Analysis of Pile Groups Embedded in Gibson Soil

Sur les Groupes des Pieux Encastrés dans le Sol Gibson

P.K.BANERJEE and
T.G.DAVIES Dept. of Civil Engineering, University College, Cardiff, U.K.

SYNOPSIS

Load displacement and load distribution behaviour of axially loaded symmetrical pile groups embedded in Gibson soil is studied by utilising the concept of the interaction factors. These factors have been derived by using the Boundary Element Method on isolated single piles and pile groups embedded in a nonhomogeneous three-dimensional solid whose modulus of elasticity increases linearly with depth. Results for settlement and load distribution characteristics have been presented in non-dimensional form. The se show that the non-homogeneity does reduce the interaction between the piles very considerably. Theoretical results agree well with the reported experimental observations.

INTRODUCTION

Although most naturally occurring soils are nonhomogeneous very few theoretical solutions of the problem of pile groups embedded in such soils have been attempted, primarily because of (a) the geometrical complexity of the problem and (b) the definition of nonhomogeneous soils. However, due to the emergence of powerful numerical methods such as the Finite Element Method and the Boundary Element Method (or the Integral Equation Method), solutions of problems involving complex geometries no longer remain a formidable task. Of these two general methods of analysis the Finite Element Method uses the discretisation of the entire volume of the body and the Boundary Element Method utilises the discretisation of the surface enveloping each homogeneous zone of the body. Therefore for three-dimensional bodies involving low surface area to volume ratios, such as those found in Geotechnical Engineering problems the Boundary Element Method is superior to the Finite Element Method. The method has already attained a high degree of performance and hence popularity. Various applications of the indirect formulation of the method to problems of elasticity, elasto-plasticity, ground water seepage and diffusion are described by Banerjee (1976) and Banerjee and Butterfield (1976). Also a direct formulation of the method for problems of elasticity, elasto-plasticity, visco-elasticity and hydrodynamics is described in Cruse and Rizzo (1975). Regarding the problem of the definition of non-homogeneous soil recent work of Gibson (1974) has shown, that a linear increase of Young's modulus with the depth given by the equation

$$E(z) = E(0) + m z$$

where $E(z)$ = Young's modulus at a depth z

$E(0)$ = Young's modulus at ground level

m = an elastic constant

with the Poisson's ratio ν remaining constant, does provide a very satisfactory representation for analysing the linear part of the response of soil deposits where the effective stresses increase linearly with depth. The model is particularly attractive because of the ease with which the medium can be defined.

In what follows the application of the Boundary Element Method to the problem of axially loaded single piles and pile groups is described in which piles have been represented by three-dimensional solids having Young's modulus E_p and Poisson's ratio = 0.15 and the soil by 2n layers of equal thickness (see Figure 1) each having a different value of the Young's modulus. Since the stresses in homogeneous soil differs negligibly from that in Gibson soil the equivalent Young's modulus of the i^{th} layer located between the depths z_{i-1} and z_i is given to a very good accuracy by

$$E^i = (i - 1.5)\, m z_{i-1} + 0.5 m z_i$$

The deformation of the soil is assumed to take place under undrained conditions, hence Poisson's ratio of soil has been chosen to be 0.5 throughout.

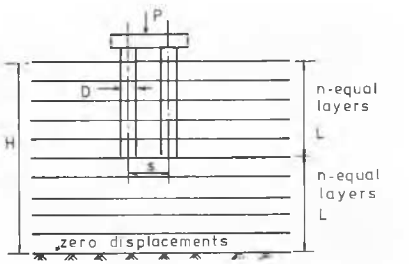

Figure 1 A Typical 2x1 Pile Group Embedded in Non-Homogeneous Soil

THE METHOD OF ANALYSIS

The general form of the Boundary Element Method for analysis of three-dimensional solids of any shape is described elsewhere (Banerjee, 1976); only a brief outline with reference to the present problem is described here. If we consider a region D bounded by a surface S, the displacements u_i and tractions p_i at a point A on S due to traction vectors $\phi_j(B)$ at B over S can be obtained from:

$$u_i(A) = \int_S K_{ij}(A, B)\ \phi_j(B)\ dS \qquad (1)$$

$$p_i(A) = \int_S \Gamma_{ij}(A, B)\ \phi_j(B)\ dS + \frac{1}{2}\phi_L(A) \qquad (2)$$

where

$$K_{ij}(A, B) = A_1\left(\frac{1}{r}\right)\left(\delta_{ij}(3-4\nu) + \frac{\zeta_i\zeta_j}{r^2}\right)$$

$$\Gamma_{ij}(A, B) = A_2\left(\frac{1}{r^2}\right)\left(\frac{(1-2\nu)}{r}(n_j\zeta_i - n_i\zeta_j) + ((1-2\nu)\delta_{ij} + (\frac{3}{r^2})\zeta_i\zeta_j)\frac{\zeta_k n_k}{r}\right)$$

$$A_1 = \frac{1}{16\pi G(1-\nu)}, \quad A_2 = \frac{1}{8\pi(1-\nu)}$$

$$\zeta_i = y_i - x_i, \quad \zeta_j = y_j - x_j, \quad \zeta_k = y_k - x_k$$

$$A(x_i)\ \&\ B(y_i); \quad r^2 = \zeta_i\zeta_i, \quad p_i = \sigma_{ij} n_j$$

$$\delta_{ij} = 1, \text{ for } i = j$$
$$= 0, \quad i \neq j$$

σ_{ij} = stresses at A

G and ν are the shear modulus and Poisson's ratio for the region D.

We can then represent the surface S by a series of triangular or rectangular flat elements (see Figure 2) and write equations (1) and (2) for the centroids of each of the elements of the surface in matrix notation as

$$\underset{\sim}{u} = \underset{\sim}{K}\ \underset{\sim}{\phi} \qquad (3)$$

$$\underset{\sim}{p} = \underset{\sim}{\Gamma}\ \underset{\sim}{\phi} \qquad (4)$$

By eliminating $\underset{\sim}{\phi}$ between these sets we obtain

$$\underset{\sim}{p} = \underset{\sim}{\Gamma}\ \underset{\sim}{K}^{-1}\ \underset{\sim}{u} = \underset{\sim}{F}\ \underset{\sim}{u} \qquad (5)$$

where the matrix $\underset{\sim}{F}$ can now be regarded as, in the terminology of the Finite Element analysis, the stiffness matrix for the region D. Instead of the conventional relationship between the nodal forces and nodal displacements we have a relationship between the surface tractions and surface displacements. Moreover because the governing differential equations are satisfied exactly by equations (1) and (2) for points within D we can make the region as large as we like.

For any layer i we can write equation (5) for the pile domain 1 and the soil domain 2 as

$$\underset{\sim}{p}_1^i = \underset{\sim}{F}_1^i\ \underset{\sim}{u}_1^i \qquad (6)$$

and

$$\underset{\sim}{p}_2^i = \underset{\sim}{F}_2^i\ \underset{\sim}{u}_2^i \qquad (7)$$

By utilising the equilibrium and compatibility at the pile-soil interface elements we can assemble these equations (in a manner similar to that used in the Finite Element Method) to form a stiffness matrix for the pile-soil system for the i^{th} layer:

$$\underset{\sim}{p}^i = \underset{\sim}{L}^i\ \underset{\sim}{u}^i \qquad (8)$$

It is interesting to note at this stage that because the thickness of each layer is assumed to be equal, the matrix $\underset{\sim}{F}_1^k$ for the k^{th} layer is obtainable from that of the j^{th} layer from

$$\underset{\sim}{F}_1^k = \lambda\ \underset{\sim}{F}_1^j \qquad (9)$$

where $\lambda = E^k/E^j$ the ratio of the Young's modulii. We can form equation (8) for each of the layers in succession and assemble the system of equations (8) for each layer of the problem by satisfying the equilibrium and compatibilities at the layer interfaces as before; we write the final system as

$$\underline{p}^{o} = \underline{L}^{o}\,\underline{u}^{o} \qquad (10)$$

where the matrix $\underline{L}^{o}$ is a blockbanded matrix. The system of equations (10) is then solved by using a Gaussian Elimination technique specially written for such block banded systems

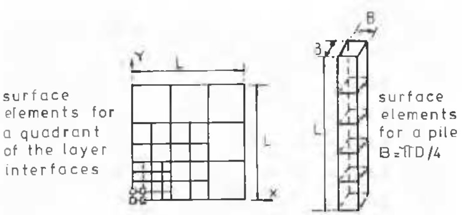

Figure 2 Discretisation of The Layer and The Pile-Soil Interfaces

Having obtained displacements of the pile-soil interface and the layer interface elements from equation (10) the stresses and displacements at any point in the interior of the respective regions can be obtained from equations similar to (6) and (7) for the respective regions (see References 1 and 2).

RESULTS OF THE ANALYSIS

Numerical results have been obtained for $\nu = 0.5$ and $H/L = 2$ throughout. The soil was represented by 10 layer elements. Figure 3(a) shows the plot of the non-dimensional stiffness $P/(mLwD)$ against the Length to diameter ratio (L/D) for various values of the pile compressibility ratios E_p/mL. Figure 3(b) shows the corresponding plots of the percentage of load carried by the base area of the pile. It is interesting to note that the base carries only marginally higher loads than those reported by Poulos (1968) and Butterfield and Banerjee (1971) for the corresponding homogeneous cases.

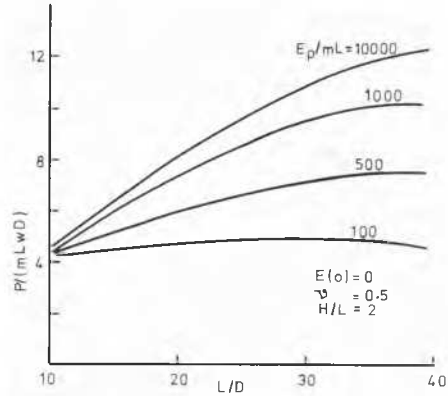

Figure 3a Load-Displacement Behaviour of Single Piles in Gibson Soil

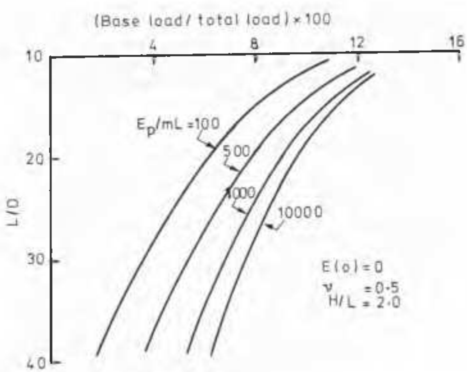

Figure 3b The Percentage of Total Load Carried by The Base

Figure 4 shows the interaction factors for various spacing to diameter ratios and the pile compressibility ratios. It can be seen that:

(i) For pile groups with spacing to diameter ratios of greater than 3, the interaction factors are much smaller than those reported by Poulos (1968) and Butterfield and Banerjee (1971).

(ii) For shorter piles $L/D < 20$ the presence of another pile beyond the spacing to diameter ratios of about 8 to 10 may help to reduce the settlement of the pile under consideration.

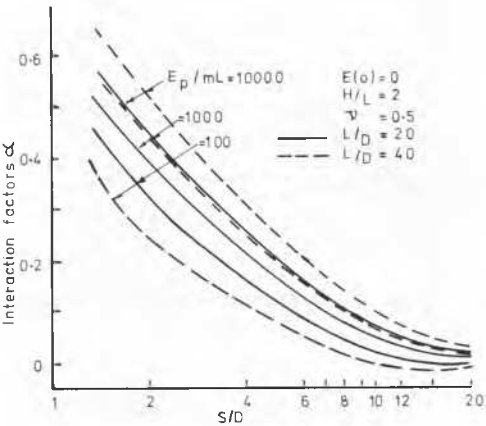

Figure 4 Interaction Factors For Pile Groups in Gibson Soil

It is also possible to calculate the settlement of any axially loaded symmetrical pile group by utilising these interaction factors, e.g. the displacement of the i^{th} pile w_i in a general pile group is given by (Poulos, 1968) as

$$w_i = w_o \sum_{j=1}^{N} \alpha_{ij} P_j \qquad (11)$$

where N = the number of piles in the group

P_j = the load on the j^{th} pile

α_{ij} = 1 for $i = j$

α_{ij} = α from Figure 4 for $i \neq j$

w_o = the settlement of the single pile due to unit load (obtainable from Figure 3a).

The system of equations (11) can then be solved for a fully rigid or fully flexible pile cap in the usual manner.

Figure 5 shows the plots of the settlement ratios (defined as the ratio of the settlement of a group under a load of NP to that of a single pile under a load P) for various pile groups with rigid caps. It can be seen that these settlement ratios are considerably smaller for groups with $S/D > 3$ than those predicted by Poulos (1968) and Butterfield and Banerjee (1971).

Typical ground displacements for a 3x3 pile group is shown in Figure 6. These were obtained by using the complete general method of analysis described earlier. The vertical displacements at a distance of about 3 times the width of the group are negligible. It is also interesting to note that (a) for a surface footing on Gibson soil these displacements become negligible outside the loaded area (Gibson 1974) and (b) for pile groups in homogeneous soils the surface displacements at this distance are about 30% of their maximum values.

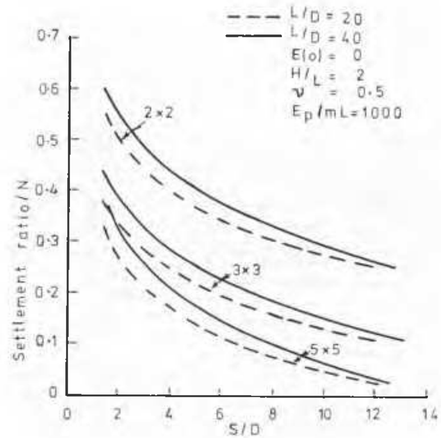

Figure 5 Settlement Ratios For Pile Groups in Gibson Soil

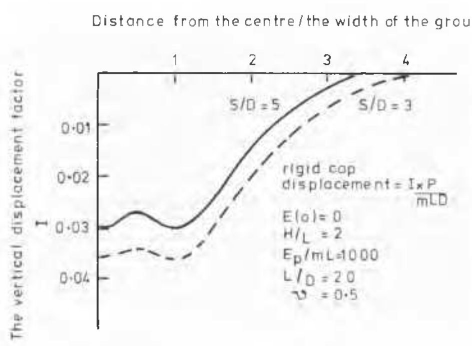

Figure 6 Ground Displacements For 3x3 Groups in Gibson Soil

COMPARISON WITH EXPERIMENTAL RESULTS

In order to investigate the applicability of the method of analysis described above to the problem of load distribution and load displacement behaviour of pile groups in real soil a series of comparisons with reported test results were undertaken. Since the present analysis is intended to describe the undrained working load behaviour, these comparisons were carried out at loads corresponding to half the ultimate load of the group. Figure 7 shows a typical comparison of the settlement ratios in which the experimental results were replotted according to the definition of the settlement ratios described above; Figure 8 shows the distribution of the loads in individual piles for a 3x3 pile group. It can be seen that beyond a spacing to diameter ratio of about 6 to 8, piles virtually act as isolated single piles in contradistinction to the load distribution in a comparable system in a homogeneous material, which at this distance is still nonuniform (see Butterfield and Banerjee 1971). However, it should be pointed out that these small scale experiments were carried out on remoulded clay samples, in which, it would appear (at least superficially) that the deformation modulii would remain constant with the depth, but as a result of the curing operation subsequent to compaction, the values of the Young's modulus at ground level would reduce very considerably due to the stress relief.

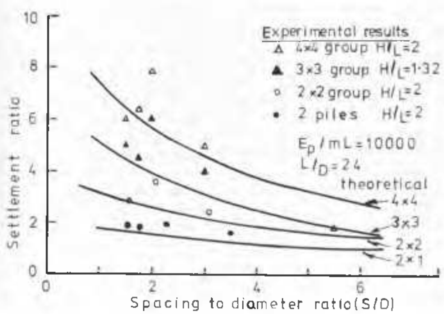

Figure 7 Comparison With the Results of Axially Loaded Pile Groups in Bentonite (Sowers Et Al, 1961)

Cooke (1975) reported the results of full scale tests on axially loaded single piles and a 3x1 pile group embedded in London clay at Hendon. Tubular steel piles 168 mm diameter and 5 metres long equipped with strain gauge load cells were used. The load-distribution in piles as well as the vertical displacements at two different levels below the ground surface were measured. Comparisons between the theoretical displacements and the experimental observations at these two levels for a 3x1 pile group with flexible and rigid pile caps are shown in Figures 9 and 10 respectively. These theoretical results were obtained by using the general method of analysis described earlier with $\nu = 0.5$ and $E(0) = 20$ MN/Metre^2 and $m = 15$ MN/Metre^2/Metre (see Marsland, 1971), $H/L = 2.0$.

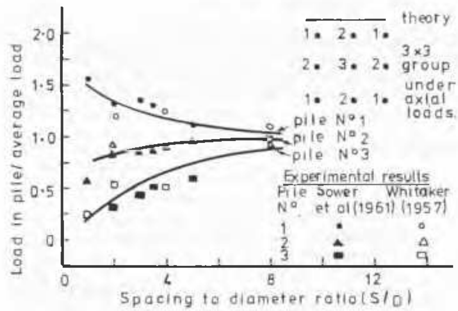

Figure 8 Comparison with the Measured Load Distribution in Piles in 3x3 Groups

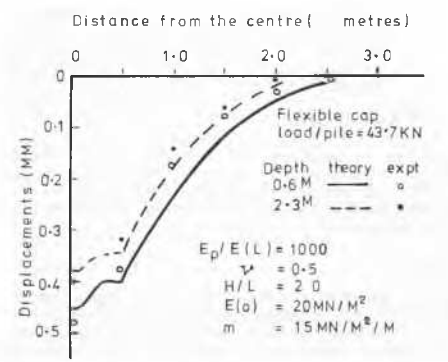

Figure 9 Comparison With Field Test Results of Cooke (1975)

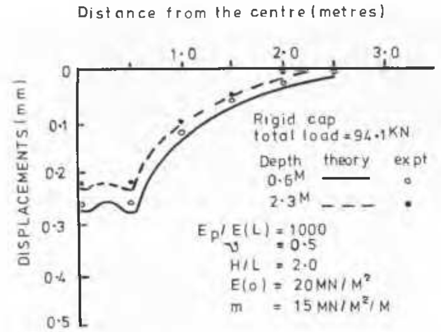

Figure 10 Comparisons With Field Test Results of Cooke (1975)

From the foregoing comparisons it can be concluded that solutions for the Gibson soil give more realistic predictions of pile group behaviour than those for homogeneous soils.

CONCLUSIONS

(i) A general method of analysis for axially loaded symmetrical pile groups of any geometry embedded in Gibson soil is described.

(ii) A comprehensive set of nondimensional plots of the load displacement relations for single piles and interaction factors for the general analysis of pile groups are presented.

(iii) It has been shown that the present analysis gives more realistic predictions of pile group behaviour than those described by Poulos (1968) and Butterfield and Banerjee (1971).

ACKNOWLEDGEMENTS

We wish to express our sincere gratitude to the Department of the Environment for their financial support and for giving their permission to publish the work described in this paper.

REFERENCES

Banerjee, P.K. (1976) "Integral Equation Methods for analysis of piece-wise non-homogeneous three-dimensional elastic solids of arbitrary shape", To be published in the International Journal of Mechanical Sciences. Pergamon.

Banerjee, P.K. and Butterfield, R. (1976) "Boundary Element Method in Geomechanics" Chapter 16 in a book entitled "Numerical Methods in Soil and Rock Mechanics". Ed.G. Gudehus, John Wiley and Sons (In print).

Butterfield, R. and Banerjee, P.K. (1971) "The elastic analysis of compressible piles and pile groups", Geotechnique, Vol. 21, No.1. p.43-60.

Cooke, R.W. (1975) "The settlement of friction pile foundations" Building Research Establishment, Department of the Environment, Report CP/12/75.

Cruse, T.A. and Rizzo, F.J. (1975), Editors of the Proceedings of A.S.M.E. Conf. on Boundary Integral Equation Method". Am.Soc. Mech.E, New York.

Gibson, R.E. (1974) "The analytical method in Soil Mechanics", Geotechnique, Vol. 24, No.2., p.113-140.

Marsland, A. (1971) "Laboratory and in-situ measurements of the deformation moduli of London clay. Proc.Sym.Int. of Structures and Foundations, Birmingham pp.7-17.

Poulos, H.G. (1968) "Analysis of the settlement of pile groups", Geotechnique, Vol.18, No.4, p.449-471.

Sowers, G.F. et al (1961) "The bearing capacity of friction pile groups in homogeneous clay from model studies", Proc. 5th Int. Conf. on Soil Mech. & Found. Engg., Vol.2, p.155-159.

Whittaker, T. (1957) "Experiments with model piles in groups", Geotechnique, Vol. 7, p.147-167.