Bearing Capacity of Unconsolidated Foundations

Force Portante des Sols de Fondation Non-Consolidés

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SYNOPSIS Under the modern speeds of loading the water saturated clay soils in structure foundations can be characterized by different state of consolidation. For unconsolidated state basic equations of plain problem of plastic non-homogeneity theory are given in this paper. On this basis problems are solved of bearing capacity of unlimited foundation with positive plastic non-homogeneity at vertical and inclined pressing-in of the rigid rough footing. Besides the problem of bearing capacity of the limited by the rigid bedding layer foundation with negative plastic non-homogeneity at vertical pressing-in of the footing was solved, which permitted the analysis of the Transcona elevator collapse. The same physical approach was used in the axial-symmetric problem of vertical pressing-in of a circular rough basement into unlimited foundation as applied to the conditions of the full-scale test. For arbitrary consolidation state the stability condition is given as well as the static equations for plain problem of the consolidating soils limiting equilibrium theory with account of incomplete stress transfer to the pore water at instant additional loading on the soil up to its limit strength. On this physical basis there are given the results of numerical solutions for different time moments of a problem of limit loading on the foundation which according the Terzaghi-Florin theory is consolidating by the uniformly distributed loading applied to its surface.

Under the modern speeds of loading the water saturated clay soils in structure foundations can occur under unconsolidated state which means that the internal friction in them is not realized that permits to consider these soils even at their genetic non-homogeneity to be subject to the condition of plastic non-homogeneity:

$$|\tau_n| = K_0 + ky\,\mathrm{sign}k, \qquad (1)$$

where K_0 and k are defined by the linear levelling of experimental epure τ_n-resistance to the soil shear along the depth of the footing; signk= +1 at positive non-homogeneity and signk= -1 at negative non-homogeneity, y is the depth from the foundation surface.

At first let us consider the plain problem. Stress components satisfying the condition (I) may be expressed as:

$$\left.\begin{matrix}\sigma_x\\ \sigma_y\end{matrix}\right\} = \sigma \pm K_0(1+\varkappa y\,\mathrm{sign}k)\cos 2\varphi, \qquad (2)$$
$$\tau_{xy} = K_0(1+\varkappa y\,\mathrm{sign}k)\sin 2\varphi,$$

where φ -is the angle of inclination to the axis x of maximum main normal stress, $\varkappa = k/K_0$.

Introducing equation (2) in the equilibrium equations of plain problem we obtain the basic system of equations of hyperbolic type:

$$\frac{\partial\sigma}{\partial x} \mp 2K_0(1+\varkappa y\,\mathrm{sign}k)\frac{\partial\varphi}{\partial x} + \left[\frac{\partial\sigma}{\partial y} \mp 2K_0 \times (1+\varkappa y\,\mathrm{sign}k)\frac{\partial\varphi}{\partial y}\right]\tan\left(\varphi \mp \frac{\pi}{4}\right) = \mp K_0\varkappa\,\mathrm{sign}k + \gamma\tan\left(\varphi \mp \frac{\pi}{4}\right), \qquad (3)$$

which provides two systems of field stress characteristics:

$$dy = \tan\left(\varphi \mp \frac{\pi}{4}\right) dx, \qquad (4)$$
$$d\sigma \pm 2K_0(1+\varkappa y\,\mathrm{sign}k)d\varphi = \mp K_0\varkappa\,\mathrm{sign}k\,dx + \gamma dy$$

which were published previously for the case of signk=+I (Stroganov,1970).

Now we consider the problem of bearing capacity of an unlimited foundation with positive plastic non-homogeneity at vertical pressing-in of the rough footing. The results of this problem's solution are given in Figure I, the form of the rigid core being obtained from the symmetry condition ($\varphi = \pi$) in its summit D; the slip sections' lengths OE are defined by the same condition; for these sections ($\varphi = 3/4\pi$) the condition of maximum shear velocity is assumed. For assumed in a numerical example (Fig.1) conditions of plastic non-homogeneity ($\bar{K}_0 = K_0/\gamma b = \bar{k} = 0.342$) dimensionless bearing capacity was obtained to be equal to $\bar{Q} = Q/\gamma b^2 = 5.23$.

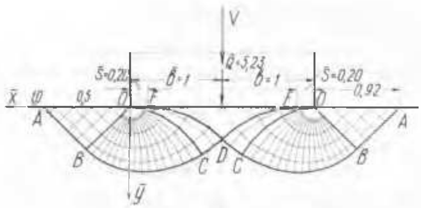

Fig. I

Let us consider a more common case of the same problem at pressing-in the rough footing by an angle to the foundation surface. The results of this problem's solution are given in Figure 2. The presence of a straight

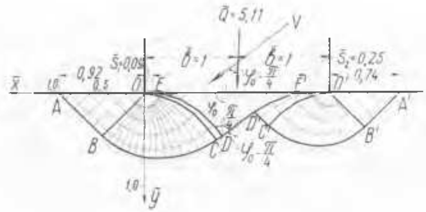

Fig.2.

isolated line of slip DD' serves as the principle basis for such solution because the presence of another form of slip line at the contact of two rigid areas and the velocity vector V of the constant direction is not possible. For the above assumed condition of plastic non-homogeneity dimensionless force of pressing-in $\bar{Q}$=5.11 was obtained, it being stated that insignificant inclination of the pressing-in force (Fig.2) sharply alters the character of the foundation collapse.

Of special interest is the consideration of bearing capacity of a foundation with negative plastic non-homogeneity limited by a rigid bedding layer, as it was in case of Transcona elevator foundation (Pek, Bryant, 1953). Dimensionless resistance to shear is characterized by the following relationship:

$$|\tau_n| / \gamma h = |\bar{\tau}_n| = 0.56(1-0.732y),$$

which was obtained on the basis of the data (Pek, Bryant, 1953) by the least squares method. The solution of this problem given in Fig.3 gave the calculated mean pressure under

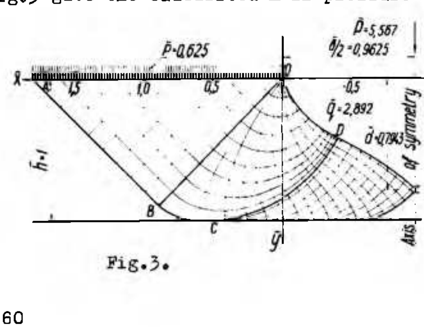

Fig.3.

the foundation toe q_c=3.33 lb/ft^2 insignificantly exceeding q_a=3.06 lb/ft^2 registed during the collapse.

Let us consider axial symmetric problem of the ideal plasticity theory closely bounded with the determination of the bearing capacity of a foundation under unconsolidated state of rough circular footing. In this particular case the condition (1) may be presented by the Saint-Venant condition

$$|\tau_n| = C \qquad (5)$$

where C is the soil cohesion which method of determination has not yet been specified as this problem is not still clear.

For the solution of this problem as being local statically determined we assume the Haar-Karman condition of full plasticity. Then the stress components satisfying condition (5) may be presented as :

$$\begin{aligned} &\left.\begin{matrix}\sigma_r\\ \sigma_z\end{matrix}\right\} = \sigma \pm C\cos 2\varphi, \\ &\tau_{rz} = C\sin 2\varphi, \qquad (6) \\ &\sigma_3, \sigma_1 = \sigma_\theta = \sigma \mp C, \end{aligned}$$

where mean normal stress is expressed by the formula:

$$\sigma = \frac{\sigma_1 + \sigma_3}{2}, \qquad (7)$$

the sign (-) minus corresponds to the flow direction from the axis z and the sign (+) plus corresponds to the flow direction to the axis z, φ is the angle between r and the direction of σ_1.

Introducing equation (6) into equilibrium equations known we obtain the basic system of equation of hyperbolic type:

$$\begin{aligned} &\frac{\partial\sigma}{\partial r} - 2C\left(\sin 2\varphi\frac{\partial\varphi}{\partial r} - \cos 2\varphi\frac{\partial\varphi}{\partial z}\right) + \frac{C}{r}(\cos 2\varphi \pm 1) = 0, \\ &\frac{\partial\sigma}{\partial z} + 2C\left(\cos 2\varphi\frac{\partial\varphi}{\partial r} + \sin 2\varphi\frac{\partial\varphi}{\partial z}\right) + \frac{C}{r}\sin 2\varphi = \gamma, \end{aligned} \qquad (8)$$

which provides two systems of field stress characteristics without auxiliary variables:

$$\begin{aligned} &\frac{dz}{dr} = -\frac{\sin 2\varphi \mp 1}{\cos 2\varphi} = \tan\left(\varphi \mp \frac{\pi}{4}\right), \\ &d\sigma \mp 2C d\varphi + \left\{C\left[\pm 1 + \text{sign}V_r\frac{dr}{dz}\right] - \gamma r\right\}\frac{dz}{r} = 0, \end{aligned} \qquad (9)$$

where the upper signs refer to the first system of characteristics and the lower signs refer to the second system; signV_r=+1 at radial plastic flow velocity from the axis z and signV_r=-1 at radial plastic flow velocity to the axis z.

The results of the problem under consideration' solution are given in Figure 4, the following simple formula being obtained:

$$q = p + 6.025C \qquad (10)$$

where q is the mean limit pressure under the toe of the foundation, p- is the additional loading on the foundation surface or equivalent to this loading deepening of the foundation.

Comparison of this result with the result of the large-scale experiment carried out by J.K.Nixon in Shell Haven (Nixon, 1949) on the basis of averaged data of unconfined test and vane test ($C=320\ lb/ft^2=0.157\ daN/cm^2$) has shown that formula (10) gives $q_c=1928\ lb/ft^2=0.94\ daN/cm^2$ while the pressure under

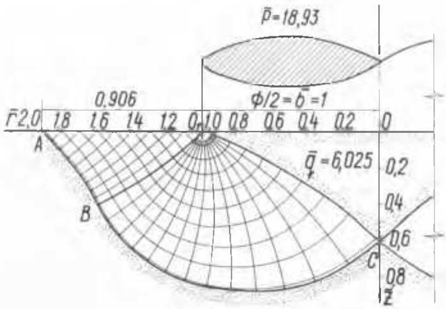

Fig.4

the tank toe at a moment of overturn has appeared to be equal to $q_a=2230\ lb/ft^2=1.09\ daN/cm^2$ that is the actual breaking loading somewhat exceeds the calculated one.

Arbitrary state of consolidation can be described by the generalized equations of the theory of consolidating soils limit equilibrium. With this end in view let us extract an infinitesimal element from the massif of such soil. At an arbitrary time moment a certain stress tensor is acting upon it, the spherical part of the tensor is the effective stress σ' and excess pore water pressure U. At the moment of time under consideration let us apply instantly a surplus spherical tensor of stresses $\Delta\sigma$ to the extracted soil element. If we designate press coefficient in pore water and β then a part of the surplus stress tensor $\beta\Delta\sigma$ is transferred to the pore water and the other part $(1-\beta)\Delta\sigma$ is transferred to the soil skeleton. Immediately after the application of the surplus stress tensor the summary pressure in pore water and the summary effective mean stress will be respectivel $U+\beta\Delta\sigma$ and $\sigma'+(1+\beta)\Delta\sigma$. Let us choose the value of $\Delta\sigma$ in such a way that summary stresses in the soil skeleton become limiting.

Then total summary stresses may be expressed by the following formulae (Solovjov, Smolin, 1976):

$$\left.\begin{matrix}\sigma_x\\ \sigma_y\end{matrix}\right\}=\sigma(1\pm\sin\alpha\cos2\varphi)-H,\quad \tau_{xy}=\sigma\sin\alpha\sin2\varphi \tag{11}$$

where the instant ultimate bearing resistance to the hydrostatic tension H, instant angle of internal friction of consolidating soil α, and the total summary averaged stress σ are defined by the following expressions:

$$H=\frac{C\,ctg\,\rho+\beta\sigma'}{1-\beta}-U,\quad \sin\alpha=(1-\beta)\sin\rho,\quad \sigma=\sigma'+U+\Delta\sigma+H \tag{12}$$

Here ρ, C-are the angle of internal friction and specific cohesion of the soil in consolidating-draining tests respectively, and the angle φ has the same sense as previously.

Special studies on determination of limit instant resistance to shear of water saturated clay soils including the measurement of the pore water pressure in the shear area (Solovjov, Smolin, 1976) were carried out to check up the obtained conditions of instant strength of consolidating soils. Comparative experimental data have shown satisfactory agreement with the theory.

Introducing relationship (II) in the differential equilibrium equation one can obtain the basic equation system of plain problem of consolidating soils limit equilibrium theory. The equations of its characteristics have the following form:

$$dy=dx\tan(\varphi\mp\mu),\quad \mu=\frac{\pi}{4}-\frac{\alpha}{2}$$

$$d\sigma\mp2\sigma\tan\alpha\,d\varphi=\left(\gamma'+\frac{\partial H}{\partial x}\right)(dx\pm dy\tan\alpha)\pm\frac{1}{\cos\alpha}(dx\cos2\varphi-dy\sin2\varphi)\frac{\partial H}{\partial y}, \tag{13}$$

where γ' is the volume weight of the soil skeleton with account of buoyancy action of the Archimedes force. Here the upper and the lower signs correspond to the characteristics of different systems. With the help of these equations as well as the numerical method of their integrating one can solve various problems of the theory of consolidating soils limit equilibrium.

In order to illustrate the above given theory let us consider the problem of consolidating foundation stability. Special static and kinematic determinations of limit loading on the foundation were carried out according to Prandtl's scheme assuming that the foundation is being compressed under the action of the uniformly distributed boundary loading q fully applied to its surface in a certain time moment t=0. The calculation was carried out in relative variables: semiwidth of the loading boundary b was taken for the unit of length measure; the value of $\gamma_w b$ (γ_w is the volume weight of water) was taken for the stress unit and value $b^2/4C_w$ (C_w is the consolidation coefficient) was taken for the time unit. Consolidating part of the problem was solved in accordance with the Terzaghi-Florin theory with the definition of pressure coefficient in pore water, which differes from unity.

The initial data for the calculation in relative values were assumed as follows:

$\rho=20^\circ$, $C\,ctg\,\rho=0.2$, $\beta=0.6$, q=1.

The relative values of limit summary loading were obtained as a result of calculations for relative time moments t=0.05; 0.1; 0.25; 0.50; 2.0; 5.0. According to static solution they are respectively 3.87; 3.92; 4.02; 4.23; 4.33. According to kinematic solution they are respectively: 6.61; 6.89; 7.02; 7.28; 7.40.

Areas of plastic deformations at time moment of 0.25 are shown in Figure 5. On the left side of the sheme these areas are given according to kinematic solution, on the right

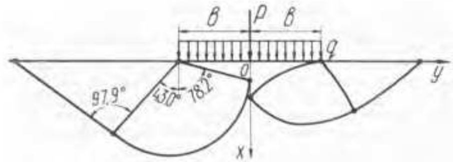

Fig.5

side - according to static solution.

In conclusion we should note that equations obtained here (11-13) summarize the equations published previously (Solovjov, Kim, 1974) for the case $\beta < 1$ that is for incomplete perception of pore water pressure.

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