Prediction of Strain Rate for Drained Triaxial Tests

Prévision de Vitesse de Déformation pour Essais Triaxiaux Drainés

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SYNOPSIS The strain rates suggested by Gibson & Henkel (1954) for drained triaxial tests are found to leave high undissipated pore water pressures in the samples. In this paper, an alternate method for determining suitable strain rates for drained triaxial tests is suggested. The magnitudes of the pore pressures that develop while shearing a specimen along the drained stress path and the corresponding axial strains are determined from the results of undrained tests.

INTRODUCTION

A drained test has to be performed in the laboratory at a strain rate which would ensure that the undissipated pore water pressure in the sample is negligible when compared to the effective stresses acting on it throughout the duration of the test. Bishop & Henkel (1962) have recommended suitable strain rates to be used in carrying out fully drained triaxial tests. These strain rates are, however, found to leave high undissipated pore pressures in triaxial specimens of kaolin sheared under fully drained condition (see Thurairajah, Balasubramaniam and Fonseka, 1975). In this paper an alternate method for predicting the strain rate for the conventional drained triaxial compression test is presented. Only normally consolidated saturated clays have been considered but, since the volume change during a drained test on a normally consolidated clay is larger than on the overconsolidated clay, the theoretical strain rates for normally consolidated clays could also be satisfactorily used for overconsolidated clays.

STRESS AND STRAIN PARAMETERS

The stress parameters used are the mean normal stress $p = \frac{1}{3}(\sigma_1' + 2\sigma_3')$ and the deviator stress $q = (\sigma_1' - \sigma_3')$, where σ_1' and σ_3' are the effective axial and radial strains. The incremental strain parameters dv and $d\epsilon$ are defined as $dv = d\epsilon_1 + 2d\epsilon_3$ and $d\epsilon = \frac{2}{3}(d\epsilon_1 - d\epsilon_3)$, where $d\epsilon_1$ and $d\epsilon_3$ are the incremental axial and radial strains.

GIBSON & HENKEL THEORY FOR ESTIMATING STRAIN RATES

In Fig. 1, NP and NQ respectively represent the stress paths for an undrained and a fully drained triaxial compression tests, on a normally consolidated clay with constant cell pressure. OX is the projection of the critical state line. The laboratory drained test has some undissipated pore water pressure in the sample and therefore the effective stress path NR for such a test lies to the left of NQ.

Let q_{uf}, q_{df} and q_f be the peak deviator stresses for the undrained, fully drained and laboratory drained tests respectively. If U_f is the pore pressure

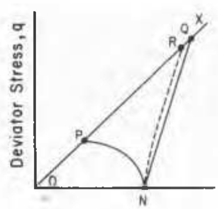

Fig. 1 Stress paths

developed under q_{uf} in the undrained test and Δu_f is the undissipated pore water pressure under q_f during the laboratory drained test, then

$$\frac{q_f - q_{uf}}{q_{df} - q_{uf}} = 1 - \frac{\Delta u_f}{u_f} = U_f \quad \text{..... (1)}$$

As U_f approaches unity, the peak point in the laboratory drained test path approaches the peak point in the fully drained test path. Gibson & Henkel (1954) derived an equation similar to Eq. (1) in which Δu_f is the average pore water pressure at failure along the shear plane in the sample and U_f is the average degree of consolidation at failure. It is apparent from their paper that their objective is to choose a strain rate so that the peak strength measured during the laboratory drained test is very close to the peak strength for the fully drained test. This objective can be achieved if the strain rate chosen is such that the undissipated pore water pressure in the sample at failure, $(1-U_f)\,u_f$ is small; u_f is the pore water pressure at failure during an undrained test on a sample consolidated under the same cell pressure as for the drained test. According to Gibson & Henkel (1954), the strain for drained triaxial tests should be such that the time to failure t_f is given by the equation

$$t_f = \frac{H^2}{\mu\, c_v\,(1-U_f)}$$; 2H is the height of the sample, μ is a factor depending on the extent and location of the drainage surface, and c_v is the coefficient of consolidation. The equations given in their paper indicate that the total pore pressure developed in a drained test is assumed to be equal to the pore pressure developed during an undrained test. But the pore water pressure developed during a drained test is much higher than the value u_f corresponding to the undrained test. For remoulded specimens of kaolin sheared under drained condition, the magnitude of pore pressure, u, developed is about 4.5 times the

value of u_f. Thus the undissipated pore pressure in the drained test specimen must be $(1-U_f)u$, instead of $(1 - U_f)u_f$. Gibson & Henkel recommended that the maximum value of the deviator stress is reached in a drained test, for the duration of t_f, corresponding to a value of U_f of 0.95. The maximum undissipated pore water pressure in the sample is then 0.05 u, where u is the total pore pressure developed in a sample sheared along a fully drained stress path and is about 4.5 times u_f in the particular case of normally consolidated kaolin specimens.

It is therefore essential that the strain rate for a laboratory drained test is such that the stress and strain parameters are determined sufficiently accurately for the entire stress path. In the approach adopted in this paper, the strain rate for a drained test is chosen so that the maximum undissipated pore pressure in the sample at any stage of the test could be controlled to be a small fraction of the cell pressure under which the sample is consolidated prior to the beginning of shear.

PORE PRESSURE DEVELOPED DURING SHEAR ALONG A FULLY DRAINED STRESS PATH

According to Roscoe and Poorooshasb (1963), the effective stress paths obtained from undrained triaxial compression tests on a normally consolidated clay are geometrically similar. Also unique relationship exists been q/p ($= \eta$) and ϵ. Thus it could be shown that u/p is uniquely related to η. Hence

$$u/p = h(\eta) \qquad \ldots\ldots (2)$$

where η denotes q/p.

Consider a specimen of clay at a state (e, p, q) corresponding to A in Fig. 3 and lying on the fully drained stress path NQ, e being the voids ratio. Let a stress increment (δp, δq) causes the state of the sample to move to B (e, $p + \delta p$, $q + \delta q$). Then the pore pressure δu developed in the specimen due to the stress increment (δp, δq) is denoted as BC in Fig. 3. It can be shown that $\delta p = (\delta q/3 - \delta u)$ and $\delta\eta = \delta q/p - \eta . \delta q/p$. Differentiating equation (2) and substituting the values of δp and $\delta\eta$, it can be shown that

$$\delta u = \frac{[h(\eta) + (3-\eta)\, h'(\eta)]\ \delta q}{3[1 + h(\eta) - \eta\, h'(\eta)]} \qquad \ldots\ldots (3)$$

where $h'(\eta) = \frac{d}{d\eta}[h(\eta)]$

Equation (3) gives the magnitude of the pore pressure, δu, developed in a specimen, while the deviator stress increased from q to $q + \delta q$ under undrained condition. If the specimen is now allowed to drain keeping the deviator stress constant at $q + \delta q$, the state of the specimen moves from B to C (in Fig. 3) when δu is fully dissipated.

For a fully drained stress path starting from a pre-shear consolidation pressure of σ_3 (same as the cell pressure), $p = \frac{3\sigma_3}{(3-\eta)}$ and $\delta q = \frac{3p}{(3-\eta)}\delta\eta$.

Substituting these values in Eq. (3), $\delta u/\sigma_3$ is given by

$$\frac{\delta u}{\sigma_3} = \frac{3[h(\eta) + (3-\eta)h'(\eta)]}{[1 + h(\eta) - \eta h'(\eta)](3-\eta)^2}\,\delta\eta \qquad \ldots\ldots (4)$$

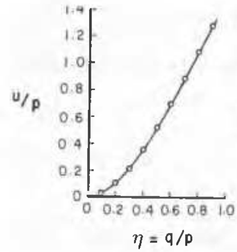

Fig. 2. Variation of u/p with q/p

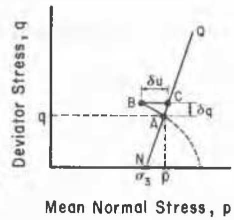

Fig. 3. Pore water pressure dissipated in a drained test

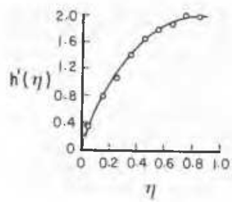

Fig. 4. Variation of $h(\eta)$ with η

The variation of $h'(\eta)$ with η as obtained from Fig. 2 is presented in Fig. 4. Knowing the values $h(\eta)$ and $h'(\eta)$, equation (4) can be integrated numerically for increments in $\delta\eta$ of 0.01. The pore pressure u, thus estimated to be developed in a fully drained test, while η changes from zero to 0.9 (corresponding to failure) is about $3\sigma_3$. In comparison to this value, the pore pressure measured during an undrained test starting from a pre-shear consolidation pressure of σ_3 is only $0.7\ \sigma_3$.

SHEAR STRAIN IN FULLY DRAINED TESTS

The plastic incremental volumetric strain, δv^P experienced by a normally consolidated clay when subjected to stress increments (δp, δq) from current stress state (q, p) can be expressed as

$$\delta v^P = \left[\left(\frac{\delta e}{1 + e}\right) - \left(\frac{k}{1 + e}\right)\frac{\delta p}{p}\right] \qquad \ldots\ldots (5)$$

Combining Eq. (5) with the energy balance equation of Roscoe, Schofield and Thurairajah (1963) and using

$\frac{\delta p}{p} = \left(\frac{\delta\eta}{3-\eta}\right)$ for a fully drained test

$$\delta\epsilon = \frac{\left[\left(\frac{\delta e}{1 + e}\right) - \left(\frac{k}{1 + e}\right)\frac{\delta\eta}{(3 - \eta)}\right]}{(M - \eta)} \qquad \ldots\ldots (6)$$

Equation (6) can be integrated numerically with respect to η. It is also possible to derive an equation similar to (6) using Roscoe & Burland (1968) energy equation.

Voids Ratio Change during a Fully Drained Test

In Fig. 5, A corresponds to the state point (e, p, q) of a normally consolidated clay subjected to a stress increment (δp, δq). Let the change in voids ratio under drained condition be δe. Curves AC and BD represent the effective stress paths under undrained conditions. Let p_c and $p_c + \delta p_c$ be the pre-shear consolidation pressures corresponding to paths AC and BD.

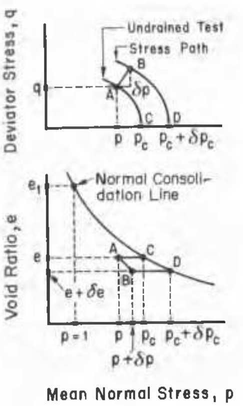

Fig. 5. Voids ratio change.

Then

$$p_c = [1 + h(\eta) - \eta/3] \quad ..(7)$$

Differentiating (7) and substituting $\delta e_1 = \frac{-\lambda}{p_c}\delta p_c$, it can be shown that for a fully drained test

$$\left(\frac{\delta e}{\lambda}\right) = \frac{h'(\eta)\ \delta\eta}{[1+h(\eta)-\eta/3]} + \frac{h(\eta)\delta\eta}{\{[1+h(\eta)-\eta/3](3-\eta)\}} \quad ..(8)$$

Equation (8) can be substituted in Equation (6) and the shear strain $\delta\epsilon$ can be integrated numerically with respect to $\delta\eta$. The voids ratio e need to be adjusted after each step of integration.

DETERMINATION OF STRAIN RATE

The chosen strain rate for the drained test should be such that the ratio of the maximum undissipated pore pressure Δu in the sample to the cell pressure σ_3 under which it is consolidated before shear is a small quantity, say x.

Fig. 6 shows the variation of u/σ_3 with ϵ during a fully drained test on normally consolidated tests. The variation seems to be linear up to about 18% shear strain. For estimating the suitable strain rate, the rate of development of pore pressure ratio u/σ_3 with ϵ for the drained test path is assumed to be a constant and equal to the slope of the straight line portion of the curve in Fig. 6. For cases where the points do not lie on a straight line, the strain rate may be calculated using the maximum gradient of the curve, and this leads to a conservative estimate.

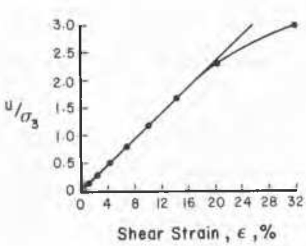

Fig. 6. Variation of u/σ_3 with ϵ for drained tests on kaolin

Consider a triaxial sample consolidated under a cell pressure σ_3 and sheared at a constant rate of shear strain y per unit time. If z is the the slope of the straight line portion of curve in Fig. 6, then

$$\frac{\delta u}{\sigma_3} = yz.\delta t \quad .. (9)$$

Equation (9) gives the quantity of pore water pressure δu developed in the sample during the drained test in an increment of time δt.

Expressions for suitable strain rates have been obtained for a triaxial sample of height 2H and radius a where 2a = H, with drainage from (i) both ends, (ii) one end only, (iii) radial boundary only, and (iv) ends and radial boundary.

Strain Rates for Different Drainage Conditions

The suitable strain rate for drained tests is given by (see Appendix)

$$y = \frac{\mu\ c_v\ x}{z\ H^2} \quad ..... (10)$$

where μ is a constant having the following values:

drainage from one end only	0.50
drainage from both ends	2.00
drainage from radial boundary only	16.10
drainage from ends and radial boundary	16.30

It should be noted that u/σ_3 may be plotted against the axial strain instead of the shear strain ϵ in Fig. 6. The value of y obtained from equation (10) will then be the axial strain rate instead of the shear strain rate.

An Approximate Method for Determining Strain Rate

For a quick estimate of the appropriate strain rate for drained tests, the following procedure may be used. The total pore water pressure developed in the sample due to loading along the fully drained stress path is first determined from the results of an undrained test by integrating equation (3) numerically. Alternatively, the total pore water pressure could be estimated graphically by making use of the fact that the undrained stress paths on (q, p) plane for different values of σ_3 are geometrically similar. z in equation (10) is then determined by making the assumption that the total pore water pressure developed during a drained test varies linearly with strain up to failure, and assuming a value for the failure strain.

Strain Rate for Specimens of Kaolin

Calculations are carried out to determine a suitable strain rate for a typical clay, Spestone Kaolin, and the result is compared with strain rate recommended by Gibson & Henkel (1954). A triaxial sample of height 3 inch and diameter 1.5 inch with drainage facilities at both ends of the sample is considered. The coefficient of consolidation, c_v, for kaolin is 0.04 in^2/min. The value of z from Fig. 6 is 0.118. For the undissipated pore water pressure in the sample to lie within 5% of the cell pressure σ_3, the shear strain rate as obtained from Equation (10) is 0.015% per minute. The shear strain for drained tests for η = 0.9 as predicted from the results of undrained tests is 31.7%. Therefore, the time required to reach the value η =0.9 by a drained test is 2110 minutes. The natural axial strain corresponding to a shear strain of 31.7% is 35.1% which is equivalent to a conventional axial strain of 29.1%. For 3 inch high triaxial sample, the rate of axial deformation is 0.00042 in/minute. The corresponding time as obtained from Gibson & Henkel method will be 375 minutes, this would correspond to a strain rate of 0.00176 in/minute for a failure strain of 22%.

CONCLUSIONS

The theoretical method derived by Gibson & Henkel to calculate the rate of strain for drained triaxial tests is found to leave high undissipated pore pressure in the sample. Expressions developed in this paper for determining strain rates for the drained triaxial compression tests satisfy the condition that the maximum undissipated pore water pressure is small when compared to the consolidation pressure. The method could easily be extended to other types of shear tests.

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APPENDIX

Drainage from Both Ends

From Terzaghi's theory for one-dimensional consolidation, the maximum undissipated pore water pressure Δu in the triaxial sample at any time T from the beginning of the test can be written as

$$\Delta u = \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}\int_0^T\left(\frac{\partial u}{\partial t}\right)\exp\left\{\frac{-(2n+1)^2\pi^2 c_v(T-t)}{4H^2}\right\}dt \quad \text{(i)}$$

where c_v is the coefficient of consolidation and $\left(\frac{\partial u}{\partial t}\right)$ is the rate of development of pore water pressure in the sample with time.

Integrating Eq. (i) gives

$$\frac{\Delta u}{\sigma_3} = \frac{16\,yzH^2}{\pi^3 c_v}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}\left[1-\exp\left\{\frac{-(2n+1)^2\pi^2 c_v T}{4H^2}\right\}\right] \quad \text{(ii)}$$

If the allowable maximum undissipated pore water pressure Δu in the sample during the test is $x\sigma_3$, the strain rate y is given by

$$y = 2.00\,\frac{c_v\,x}{zH^2} \quad \text{..... (iii)}$$

If drainage is permitted from one end of the sample only, the strain rate is given by

$$y = 0.50\,\frac{c_v\,x}{zH^2} \quad \text{..... (iv)}$$

Drainage from Radial Boundary Only

The undissipated pore water pressure at the centre of the triaxial sample at any time T is given by the expression (Carslaw and Jaeger, 1959).

$$\Delta u = 2\sum_{m=1}^{\infty}\frac{1}{\beta_m J_1(\beta_m)}\int_0^T\left(\frac{\partial u}{\partial t}\right)\exp\left\{\frac{-c_v}{a^2}\beta_m^2(T-t)\right\}dt \quad \text{...(v)}$$

where β_m are the positive roots of $J_0(\beta) = 0$, J_0 being Bessel's function of the first kind and order zero.

On integrating Eq. (v) and substituting 2a = H,

$$\frac{\Delta u}{\sigma_3} = \frac{yzH^2}{2c_v}\sum_{m=1}^{\infty}\frac{1}{\beta_m^3 J_1(\beta_m)}\left[1-\exp\left(-4\beta_m^2\frac{c_v T}{H^2}\right)\right] \quad \text{..... (vi)}$$

The strain rate is given by

$$y = 16.1\,\frac{c_v x}{zH^2} \quad \text{..... (vii)}$$

Drainage from Ends and Radial Boundary

$$\Delta u = \frac{8}{\pi}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^n}{(2n+1)\,\beta_m J_1(\beta_m)}\int_0^T\left(\frac{\partial u}{\partial t}\right)\exp\left[\frac{-c_v}{a^2}\left\{\beta_m^2 + \frac{(2n+1)^2\pi^2 a^2}{4H^2}\right\}(T-t)\right]dt \quad \text{..... (viii)}$$

Integrating Eq. (viii) and substituting 2a = H gives

$$\frac{\Delta u}{\sigma_3} = \frac{2yzH^2}{\pi c_v}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^n\left[1-\exp\left\{-\frac{4c_v T}{H^2}\left[\beta_m^2+\frac{(2n+1)^2\pi^2}{(16)}\right]\right\}\right]}{(2n+1)\beta_m J_1(\beta_m)\left[\beta_m^2+\frac{(2n+1)^2\pi^2}{16}\right]} \quad \text{(iv)}$$

The strain rate is given by

$$y = 16.3\,\frac{c_v x}{zH^2} \quad \text{..... (v)}$$