Methods of Analysis for Structures on Settling Ground

Mèthodes pour la Calculation des Bâtiments sur le Sol

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SYNOPSIS Three different methods of analysis for slab foundations on a settling ground can be distinguished among the well known analytical procedures: the method of subgrade reaction, the method of the modulus of volume change (Steifemodulverfahren) and combinations of the two methods. This paper presents formulas for the direct calculation (without iterations) of the pressure distribution at the base of a foundation slab, bending moments, shear forces and deformations of structure. It is an outstanding method for the treatment of partical analytical problems because of the following reasons:
The entire structural system (foundation slab and structure) can be treated in one calculation. The nonlinear deformational behaviour of the soil is analysed in a computational procedure, clearly separated from the structure. Structures with or without joints deviating from rectangular shape can be analysed.
The methods of analysis are programmed in FORTRAN IV for computer calculations.

INTRODUCTION The problem in the calculation of elastic slab foundations on settling soil is to define the contact pressure in such a manner, that the deformation of the structure at the contact area coincides with the subgrade deformation as closely as possible.

Elastic foundation beams (two dimensional beam problem) can be analysed with three distinct methods. After Winkler the beam as part of the statically indeterminate structure is taken correctly in account. However the ground is simulated by independent elastic springs.

This method of elastic springs does not simulate the behaviour of the soil appropriately. When a load is applied to a part k of the beam, the vicinity i near the beam will also settle.

This observation suggests the use of the theory of elasticity as a model for the soil behaviour. Only for special cases a rigorous solution of the integral equation is known. Ohde (1942) and Kany (1959) and (1974) 2nd edition, also de Beer/Kosmanovic (1951), Gorbunov-Posadov (1954) de Beer/ Lousberry (1966) Sherif (1974) and other authors have developed tables, to present numerical solutions to systems of linear equations which give similar deformations of the structures and of the soil. These methods are called the methods of the modulus of volume change (Steifemodulverfahren). Computer programs have been developed by Kany (1974), Sherif (1974) and others to simplify the use of this method.

Ohde (1942) proposed a solution for foundation slabs (three dimensional problem). Due to the extensive arithmetic effort, it only became possible to use this method for practical purposes on rectangular slabs with the advance of electronic data processing; Denninger (1964), Kany (1974), Hertwig Seiffert (1975). A computer program was developed at the authors institute by Meyer (1975) to calculate rectangular slabs using the method of finite elements. The development was continued by Ellner (1976), to take the stiffening of the superstructure by reinforced concrete cellars in account.

A survey of the recent literature on the subject can be taken from the mentioned literature as well as from the report by Gorbunov-Posadov (1973) submitted at the 8th International Conference at Moscow.

An analysis of measurements taken on structures - Schultze (1961), Kany (1974) Seiffert (1974) Seiffert-Hertwig (1975) - showed, that a reduction of the calculated stress maxima at the edges of heavily loaded small slabs occured due to plastic deformation. Schultze (1970) proposed a combination of the modulus of volume change and the modulus of subgrade reaction for the calculation of foundation beams.

A linear modulus of elasticity,increasing with depth, can also be taken in account. The stiffening of the slab caused by the structures (high buildings) has to be considered too. For the recording of the influence of stiffnes, the total structure does not have to be accounted for, as has been shown in investigations by Heil (1971) Aman-Breth (1972), Kany-Stenzel (1965), Muns-Weiß (1969) and Sommer (1959). Yet, it is enough, to consider only the slab for itself.

GENERALLY ACCEPTED METHODS FOR THE ANALYSIS OF ELASTIC FOUNDATION SYSTEMS.

We consider (figure 1) a statically, not completely rigid structure of arbitrary shape. We suppose, that the structure is errected on completely rigid ground. Only column 5 has no support. The force $P_{5.5}$ causes a displacement of the column 5 of $s_5 = 1$, at the supports 1. to 4 and 6 to 9 reaction forces $P_{i.5}$ appear. These forces can be calculated according to standard methods of structural analysis. The geometry of the structure and the properties of the substructure have to be considered. If the calculation is performed for all 9 supports a matrix $\Delta P_{i.k}$[kN] for the structure of system will be attained. With n supports, the matrix has n^2 elements.

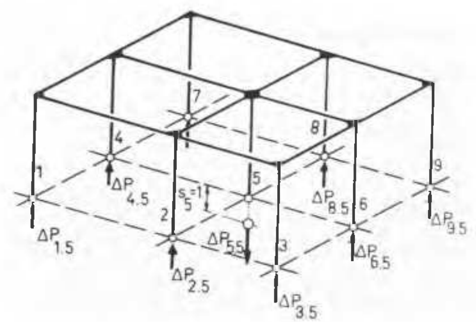

Fig. 1 Structure (example) with settlement $S_5 = 1$, of rank 5 (9 contact points)

If several structures are next to each other (fig. 2), and in addition there are external foundations (46 and 47 in picture 2), we get a biquadratic matrix (figure 3) with some elements that are 0.

$P_{i.o}$[MN] is the vertical load on the foundation i which we get from the static calculation of the structure, arrived at if the external load is applied to the 9 supports of the rigidly supported building. $P_{i,F}$ [KN] is the weight of the foundation.

If the structure is supported by a foundation which is on settling soil the foundation i will settle due to load $P_{i.o}$, the weight of the foundation $P_{i.E}$ and because of the stress distribution in the subsoil caused by the load P_k of the other n - 1 foundations.

In addition, we get the sums $s_{i,A}$ of the settlement of the foundation i, due to the m external foundations.

$$s_{i.A} = \sum_{l=1}^{m} s_{i.A.l} \quad [m]$$

Under consideratation of the constraining forces ΔP_i[kN/m] we get the total vertical loading P_i of the foundation i from equation (1)

$$P_i = P_{i.F} + P_{I.o} + \sum_{\mu=1}^{n} s_\mu \cdot \Delta P_{i.\mu} \quad [kN] \quad \ldots (1)$$

The settlement s of the foundation is given in equation 1 from the sum of n members:

$$s_\mu = \sum_{\nu=1}^{n} s_{\mu.\nu} \quad [m] \quad \ldots (2)$$

BILINEAR CONSTITUTIVE LAW OF THE GROUND

Below the criterion of failure q_G the ground shows a bilinear behaviour (fig. 3): After reaching the load q_v of the overburden at the bottom of the foundation before excavation

$$q_v = \gamma \cdot t_F \quad [kN/m^2] \quad \ldots (3)$$

the fround will settle more under a constant pressure q (modulus of volume change E_s [KN/m²] for initial loading) then before reaching the overburden load q_v (modulus of elasticity W_s). After reaching ultimate loading q_G (load of base failure) the foundation will settle, without further load increase until failure occurs. In the

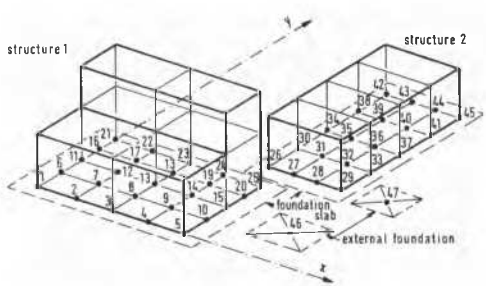

Fig. 2 Building complex with 2 structures (25 + 20 = 45 contact points) and 2 external foundations. (46 + 47)

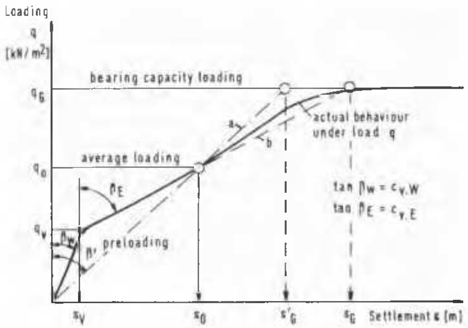

Fig. 3 Load Settlement diagram (bilinear deformation)

following derivation the stress settlement - curves until failure q_G are divided into two regions (fig. 3). This simplification of the settlement behaviour is a considerable improvement compared to the previously used singular linear deformation behaviour.

The settlement $s_{\mu.\nu}$ of the foundation (equation 2) can be derived from two variations of the equation. Equation 2 is therefore extented as follows:

$$s_{\mu} = \sum_{\nu=1}^{n} s_{\mu.\nu.W} + \sum_{\nu=1}^{n} s_{\mu.\nu.E} \quad [m] \quad ...(4)$$

where

$$s_{\mu.\nu.W} = \sum_{\nu=1}^{n} \left(c_{\mu.\nu.W} \cdot \frac{P_{\nu.W}}{A_{\nu} \cdot B_{\nu}} \right) \quad [m] \quad ...(4a)$$

and

$$s_{\mu.\nu.E} = \sum_{\nu=1}^{n} \left(c_{\mu.\nu.E} \cdot \frac{P_{\nu.E}}{A_{\nu} \cdot B_{\nu}} \right) \quad [m] \quad ...(4b)$$

It can further be generally said that the total load on the foundation

$$P_{\nu} = P_{\nu.W} + P_{\nu.E} \quad [kN] \quad ...(4c)$$

In equation 4 a and 4 b the symbols stand for

$c_{\mu.\nu.W}$ = settlement caused by the reloading part[m³/kN]under load q = 1[kN/m²]on the foundation.

$P_{\mu.W}$ = part of reloading[kN]of the foundation

A_{ν} = length [m] of foundation ν in x-direction

B_{ν} = width [m] of foundation ν in y-direction

$c_{\mu.\nu.E}$ = settlement of the foundation caused by primary loading[m³/kN] under load q = 1[kN/m²]on the foundation.

$P_{\nu.E}$ = primary loading part [kN] of the foundation ν.

CONTINUITY EQUATION

To establish continuity between the structure and the ground the bilinear law of deformation (4) , described in section 3, shall be inserted into equation (1). It has to be decided, if the preload $q_{v.i}$ at the point i is exceded or not (equation 3). If the answer is yes, then exit E otherwise exit W of the flow diagram. We get a matrix of n linear equations with the unknowns P_{ν} . In the n^2 elements of the matrix, we distinguish the cases E and W.

Tabel 1(next page)shows equation i of the n equations as an example. Either the equation on the left (case W) or on the right (case E) is valid. Only the first three terms of the equation (on the right side) are similar. In the additional terms we have to decide between E and W.

Using equation 5, we make a preliminary distinction of cases. After the solution of the equations we have to decide if the choice was correct. It may be necessary to improve the results by further iterations, if individual elements deviate from the preliminary distinction of cases.

PROGRAMMING

The calculations have been programmed in FORTRAN IV for a computer solution. In the first segment (see flow chart) the matrix $P_{i.K}$ is defined, in the second segment the vector Pi.o. A modified version of STRESS is used. Figure 4 shows a generalized flow diagram of the computations. The code name of the programm is ELPLA.
Input is made on forms. The slab foundation is divided into rectangles. The subgrade can consist of different layers and have different moduli under each subfoundation. The input includes boring profiles and soil parameters E_s, W_s and γ . It is also possible, to introduce stress-dependent elastic moduli. Bearing pressures and settlements are the results. The bending moments and shear forces of the structure can be determined as well.

A detailed programming and users manual (ELPLA, 1975) is available. Therefor the practical application of the method is greatly facilitated.

Table I (Differences in Equation 5)

case W (preload q_v is not exceeded)

$$\sum_{v=1}^{i-1}\left[\frac{1}{A_v\cdot B_v}\cdot\sum_{i=1}^{n}\left(c_{i.v.E}\cdot\Delta P_{j.i}\right)\right]\cdot P_v +$$

$$+\left[\frac{1}{A_i\cdot B_i}\cdot\sum_{i=1}^{n}\left(c_{i.i.E}\cdot\Delta P_{j.i}\right)-1\right]\cdot P_i +$$

$$+\sum_{v=i+1}^{n}\left[\frac{1}{A_v\cdot B_v}\cdot\sum_{i=1}^{n}\left(c_{i.v.E}\cdot\Delta P_{j.i}\right)\right]\cdot P_v =$$

case E (preload q_v is exceeded)

$$\sum_{v=1}^{i-1}\left[\frac{1}{A_v\cdot B_v}\cdot\sum_{i=1}^{n}\left(c_{i.v.W}\cdot\Delta P_{j.i}\right)\right]\cdot P_v +$$

$$+\left[\frac{1}{A_i\cdot B_i}\cdot\sum_{i=1}^{n}\left(c_{i.i.W}\cdot\Delta P_{j.i}\right)-1\right]\cdot P_i +$$

$$+\sum_{v=i+1}^{n}\left[\frac{1}{A_v\cdot B_v}\cdot\sum_{i=1}^{n}\left(c_{i.v.W}\cdot\Delta P_{j.i}\right)\right]\cdot P_v =$$

$$= - P_{F.j} - P_{o.j} - \sum\left(s_{A.i}\cdot\Delta P_{j.i}\right) -$$

case W:

$$- 0 \qquad [kN] \qquad (5W)$$

case E:

$$- \sum_{v=1}^{n}\left\{\frac{P_{W.v}}{A_v\cdot B_v}\cdot\sum_{i=1}^{n}\left[\left(c_{i.v.W}-c_{i.v.E}\right)\cdot\Delta P_{j.i}\right]\right\} \qquad [kN] \qquad \ldots (5E)$$

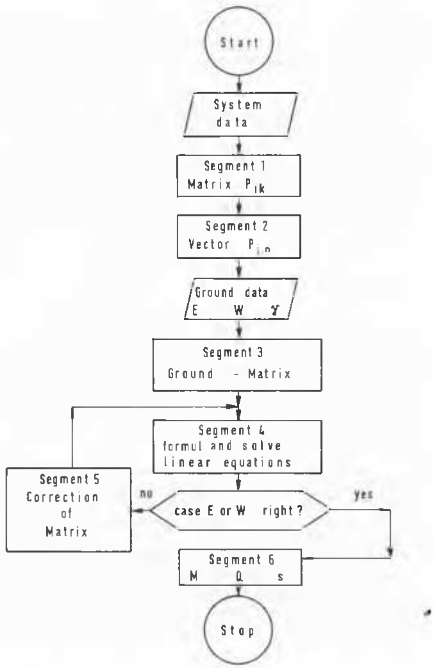

Fig. 4 Flow diagram of the computer program

SYSTEMS WITH RIGID SLABS

The described method may also be used to calculate rigid slab foundations. In this case segment 4 of the flow chart is not used since the deformation conditions can easily be defined with a geometrical method. We refer also to our program documentation STAPLA (1975).

CONCLUSION

This paper presents new formulas for the direct calculation (without iterations) of the pressure distribution at the base of the foundation slab, bending moments, shear forces and deformations. It is an outstanding method for the treatment of practical analytical problems because of following reasons:

1.) The entire structural system (foundation slab and structure) can be treated in one calculation.

2.) The deformational behaviour of the ground is analysed in a computational procedure, clearly seperated from the structure.

3.) Nonlinear deformational behaviour of the ground can be taken into account.

4.) It causes no difficulties to account for important side effects such as local plastic yielding of the soil and balancing of computational negative pressures at the base of the footing.

5.) Structures deviating from rectangular shape can be analysed.

6.) Adjacent buildings, seperated by joints can be studied.

Generally applicable formulas are presented for the calculation of nonlinear deformational behaviour of the ground.
The methods of analysis are programmed in FORTRAN IV for computer calculations. The practical application of the method is greatly facilitated by comprehensive documentation.

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