of the Lode parameter. The response of the soil is evaluated in terms of elastic models whose constants of compliance and stiffness were calculated using statistically oriented methods. The experimental results demonstrated that changes in the Lode parameter had a marked influence on the constitutive relationships.

INTRODUCTION

In recent years many studies have been made of the influence of the intermediate principal stress on soil properties. These include investigations of failure criteria (e.g. Lade and Duncan, 1973; Lomize et al, 1969; Sutherland and Mesdary, 1969; Yong and McKyes, 1967) and stress-strain relationships (e.g. Bishop et al, 1973; Shibata and Karube, 1965). These studies have required the development of equipment enabling the three principal stresses to be independently varied. Examples of this type of equipment have been critically evaluated elsewhere (Makiuchi and Shackel, 1976). Most of the equipment has been designed for static tests leading to failure. However, in highway and earthquake engineering and in the design of machine bases static tests are inappropriate and failure criteria are of lesser importance than the delineation of constitutive relationships. This paper reports a study of consummate stress-strain relationships based on data obtained using a true triaxial apparatus which was capable of simulating a variety of dynamic stress histories.

CONSTITUTIVE RELATIONSHIPS AND DEFORMATION LAWS

A constitutive relationship is an equation or law which includes all of the factors necessary to predict the response of a material to some input or forcing function (Lazan, 1968). Ideally, the constitutive relationship should predict the response that will result from any conceivable input. For soils, this implies that the relationship must necessarily include measures of every factor that may modify the soil response including stress magnitude, stress history, time, state parameters such as density and saturation and environmental parameters such as temperature.
At present the testing procedures used in geomechanics are not adequate to delineate the functional relationships between the various parameters contributing to constitutive relationships for soils. For this reason it is customary to employ the techniques of simulation service evaluation (Lazan, 1968) to simplify the problem. Here, special ad-hoc laboratory tests are employed to characterise the soil under conditions which simulate the stress histories and environmental conditions expected in the prototype. In this case samples are prepared to those specific combinations of density and saturation relevant to the problem under consideration. No prior assumptions concerning the nature of the soil response are then necessary. Rather, it is assumed that the behaviour of the soil in simulation will be similar to its behaviour in service and it becomes possible to define a deformation law relating stress and strain which may be used to analyse and characterise the prototype. It is important to note, however, that such a deformation law represents just a special case of the more general constitutive relationship for the soil because it only applies to those particular combinations of factors simulated in the laboratory tests. The problem of defining deformation laws is now considered in more detail. In the case of a non-linear, rate independent, anisotropic material each component of strain is usually expressed as some complex functional relationship of every component of stress (e.g. Shackel, 1974). However, at present there is no reliable information on the form of these functionals for soils and it is customary to characterise soils in terms of linear models adapting these, where necessary, to non-linear problems by applying incremental methods of analysis (e.g. Dehlen, 1969). For an anisotropic material, the linear model takes the form

$$\{\epsilon_{ij}\} = C_{ij} \{\sigma_{ij}\} \tag{1}$$

where ϵ_{ij} and σ_{ij} are the strain and stress tensors respectively and C_{ij} is a matrix of compliances. The tensors, ϵ_{ij} and σ_{ij} each contain six discrete elements (i.e. 3 normal stresses or strains plus 3 shear stresses or strains). Consequently C_{ij} contains 36 coefficients (compliances). Green's approach to elasticity based on the conservation of strain energy requires, however, that the matrix C_{ij} be symmetrical and only 21 of the coefficients are independent. All of these coefficients must be measured if an anisotropic soil is to be fully characterised. This is not possible because to date no system for superimposing 3 independent shear stresses onto 3 independent normal stresses has been developed. It therefore follows that soils may only be characterised in terms of simpler models than that represented by equation 1.

One way to simplify the deformation model is to assume that axes of symmetry exist in the anisotropy. Following this approach, considerable progress has been made in the characterisation of cross-anisotropic materials (e.g. Dehlen, 1969; Morgan and Gerrard, 1973). However, most investigations have been restricted to the evaluation of isotropic models of soil behaviour. For example Scott and Ko (1969) have shown that a deformation law for soils may be postulated in the form:

$$\sigma_{ij} = C_1\ \delta_{ij} + C_2\ \ \varepsilon_{ij} + C_3\ \ \varepsilon_{ij}\ \ \varepsilon_{Ij} \qquad (2)$$

where C_1, C and C_3 are functions of the invariants I_1, I_2 and I_3 of the strain tensor, ε_{ij}, and δ_{ij} is the Kronecker delta.

The alternative to deformation laws such as that given in Equation 2 is to adopt a heuristic approach to defining stress-strain relationships. Thus Newmark (1960) has suggested that it may be possible to express deformation laws for soils in the forms:

$$\varepsilon_{oct} = f_1\ (\sigma_{oct}) + f_2\ (\tau_{oct}) + f_3\ (\phi)$$

$$\gamma_{oct} = f_4\ (\sigma_{oct}) + f_5\ (\tau_{oct}) + f_6\ (\phi) \qquad (3)$$

and $\quad \theta = f_7\ (\sigma_{oct}) + f_8\ (\tau_{oct}) + f_9\ (\phi)$

where f_1 to f_9 are arbitrary functions, σ_{oct} and τ_{oct} are the octahedral normal and shear stresses and ε_{oct} and γ_{oct} are the corresponding normal and shear strains. The parameters, ϕ and θ are functions of the third stress and strain invariants respectively.

Similarly, it has been postulated (Lomize and Kryzhanovsky, 1967; Lomize et al, 1969) that deformation laws for soils may be expressed in terms of the first and second invariants, I_1, and I_2, of the strain tensor and all three invariants, J_1, J_2 and J_3 of the stress tensor. The law can then be written as

$$\begin{aligned} I_1 &= f_{10}\ (J_1,\ J_2,\ J_3) \\ I_2 &= f_{11}\ (J_1,\ J_2,\ J_3) \end{aligned} \qquad (4)$$

where f_{10} and f_{11} are empirically derived functions.

In order to evaluate the relationships expressed in Eqns. 2, 3 and 4 it is necessary to independently control only the three invariants of the stress tensor, σ_{ij}. This can be accomplished by independently varying just the three principal stresses, σ_1, σ_2 and σ_3. Thus the heuristic stress-strain relationships have the important advantage over the deformation law represented by Eqn.1 that they do not require the application of shear stresses or deformations to the test specimen.

Methods for evaluating the functions f_1 to f_{11} in Eqns. 3 and 4 have been described elsewhere (Shackel, 1973a, 1973b) and the forms that these functions may assume have been experimentally examined for a cyclically stressed sand-clay (Shackel, 1972a, 1972b,1973a, 1973c). However these studies employed a conventional triaxial stress system ($\sigma_1 > \sigma_2 = \sigma_3$) and all three stress invariants could not be independently controlled. This deficiency was rectified in the work reported here which employed a specially developed true triaxial apparatus to study the forms that consummate deformations laws may assume.

THE TRUE TRIAXIAL APPARATUS

Between 1973 and 1975 the authors developed a unique triaxial apparatus to subject cubical soil specimens to repeated loading with independent control of all three principal stresses. This device has been described in detail elsewhere (Makiuchi and Shackel,1976) and only a summary of its main features will be given here. As shown in Figure 1, the specimen was loaded through three rigid platens constrained to move so that shear strains were not permitted. Small clearances were provided between the loading platens along three edges of the specimen (ab, ac and ad in Fig.1) to accomodate changes in the specimen dimensions. Load was applied to the platens by three mutually perpendicular hydraulic rams reacting against the main frame of the apparatus. The rams were connected to a pump driven hydraulic system via electrically actuated directional valves. The operation of these valves was controlled by a timer so that load could be repetitively applied and removed in such a manner that the ratio $\sigma_1 : \sigma_2 : \sigma_3$ was maintained constant during both loading and unloading. The loads applied to the specimen were measured by load cells placed between the loading platens and the hydraulic rams. Deformations were measured by displacement transducers mounted coaxially with the load cells and bearing against reference arms fixed to the frame of the apparatus. The signals from the load cells and displacement transducers were recorded on a UV recorder.

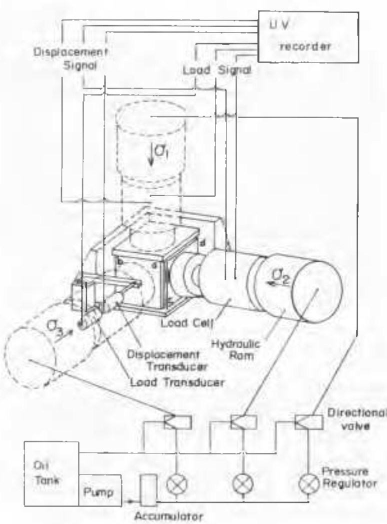

Fig.1 The True Triaxial Apparatus

THE EXPERIMENTAL WORK

a) Soil and Specimen Characteristics

An artificial soil was selected for study. This comprised a mixture of 40% sand, 35% silt and 25% kaolin by weight. The properties of the components and of the blended soil are summarised in Table 1. Cubical specimens, each 100 mm x 100 mm x 100 mm, were prepared to designated values of density and saturation by floating mould compaction; a variant of static compaction known to give specimens of exceptional uniformity (Shackel, 1970). The dry density lay on the line of optimums midway between the maximum dry densities given by the standard and modified AASHO compaction tests and had a value of 2.06 gm/cm^3. The moulding moisture content was 9.06% corresponding to a degree of saturation of 85%.

TABLE I

Properties of the Experimental Soil

(1)	CLAY (Kaolin)	
	Liquid limit	w_L = 71.6%
	Plasticity index	I_p = 35.4%
	Activity	A_c = 0.43
	Ignition loss (Weight loss)	14.5
	Specific gravity	G_s = 2.57
(2)	SANDY SILT	
	Grain shape	semi-round
	Effective grain size	D_{10} = 0.033 mm
	Uniformity coefficient	U_c = 2.8
	Specific gravity	G_s = 2.66
(3)	SAND (Botany Bay sand)	
	Grain shape	sub-round
	Effective grain size	D_{10} = 0.20 mm
	Uniformity coefficient	U_c = 1.7
	Specific gravity	G_s = 2.66
(4)	MIXED SOIL (sand:silt:clay = 40:35:25% by weight)	
	Pretreatment	Oven dry at 110°C, 24 hr.
	Liquid limit	w_L = 21.9%
	Plasticity index	I_p = 9.7%
	Linear shrinkage	S = 0.056
	Specific gravity	G_s = 2.64

After compaction the specimens were wrapped in 0.02 mm plastic film which was found to inhibit moisture losses over periods in excess of 70 days. This plastic wrapping was left on the specimen during all subsequent curing and testing procedures. Each specimen was cured at 20°C for 3 days before being subjected to repeated true triaxial loading.

b) Design and Execution of the Experiment

The prime objective of the experimental work was to delineate the influence of the intermediate principal stress on the rheological characteristics of the soil. It has been customary in geomechanics to divide the state of stress experienced by a soil into a spherical or hydrostatic component plus a deviatoric or shearing component. These components are usually expressed in terms of the octahedral normal and shear stresses, σ_{oct} and τ_{oct}, or related parameters such as p and q. These parameters do not, however, include any indication of the magnitude of the intermediate principal stress. For this reason, in the work reported here, it was decided to supplement the use of the octahedral stress parameters by an intermediate principal stress parameter, ζ ; the so-called Nadai-Lode stress parameter where

$$\zeta = (2\sigma_2 - \sigma_1 - \sigma_3)/(\sigma_1 - \sigma_3) \qquad (5)$$

The parameter ζ can assume values between +1 (triaxial extension $\sigma_1 = \sigma_2$) and -1 (triaxial compression where $\sigma_2 = \sigma_3$) and, as shown in Fig.2, can be varied by altering σ_1, σ_2 and σ_3 without changes in the octahedral stresses, σ_{oct} and τ_{oct}.

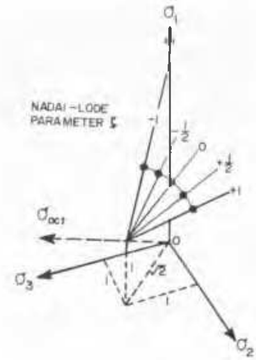

Fig.2 The Stress States Selected for Study

In order to study the effects of the intermediate principal stress an incomplete fully randomised factorial experiment without replication was implemented. The factors studied were σ_{oct}, τ_{oct} and ζ. Levels of σ_{oct} varied from 49 kPa to 196 kPa whilst τ_{oct} varied from 17 kPa to 137 kPa and ζ varied from -1 to +1. For each combination of these factors a repeated loading true triaxial test was performed. Here the frequency of load application was 0.5 Hz and each test proceeded until either the sample had withstood 10^5 stress repetitions or, alternatively, a principal strain of 4% had accumulated. The test temperature was maintained at 20°C.

ANALYSIS OF THE EXPERIMENTAL DATA

Unless stated to the contrary, only quasi-elastic strains are considered here i.e. those strains which were fully recoverable upon removal of load. These elastic data were used to evaluate a hierarchy of deformation models using both the anisotropic approach represented by Eqn.1 and the quasi-isotropic approach represented by Eqn.3.

a) The Anisotropic Model

In the case of a material experiencing only principal stresses and strains Eqn.1 may be simplified to

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} \qquad (6)$$

where C_{ij} is the compliance matrix.

Similarly the deformation law may be expressed in terms of stiffness viz:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{Bmatrix} = \begin{vmatrix} S_{11} & S_{12} & S_{13}\\ S_{21} & S_{22} & S_{23}\\ S_{31} & S_{32} & S_{33}\end{vmatrix} \begin{Bmatrix}\epsilon_1\\ \epsilon_2\\ \epsilon_3\end{Bmatrix} \qquad (7)$$

where S_{ij} is the stiffness matrix.

The values of the coefficients of the compliance and stiffness matrices were evaluated over all values of the intermediate stress parameter, ζ, by multiple linear regression using procedures described elsewhere (Shackel,1973a, 1973b). The results of these analyses are shown as functions of the number of stress applications, N, in Figures 3 and 4. From these figures it may be seen that C_{ij} and S_{ij} varied slightly with stress repetition.

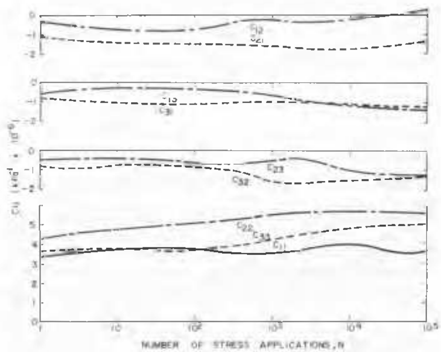

Fig.3 The Effects of Load Repetition on the Compliances

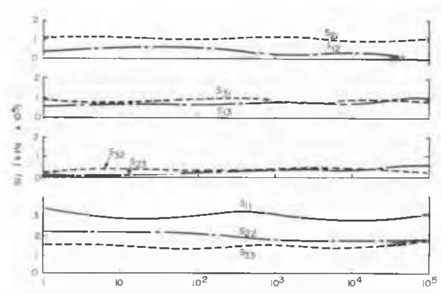

Fig.4 The Effects of Load Repetition on the Stiffnesses

For an anisotropic elastic material the compliance and stiffness matrices given in eqns. 6 and 7 should be symmetrical about their main diagonals. Inspection of Figures 3 and 4 shows that whereas the stiffness matrix was almost symmetrical the compliance matrix exhibited a lack of symmetry. However, it was not clear whether this might be attributed to experimental error or whether it represented true stress induced anisotropy.

In order to further examine whether the soil was behaving isotropically it was decided to study the relation between the Nadai-Lode stress parameter, ζ, defined earlier and the corresponding strain parameter, η, where

$$\eta = (2\,\epsilon_2 - \epsilon_1 - \epsilon_3)/(\epsilon_1 - \epsilon_3) \qquad (8)$$

and ϵ_1, ϵ_2, and ϵ_3 were the principal strains. In this case both elastic and plastic components of strain were considered, represented by η^e and η^p respectively. Values of η averaged over all values of the number of stress repetitions, N, are plotted against the corresponding values of the stress parameter, ζ, in Fig.5 for σ_{oct} = 196 kPa and τ_{oct} = 137 kPa. Similar relationships to those given in Fig.5 were observed at the other stress levels studied.

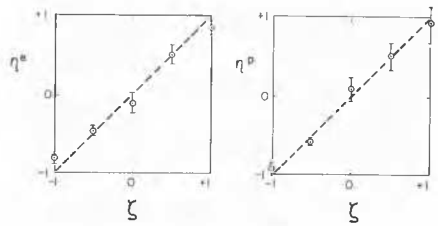

Fig. 5 Relationship between the Intermediate Principal Stress and Strain Parameters.

From Fig.5 it may be seen that within the confidence intervals plotted for η ($\pm$1Std. dev.) the relationship between η and ζ was linear. This indicated that the soil behaviour, averaged over the duration of each test, was essentially isotropic.

b) The Isotropic Models

As shown in the preceding section, the soil responded in a quasi-isotropic manner to repetitive loading. This meant that the heuristic models represented, for example, by Eqns. 3 and 4 could be used to characterise the soil. This required the experimental data to be expressed in terms of invariant parameters. The octahedral stresses and strains were the parameters selected for this purpose.

The elastic octahedral volumetric and shear strains, ϵ^e_{oct} and γ^e_{oct}, averaged over all values of the number of load repetitions, N, are shown in Fig.6 as functions of the octahedral stresses. As shown in this figure, the stress-strain curves tended to be nonlinear irrespective of the value of the intermediate principal stress parameter, ζ. The forms of these relationships resembled those observed in conventional static triaxial tests and, in the case of the ϵ_{oct} - σ_{oct} relationships, were similar to those reported earlier by Shackel (1973c) for a repetitively stressed sand-clay.

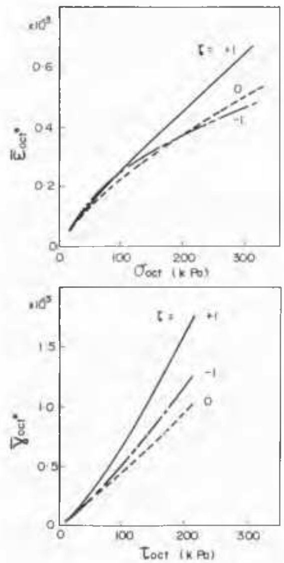

Fig. 6 Relationship Between the Octahedral Stresses and Strains

To facilitate further studies of the influence of the intermediate principal stress, the stress-strain relationships were approximated by the following deformation laws:

$$\varepsilon_{oct} = b_1 \; \sigma_{oct}^{\;m_1} \tag{9}$$

and

$$\gamma_{oct} = b_2 \; \tau_{oct}^{\;m_2} \tag{10}$$

where b and m were empirical constants. The manner in which these constants varied with changes in the intermediate principal stress, expressed in terms of ζ, is shown in Fig.7.

The data plotted in Figures 6 and 7 indicate that the soil exhibited a non-linear stress-strain response. It followed that it could not be assumed that there was no coupling between the shearing and volumetric components of stress and strain. Consequently the simple relationships given as Eqns. 9 and 10 could not adequately characterise the deformation response of the soil. It was decided, therefore, to evaluate deformation laws in which each component of strain depended on both the spherical and the deviatoric components of stress. For this purpose, a simple linear law similar to that given in Eqn.3 was selected for study. This model took the form:

$$\begin{Bmatrix} \varepsilon_{oct}^{e} \\ \gamma_{oct}^{e} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{Bmatrix} \sigma_{oct} \\ \tau_{oct} \end{Bmatrix} \tag{11}$$

and was similar to linear models studied earlier under conventional triaxial stress states (ζ= -1) by Shackel (1972b, 1973c).

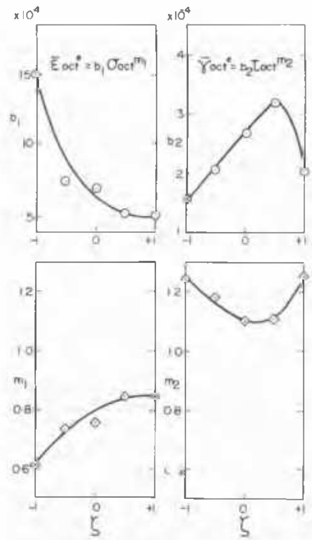

Fig. 7 The Hyperbolic Stress-strain Relationships

The coefficients of the matrix a_{ij} were determined for specific values of the number of stress repetitions, N, using statistically oriented regression techniques described earlier (Shackel, 1973a, 1973b). In these regression analyses, values of the multiple correlation coefficient, r^2, ranged between 0.91 and 0.99. This indicated that the model represented by Eqn.11 provided an excellent description of the experimental data. Typical values of a_{ij} are plotted in Figure 8 as functions of the intermediate stress parameter, ζ.

From Figure 8 it will be seen that the changes in the intermediate principal stress profoundly modified the values of the compliances, a_{ij}, needed to define the deformation law. In general, the compliances were either a maximum or a minimum at values of the Nadai-Lode parameter, ζ, close to zero. In this respect the variation of a_{ij} with ζ was similar to the manner in which the failure parameter, ϕ, has been reported to vary with the intermediate principal stress (Green and Bishop, 1969; Lomize et al, 1969, Mesdary and Sutherland, 1970; Ramamurthy and Rawat. 1973).

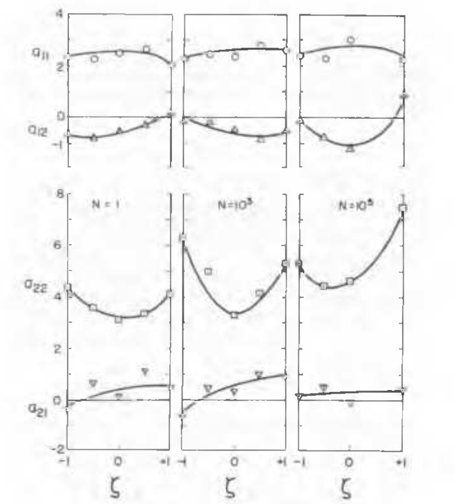

Fig. 8 The Effects of the Intermediate Principal Stress on the Compliances Needed to Define the Heuristic Deformation Law.

SUMMARY AND CONCLUDING COMMENTS.

The investigation described in this paper showed that it was practical to develop a true triaxial apparatus which could be used to simulate dynamic and repetitive stress histories similar to those associated with highway pavements and machine bases. Using this device it was possible to derive consummate stress-strain laws and to delineate the effects of both the intermediate principal stress and a repetitive stress history on the compliance and stiffness matrices. In addition it was possible to study the degree of anisotropy of the soil.

For the particular soil examined, a compacted sand-clay, it was determined that the response of the material was essentially quasi-isotropic although some evidence of slight stress induced anisotropy was noted. A linear, isotropic deformation law was postulated in a form similar to that proposed by earlier workers which incorporated coupling between the spherical and deviatoric components of stress and strain. This model was found to provide a good characterisation of the soil. Finally, the effects of the intermediate principal stress upon the model parameters were studied and were shown to be similar to its effects, rpeorted earlier, upon the failure parameter, ϕ.

BIBLIOGRAPHY

Bishop,A.W., Green,G.E. and Skinner,A.E.(1973), "Strength and Deformation Measurements on Soils." Proc. 8th Int. Conf. SMFE. Moscow, Vol. 1.1,pp.57-64.

Dehlen,G.L. (1969),'The Effect of Non-Linear Material Response on the Behaviour of Pavements Subjected to Traffic Loads". Thesis (Ph.D) Univ. of Calfornia.

Green,G.E. and Bishop,A.W.(1969),"A Note on the Drained Strength of Sand under Generalized Strain Conditions". Geotechnique,Vol.19,No.1,pp144-149.

Lade,P.V. and Duncan,J.J. (1973),"Cubical Triaxial Tests on Cohesionless Soil". Journal ASCE,No.SMIO, pp.793-812.

Lazan,B.J. (1969), "Damping of Materials and Members in Structural Mechanics". Pergamon Press.

Lomize,G.M. and Kryshanovsky,A.L. (1967), "On the Strength of Sand". Proc. Geotechnical Conf. Oslo, Vol.1, pp.215-219.

Lomize,G.M. et al (1969), "Study on Deformation and Strength of Soils under Three-Dimensional State of Stress". Proc. 7th Int. Conf. SMFE, Mexico,Vol.1, pp.257-165.

Makiuchi,K and Shackel, B. (1976), "Soil Characterization Using a Repeated Loading Cubical Triaxial Apparatus". Proc.8th Conf.Aust.Road Res. Board.

Mesdary,M.S. and Sutherland,H.B. (1970), "Correspondence on A Note on the Drained Strength of Sand under Generalized Strain Conditions". Geotechnique,Vol.20, No.2, pp. 210-212.

Morgan,J.R. and Gerrard,C.M. (1973),"Anisotropy and Non-linearity in Sand Properties". Proc.8th Int. Conf. SMFE. Moscow,Vol.1.2, pp. 287-292.

Newmark,N.M. (1960),"Failure Hypotheses For Soils". Proc. ASCE Res. Conf. on Shear Strength of Cohesive Soils. Univ. of Colorado, pp. 17-32.

Ramamurthy,T and Pawat,P.C. (1973), "Shear Strength of Sand Under General Stress System". Proc. 8th Int. Conf. SMFE,Vol. 1.2, pp. 339-342.

Scott, R.F. and Ko,H.Y.(1969),"Stress-Deformation and Strength Characteristics". Proc. 7th Int.Conf. SMFE. Mexico,Vol.3,pp.1-47.

Shackel,B.(1970),"The Compaction of Uniform Replicaté Soil Specimens".Aust.Rd.Res.Vol.4,No.5,pp. 12-31.

Shackel,B.(1972a),'The Deformation Response of a Sand-Clay Subjected to Repeated Triaxial Compressive Stress' Proc.Symp.on Strength and Deformation Behaviour of Soils. Bangalore, Vol.1, pp. 145-149.

Shackel,B.(1972b),"Linear and Non-Linear Models of the Stress-Strain Response of a Cyclically Stressed Soil". Proc.3rd SEACSE, Hong Kong,pp. 349-355.

Shackel,B.(1973a),"The Derivation of Complex Stress-Strain Relationships".Proc. 8th Int.Conf.SMFE.Vol.1.2.

Shackel,B.(1973b),"An Engineering Approach to Defining Three Dimensional Stress-Strain Relationships".Proc. Conf.on Stress and Strain in Engineering,Inst.of Eng. (Aust.),Brisbane, pp. 26-31.

Shackel,B.(1973c),"The Three Dimensional Stress,Strain and Damping Response of a Cyclically Stressed Sand-Clay".Proc.5th World Conf. on Earthquake Eng. Rome.

Shackel,B.(1974),"Some Effects of Repeated Triaxial Stresses on Road Pavement Materials". Thesis(Ph.D.), Univ. of New South Wales.

Shibata,T. and Karube,D.(1965),"Influence of the Variation of the Intermediate Principal Stress on the Mechanical Properties of Normally Consolidated Clays", Proc.6th Int. Conf.SMFE.Montreal,Vol.1,pp 359-363.

Sutherland,H.B. and Mesdary,M.S.(1969),"The Influence of the Intermediate Principal Stress on the Strength of Sand', Proc.7th Int.Conf.SMFE,Mexico,Vol.1, pp. 391-399.