Analysis of Interference of Strip Footings by FEM

L'Analyse de l'Interférence des Fondations sur Semelle par la Méthode de l'Element Fini

B.S.KHADILKAR Professor,
B.S.VARMA Research Scholar, Dept. of Civil Engineering, Indian Institute of Technology, Bombay, India

SYNOPSIS The problem of interference of two strip footings resting on cohesionless soil is attempted by the finite element method, considering the nonlinear stress dependent and inelastic soil behaviour. The stress deformation study is made first for an isolated footing and subsequently, it is extended to the interfering footings at various spacings for the cases of rigid and flexible foundations. The influence of friction between the rigid footing and the soil is also considered for few cases by incorporating special joint elements in plane strain in this nonlinear analysis. The magnitudes of total settlements and directions of footing tilts at various stages of loading for different spacings of interfering foundations are reported. Analytical data for isolated and pair of strip footings is compared and the results of the investigations are briefly presented and discussed. These are helpful in arriving at certain conclusions and in understanding the behaviour of such footings.

INTRODUCTION

Various available methods in vogue, for the determination of the ultimate bearing capacities of footings are evolved mainly for isolated footings. In practice, the footings are not isolated and their relative positions within a certain distance from each other lead to interference between them.

In this respect, contributions have been made by Stuart and Hanna (1961), Stuart (1962), Mandel (1963) and other research workers. In general, these earlier researchers have not considered the settlement aspects so much but essentially paid attention to the determination of the ultimate bearing capacity of the interfering footing, based on limit equilibrium methods assuming the soil to behave as a rigid plastic material. However, it is found that the soil behaviour over a wide range of stresses is nonlinear, inelastic and dependent on the magnitude of the confining pressure.

In practice, the design of foundations is generally governed by the total and differential settlements undergone by them and thus it is necessary to evaluate the deformations caused due to interference. Skempton and MacDonald (1956) have summarized limiting values of tilts in the footings which cause structural distress and Burland (1974) has reported types of movements and deformations experienced by foundations. Therefore, with the help of a powerful numerical technique (Desai, 1968) - the finite element method, the settlement behaviour of a pair of identical strip footings placed at different spacings is investigated in this work (Varma, 1976).

F.E. DISCRETIZATION

The finite element discretizations are evolved for the cases of rigid and flexible strip footings representing the isolated behaviour and also the interference between footings at various spacings, by specifying appropriate boundary and displacement conditions. A quadrilateral finite element composed of four constant strain triangles has been adopted (4 C.S.T. Quad element). Typical discretization for the problem of interference between two rough rigid footings spaced at 2 B apart is shown below in Fig. 1.

STRESS-STRAIN BEHAVIOUR OF SANDS

The deformation behaviour of sands was approximated by using the hyperbolic simulation as indicated by Duncan and Chang (1970). The

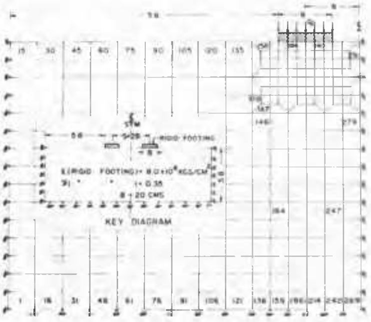

Fig. 1 Finite element discretization for spacing = 2B.

Poisson's ratio was kept at a constant value of 0.35. The stress-strain matrix of this plane strain problem is given by Eq.1, which utilizes the tangent modulus, E_t and Poisson's ratio, ν (using standard notations)

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ T_{xy} \end{Bmatrix} = \begin{bmatrix} M_B + M_D, & M_B - M_D, & 0 \\ & \text{Sym} \quad M_B + M_D, & 0 \\ & & M_D \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xy} \end{Bmatrix} \quad \ldots(1)$$

where M_B and M_D are the bulk and shear moduli of soil element respectively.

HYPERBOLIC PARAMETERS

The initial tangent modulus (zero shear stress) is assumed to vary with the confining pressure (minor principal stress, $_3$) as follows (3)

$$E_i = K.P_a.\left(\frac{\sigma_3}{P_a}\right)^n \quad \ldots(2)$$

P_a = atmospheric pressure measured in the same units as the stress

K and n are empirical curve fitting constants.

The tangent modulus decreases progressively along an assumed hyperbolic stress-strain curve which reaches an upper asymptote limit of stress as defined by soil strength, expressed in terms of parameters, cohesion c, angle of internal friction ϕ and R_f which relates the asymptote of the fitted hyperbola to the maximum measured strength. Thus the tangent modulus (3)

$$E_t = \left[1 - \frac{R_f(1-\sin\phi)\ (\sigma_1 - \sigma_3)}{2C\cos\phi + 2\,\sigma_3 \sin\phi}\right]^2 E_i \quad \ldots(3)$$

For this analysis, the hyperbolic parameters were assumed as those for a dense silica sand reported by Duncan and Chang (1970) and given in Table-I below.

Relative density	ϕ in degrees	R_f	K	K_{ur}	n
100 p.c. (dense)	36.5	0.91	2000	2120	0.54

$C = 0$ for sand, $\gamma = 0.17\times10^{-2}$ kgs/cm^3.

INCREMENTAL LOADING

The initial stresses were first introduced in the soil as per rest conditions. The coefficient of earth pressure at rest is taken as 0.5. The nonlinear, stress dependent and inelastic behaviour of the soil is effected in the analysis by following an incremental procedure wherein the stress components of the elements are accumulated at the end of each load step and from the resulting principal stresses the tangent moduli for the successive load increment are computed after ascertaining the strength criterion based on Mohr-Coulomb failure hypothesis.

In the soil elements where failure sets in, the tangent modulus is kept the same as for the increment prior to failure and the bulk modulus is evaluated from this value, while the shear modulus is assigned a low value, until the element satisfies the strength criterion at a subsequent load increment. If an element develops tensile stresses it was assigned a small value, i.e. 0.1 kg/cm^2 for E_t.

The inelasticity of the soil behaviour is considered in the analysis by adopting the appropriate unload and reload modulus for elements where the major principal stress value, σ_1, decreases for the progressive load increments on the footing. The modulus is calculated from Eq.4 (3) until the element develops a value of σ_1 which exceeds the corresponding value prior to unloading.

$$E_{ur} = K_{ur}\left(\frac{\sigma_3}{P_a}\right)^n \quad \ldots(4)$$

The load intensity on the footing is incremented to 1.4 kg/cm^2 by steps and this value exceeds the ultimate bearing capacity of an isolated footing on this sand as obtained from standard formulas (Terzaghi, 1943).

RIGID FOOTING WITH INTERFACE ELEMENTS

In order to study the influence of friction between a rigid footing and the soil, the analysis is further extended using special joint elements in plane strain (Goodman,et.al 1968) for few selected cases. Appropriate values are used for the parameters k_n and k_s which describe the stiffness matrix of the joint element. The normal and shear stresses at the footing-soil contact interface are computed after each load increment and a friction rule is operated between them by prescribing a coefficient of sliding friction, $U_f = 0.5$. The joint elements wherein the coefficient of friction is exceeded, are identified. The values of k_n and k_s (4) are taken as 10^9 kg/cm^3 for no slip and for join elements where slip occurs the value of k_s is assigned a low value while maintaining the k_n value as per no slip condition.

PRESENTATION OF DATA AND DISCUSSION

The load displacement diagrams obtained from the analysis for the case of an isolated and interfering footings are shown in Figs. 2 and 3 for the cases of rigid and flexible strip footings respectively. It is found that interfering footings in certain cases indicate an increase in bearing capacity governed by the settlement consideration. However, it is observed that at smaller spacings the interference causes greater differential settlements. The settlement pattern of interfering footings is indicated in Fig.4 and their magnitudes are given in Tables II and III.

The greater horizontal stress components in the soil continuum under the closer vicinity of the footing on the interfering side causes larger soil moduli due to confinement. Hence the interfering rigid footings tilt away from each other during the initial stages of loading (α is -ve). As the loading is progressed, these increased soil moduli in this region build up greater vertical stress components which in the case of smaller spacings (2B and below) are large enough to set many soil elements to failure condition, resulting in the titlt of footings towards each other (α is +ve).

At greater spacings, e.g. 3B and above the vertical stress components in the region as mentioned above do not build up to such concentrations to set in failure while the increased horizontal stress component prevails. Hence the footing tilt progressively away from each other (Table II) as the load is incremented (α is -ve).

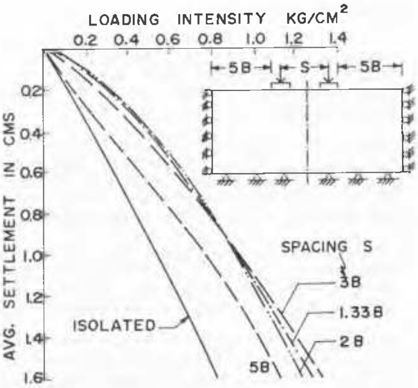

Fig. 2 Load-settlement diagram for interfering rigid footings

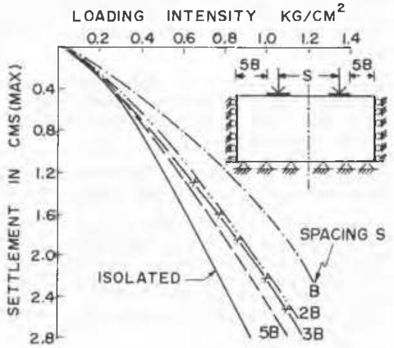

Fig.3 Load-settlement diagram for interfering flexible footings

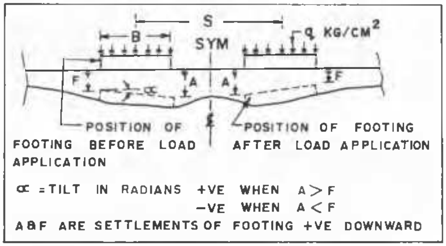

Fig.4 Settlement pattern of interfering footings.

Results of tilts for the interfering footings of the corresponding spacings when the influence of friction at the contact interface is considered are given in Table III. It shows that friction plays an important role in the case of interfering footings. The interfering footings for $U_f = 0.5$ case yield greater total and differential settlements than rough rigid interfering footings.

The development of stress conditions while the loading is incremented on the footing is shown in Table IV and discussed below for only two typical pairs of elements due to limitation of space. The elements in each pair are located symmetrically w.r.t. the centre line of one of the interfering footings at spacing = 1.33 B. The first pair consti-

Table II. Settlement pattern for interfering rigid footings (rough interface)

q	SETTLEMENT (CMS) S = 2B		TILT α	SETTLEMENT (CMS) S=3B		TILT α
	F	A		F	A	
0.4	0.27	0.34	0.0035	0.34	0.33	-0.0005
0.6	0.51	0.49	-0.001	0.65	0.54	-0.005
0.8	0.83	0.80	-0.0015	0.88	0.79	-0.0045
1.0	1.19	1.15	-0.0020	1.18	1.03	-0.0075
1.2	1.36	1.53	0.0085	1.47	1.17	-0.015

Table III. Settlement pattern for interfering footing ($U_f = 0.5$).

q	SETTLEMENT (CMS) S=2B		TILT α	SETTLEMENT (CMS) S=3B		TILT α
	F	A		F	A	
0.4	0.22	0.27	0.0025	0.53	0.50	-0.0015
0.6	0.50	0.63	0.0068	0.65	0.58	-0.0035
0.8	0.56	0.69	0.0065	0.81	0.65	-0.008
1.0	0.65	1.00	0.0175	0.90	-0.11	-0.0505
1.2	0.92	1.32	0.02	1.00	-0.09	-0.0845

Table IV. Net stress components and moduli in soil medium (in kg/cm^2)

q IN KGS/CM^2	ELEMENT 204			ELEMENT 240			ELEMENT 164			ELEMENT 247		
	σ_z	σ_x	E_t	σ_z	σ_x	E_t	σ_z	σ_x	E_t	σ_z	σ_x	E_t
0.2	0.17	0.077	177	0.167	0.067	188	0.067	0.018	151	0.083	0.01	116
0.6	0.39	0.12	10	0.57	0.06	102^F	0.20	0.09	94	0.26	0.05	41
1.0	0.39	0.10	11	0.68	0.16	0.08	0.33	0.17	87	0.45	0.11	38
1.4	0.11	0.57	567^F	0.68	0.16	0.09	0.46	0.25	69	0.68	0.18	46

(F - INDICATES THE FAILURE CONDITION)

tute the elements 204 and 240 (Fig.1) and the next pair consists of elements 164 and 247.

It is noticed that the element 240 has built up higher stresses in the initial loading stages compared to element 204 and this has resulted in failure condition which is associated with a low value of shear modulus. It is interesting to note that this element has attained the stability condition due to stress redistribution in later loading stages. This study of the stress developments is helpful in understanding the failure mechanisms of such interfering footings.

The finite element analysis is further extended to obtain the displacement and stress pattern for few cases of interfering foundations of greater widths and as a result of this analysis, it is noticed that there is a qualitative agreement in the settlement and tilt patterns with those obtained for smaller footing widths. However, for same spacing of interfering footings, the magnitudes of tilts associated with wider footings have been found to be smaller when compared with footings of smaller widths for the applied load intensity having the same proportion to the ultimate bearing capacities for isolated footings corresponding to these respective widths.

CONCLUSIONS

Interfering footings are associated with stress concentrations at smaller spacings and depending on the change in soil moduli the differential settlement and tilt is affected. However, the average settlement of an interfering footing is found to decrease with the decrease in spacing upto 3B and there is not much change when they are brought further close. The friction at the footing-soil interface has a great influence in the determination of tilts and the estimation of heave.

Furthermore, it is interesting to note that the pattern of tilt gets altered depending on the width, the spacing and the intensity of loading on the interfering footings. These analytical investigations bring out clearly the interesting behaviour of two strip footings at various spacings as influenced by the assumed constitutive response of the soil and it is helpful in understanding the problem of interference in a better manner.

ACKNOWLEDGMENTS

The authors wish to thank the Director, Indian Institute of Technology, Bombay for his encouragement. Thanks are also due to the authorities of Tata Institute of Fundamental Research, Bombay for computer facilities.

SELECTED REFERENCES

1. Burland, J.B. (1974). 'Settlement of Buildings and Associated Damage,' Review paper: Session V, Settlement of Structure Pentech Press, London, pp. 611-654.

2. Desai, C.S. (1968). 'Solution of Stress-deformation Problems in Soil and Rock Mechanics using Finite Element Methods,' Ph.D. dissertation presented to the University of Texas at Austin, Texas.

3. Duncan, J.M., and Chang, C.Y. (1970). 'Nonlinear Analysis of Stress and Strain in Soils,' Proc. A.S.C.E., J. Soil Mech. and Found.Div., 96, No. SM5,pp.1629-1653.

4. Goodman, R.E., Taylor, R.L. and Brekke, T.L. (1968). 'A Model for the Mechanics of the Jointed Rock,' J.Soil Mech. and Found. Div., A.S.C.E., Vol.94, No. SM5, pp. 635-659.

5. Skempton, A.W. and MacDonald, D.H. (1956) 'Allowable Settlement of Buildings,' Proc. Instn. Civ. Engrs., Pt.III, Vol.5, pp. 727-768.

6. Stuart, J.G. (1962). 'Interference betwee Foundations with special reference to Surface Footings in Sand,' Geotechnique, 1962, pp. 15-22.

7. Terzaghi, K. (1943). 'Theoretical Soil Mechanics,' J. Wiley and Sons, New York.

8. Varma, B.S. (1976). 'Analysis of the Interference of Strip Footings and the Interaction of Circular Footings with various underlying Soil Media using Finite Element Method,' Ph.D. Thesis, Indian Institute of Technology, Bombay (Unpublished).