Stressed-Strained State of Soil Masses

L'Etat Tente et Déforme des Massifs des Sols

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SYNOPSIS The formulation and solution are given for problems, within the scope of the plane problem of elasticity, for evaluating the stressed-strained state of soil masses with a curvilinear boundary subject to the action of surface and body (gravitational and inertia) forces. Laws of stress distribution, presented in the form of stress isolines, are analysed. The applicability of the obtained solutions is demonstrated by specific examples.

The most important problems of applied geomechanics concern the evaluation of the natural stressed state of soil masses and predictions of its redistribution due to the action of various natural factors and those associated with man's activities. The first of these is the more complex and problematic. Its solution requires a knowledge of the history of the formation of the soil massif and the factors responsible for the stressed state. Natural stresses are developed as the result of the action of gravitational, seismic and other forces. In the present stage of development of geomechanics, it is impossible to take all these factors into account. Therefore, it becomes necessary to resort to some hypothesis that can be tested by field investigations. The second task is less problematic and can be successfully realized on the basis of the mechanics of continuous media under the condition that the history of the formation of the massif is given.

Actually, an assessment of the influence of such factors as the construction of large hydraulic, diversion and other structures, open-cut mining of minerals, the cutting of rivers into the bottoms of valleys, and many others, can be reduced to the evaluation of the redistribution of the stressed strained state of soil masses under the effect of external loads or as a result of a change in the surface relief. It can be shown that in many cases the assessment of the action of a seismic force can be reduced to the statical problem of redistribution of soil masses due to the action of body forces applied to each point of the massif. This holds for the case when the length of the seismic wave is commensurate with the linear dimensions of the curvilinear portion of the massif being investigated.

The possibility of taking into account all of the foregoing factors on the basis of the mechanics of linearly deformable media is shown below. The restriction to linear deformability is not essential if the additional stresses due to the redistribution of the stressed state are not very large.

A soil massif is dealt with as a continuous, linearly deformable homogeneous, isotropic semi-infinite region S which approaches a half-plane at infinity. The boundary $\mathcal{L}$ of the region describes either the existing profile of the massif or that anticipated after the action of the relief-forming factor. Acting at each point in the region is a certain resultant body force γ_c, directed at an arbitrary angle to the vertical (Fig.).

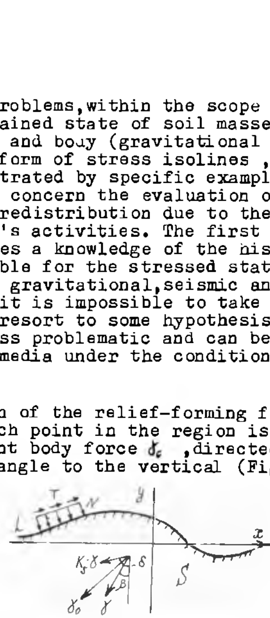

Fig.1. Schematic diagram of the problem

It is also assumed that uniform loads N (Normal to $\mathcal{L}$) and T (tangent to $\mathcal{L}$) act at the boundary. The problem is dealt with on the basis of plane strain. The stress components $\sigma_x(x,y)$, $\sigma_y(x,y)$ and $\tau_{xy}(x,y)$ are to be determined. In the past, the authors have been able to solve a number of special problems stated in this way (Ter-Martirosyan and Akhpatelov, 1971, 1972 and 1975). Consequently, we shall not dwell here on the solution of the problem, but only note the following. The general solution is sought for in the form of the sum of certain specific and supplementary solutions. The specific solutions are selected so that, in the first place, they satisfy the nonhomogeneous equation of equilibrium and, in the second place, approach infinity, i.e. they describe the natural stress field in receding from the boundary (according to our chosen hypothesis). Evidently, a specific solution is precisely the one which carries in itself information on the history of the formation of natural stresses. In this way we dealt with the body forces of gravitation γ and inertia $K_s\gamma$ due to seismic action, where $K_s=a_s/\gamma$ is the seismic factor, a_s is the seismic acceleration and γ is the unit mass of the rock of which the region is composed. Assuming the direction of the body forces to be according to Fig.1, the specific solutions are taken in the following form:

$$\sigma_x^{sp} = K\sigma_y^{sp}$$
$$\sigma_y^{sp} = \gamma(\cos\beta + K_s\cos\delta)y \qquad (1)$$
$$\tau_{xy}^{sp} = \gamma(\sin\beta + K_s\sin\delta)y$$

At $K_s=0$ and $\beta=0$, we obtain $\sigma_x=K\sigma_y$, $\sigma_y=\gamma y$ and $\tau_{xy}=0$. It follows that a ponderous half-plane has been taken as the natural (initial) stress field. In equations (1), $K = \mu/(1-\mu)$ is the coefficient of lateral pressure, and μ is Poisson's ratio. The acceptance of the conditions of equations (1) leads to a new problem with new boundary conditions for determining additional stresses, but without the body forces. This problem was solved by the Kolosov-Muskhelishvili method of complex potentials, making use of conformal mapping and the properties of Cauchy-type integrals.

The function which conformally maps the lower half-plane on the survilinear semi-infinite region being investigated has the form

$$z=\omega(\zeta)=h\left[\zeta+\frac{A-B\zeta}{\zeta+a\cdot i}-\frac{Ci}{(\zeta-i)^2}+\frac{K}{(\zeta-i)^3}\right] \qquad (2)$$

where $z=x+iy$, $\zeta=\xi+i\eta$, $\eta\le 0$

A, B, C, K and a are constants: and h is the proportionality factor.

Separating the real and imaginary parts of equation (2), we obtain for $\xi=t$ and $\eta=0$, the parametric equations

$$x=t+\frac{(A-Bt)(t+a)}{(t+a)^2+1}+\frac{2Ct}{(t^2+1)^2}+\frac{K(t^2-3)t}{(t^2+1)^3}$$
$$y=-\frac{A-Bt}{(t+a)^2+1}-\frac{C(t^2-1)}{(t^2+1)^2}+\frac{K(t^2-1)}{(t^2+1)^3} \qquad (3)$$

which describe the boundary curves of the region being investigated.

Figure 2 shows the external contours of the semi-infinite regions for various values of the parameters A, B, C, K and a. As is evident from Fig.2, the conformal mappinf function (2) enables the stressed state to be investigated for a very extensive class of curvilinear semi-infinite regions. The solution is obtained in the closed form. It is, however, so unwieldy that it proves expedient to calculate the stress components at the points of interest in the region by means of an electronic computer. It should be underlined that the computer does not solve the problem, but

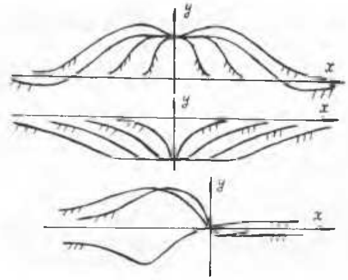

Fig.2. Typical boundaries of semi-infinite regions, described by parametric equations (3).

only calculates the stress values to programmed formulas. Hence, a unified program has been composed for all regions whose boundaries are described by equations (3).

The isolines of the stress components were plotted from the values calculated by the computer. In addition, isolines of strength factors η_s were plotted after being calculated by the formula

$$\eta_s=\frac{(\sigma_1+\sigma_2)\tan\varphi+2c}{2\tau_{max}\cos\varphi}-\tan^2\varphi \qquad (4)$$

where σ_1, σ_2 and τ_{max} are the principal stresses, φ is the angle of internal friction and c is the specific cohesion.

Equation (4) expresses the ratio of the tangential stress, acting at a specific point on the most dangerous element of area, to the limiting value (according to the Coulomb-Mohr criterion). Evidently, at $\eta_s<1$, the point being considered is in the sub-limiting state, at $\eta_s>1$, it is in the super-limiting state, while $\eta_s=1$ corresponds to the limiting state. Certain results obtained in plotting the isolines are illustrated in Figs.3 and 4.

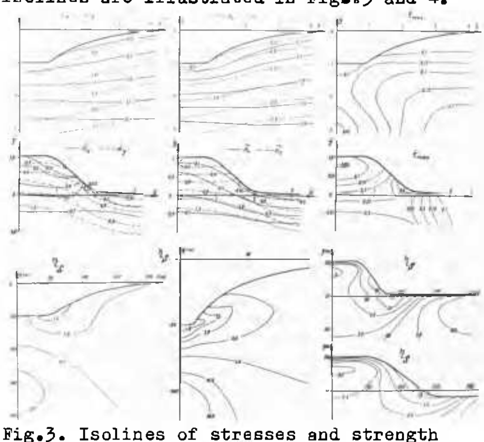

Fig.3. Isolines of stresses and strength factors: $K=0.8$, $\varphi=30°$, $\gamma=19$ N/m^3, $C=8$N/m^2

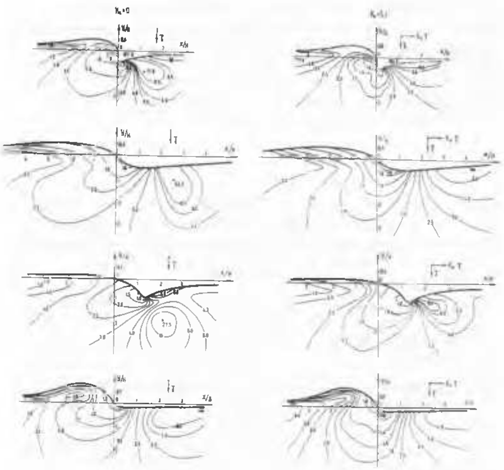

Fig.4. Isolines of strength factors: $K=0.8$, $\varphi=20^{\circ}$, $C=20N/m^2$, $\gamma=20N/m^3$, $H=100m$

On the basis of the obtained solution, computer calculations and the plotted stress and strength factor isolines, the stressed state was investigated for thirteen types of curvilinear semi-infinite regions subject to the action of gravitational, seismic and surface forces. An analysis of these investigations enabled certain general rules to be derived for the distribution of stresses in curvilinear semi-infinite regions. Under the action of body forces, the curvilinearity of the boundaries of the semi-infinite region does not essentially influence the vertical normal stress, calculated from the weight γy of the overlying stratum. The horizontal normal stresses are subject to substantial deviation from the value $K\gamma y$. Tangential stresses are developed on the vertical and horizontal elements of area. The values of these stresses tend to zero in receding from the curvilinear portions of the region, approaching zero much faster in the horizontal direction that in the vertical one. Considerable concentrations of the maximum tangential stresses are observed on the boundaries around points characterizing inflections and maximum curvature of the boundaries. The most dangerous zones with respect to strength are also located here (see Figs.3 and 4). In semi-infinite regions characterized by sharply defined projections and cuts there are compacted cores which are in a firm state and are located either under the ridge of the projection or under the base of the cut. The most dangerous zones, whose development is initiated at the bottom of the massif, extend into those of its regions that are characterized by a large projection. The dimensions of the dangerous zones depend essentially on whether the curvilinearity is due to a projection above the half-plane, the deeper the extent of the dangerous zones (Fig.3). The horizontal seismic force which, as a whole, impairs the stressed state of semi-infinite regions, differently affects regions divided by a depression and those divided by aprojection (Fig.4). When there is a depression in a region which is closer along the direction of the seismic forces, the stressed state is impaired; if it is in one which is farther away along the direction of these forces, the stressed state is improved. On the contrary, when there is a projection in a region which is closer along the direction of the seismic forces, the stressed state is improved; if it is in one which is farther away along the direction of these forces, the stressed state is impaired.

The coefficient of lateral pressure has a substantial effect, both on the type of stress distribution and the magnitude of the stresses. The higher this coefficient, the greater the region influenced by the curvilinearity of the boundaries. On the other hand, the higher the coefficient, the smaller the gradients of tangential stresses.

A comparison of the patterns of isolines of stresses and strength factors in semi-infinite regions, having boundaries which concide at infinity and differ in near-slope regions, enables the redistribution of the stressed state in soil masses to be studied. Fig.5 shows the isolines of the maximum tan-

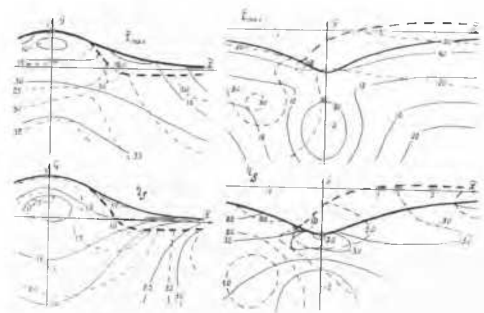

Fig.5. Redistribution of the maximum tangental stresses and strength factors when the relief of the soil massif is changed.

gential stresses and strength factors in soil masses having boundaries of curvilinear configuration (continuous lines) and the redistribution of these isolines when the relie f of the massif is changed (dash lines). Without dwelling in detail on the interrelation of the changes in relief and various factors of relief formation, we point out only that the change in relief essentially affects the stressed state of the massif. Thus, the industrial development of the foot of a mountain massif considerably impairs the stressed state of the massif and extends the zones which are in the limiting state. The stressed state can also be impaired by changes in the surface relief caused by the deposits of soil on the surface.

The solution obtained for the problem on the stressed state of curvilinear semi-infinite regions subject to body and surface forces enables a more accurate value to be found for the first critical load on foundation bases, calculated at present by the Puzyrovsky-Gersevanov formula. It is known (Tsytovich, 1963), firstly, that the coefficient of lateral pressure is taken as unity in this formula, which is not true, and secondly, that the above-foundation part of the massif is replaced by a uniform load of the intensity γh, which distorts the real model of the foundation base. A comparison is shown schematically in Fig.6 of the approach of the

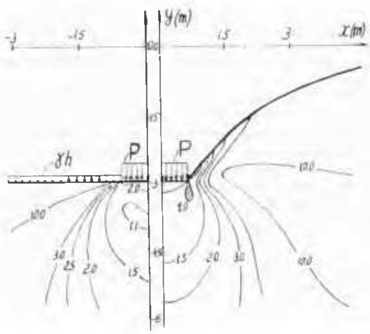

Fig.6. Isolines of strength factors: $K=0.7, C=8\ N/m^2, \varphi=20^\circ, \gamma=19\ N/m^3$

authors of the present paper and of that of the Puzyrovsky-Gersevanov formula. The coefficient of lateral pressure is taken equal to 0.7. As is evident from the figure, in taking the pit into account, the zones of limiting equilibrium are increased, consequently reducing the value of the critical load. It should be pointed out that a reduction in the coefficient of lateral pressure also leads to a reduction in the critical load. Hence, the application of the Puzyrovsky-Gersevanov formula leads to the assumption of an excessive value for the critical load on foundation bases.

Mention may be made of still another application of the solution obtained for the problem being investigated. A knowledge of the stress distribution in soil masses enables the stability of a soil massif as a whole to be calculated by applying existing procedures for evaluating the stability of slopes, but with more refined values for the acting forces on the surfaces of displacement.

The problems in applied geomechanics discussed here by no means exhaust all the possible cases in practice in which it proves expedient to make use of the obtained solution of the problem on the stressed state of curvilinear smi-infinite regions subject to the action of body and surface loads.

REFERENCES

Akhpatelov, D.M. and Z.G.Ter-Martirosyan (1971) "On the Stressed State of Ponderable Semi-Infinite Regions", Publishing House of the Armenian Academy of Sciences, Mechanics, Fol.XXIV, No.3, (in Russian).

Muskhelishvili, N.I. (1966), "Certain Principal Problems of the Mathematical Theory of Elasticity", Nauka Publishers, Moscow (in Russian).

Ter-Martirosyan, Z.G. and D.M.Akhpatelov (1972) "On the Stressed State of an Infinite Slope With a Curvilinear Boundary Within a Field of Gravitation and Percolation", Jour.Problems of Geomechanics, No.5, Erevan (in English).

Ter-Martirosyan, Z.G. and D.M.Akhpatelov (1975) "The Stressed State of Mountain Masses in a Gravitational Field", Doklady USSR Academy of Sciences, No.2 (in Russian).

Tsytovich, N.A. (1963). "Soil Mechanics", Gosstroiizdat Publishers, Moscow (in Russian).

Tsytovich, N.A. (1973), "Problems of Soil and Rock Mechanics in Geomechanics", Special Lecture at the VIII International Conference on Soil Mechanics and Foundation Engineering, Moscow (in English).