On the Scale Effect of Footings in Dense Sand

L'Effet d'Echelle des Semelles dans Sable Ferme

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SYNOPSIS A series of loading tests using Toyoura sand is carried out in a centrifuge. Deformations of the sand are measured by a set of electronic devices and slip lines are observed by an X-ray apparatus. It is found that the shearing strains in the sand differ very mcuh from place to place and that the progressive failure does exist in dense sand, which leads to a conclusion that the assumption of constant shearing strain adopted in existing bearing capacity theories cannot be accepted. Kötter's equations are solved by combining a unique relationship between the angle of shearing resistance and shearing strains with observed shearing strains in models. Computed bearing capacities compare well with measured values for the dense sand in which shearing strains differ from place to place along the slip line.

INTRODUCTION

Many theoretical procedures have been proposed for evaluating the bearing capacity of soils by assuming the soil to be a rigid plastic material. It has been pointed out by many investigators that the theories based on the assumption of rigid plasticity generally overestimate the actual bearing capacities of soils. This discrepancy is considered to owe to the fact that the behavior of soils is more suitably described by elasto-plasticity with hardening than by rigid plasticity. For this type of material, the failure takes place inevitably in a progressive way.

As far as the authors are aware, Muhs (1963) is the first to have realized the importance of the progressive failure in connection with the failure of sand. From the results obtained with a series of large scale experiments on shallow foundations in dense sand, he drew a conclusion that the maximum shear strength could not be mobilized at the same time everywhere along a final slip line. De Beer (1965 B) discussed the effect of the scale of footings on the bearing capacity of dense sand in the light of progressive failure. Although the discussion by de Beer is considered to be very important, the full experimental evidence has not yet been given to it.

In this paper the authors attempt to explain the scale effect of footings on the bearing capacity of dense sand based on the differences in shearing strains observed for "narrow" and "wide" footings in the centrifuge experiments. And it is shown that the bearing capacities calculated by taking into account the variation of the angle of shearing resistance along the final slip line are good approximations of those of actual foundations.

EXPERIMENTS

(A) Problems concerning a centrifuge It has been known that the stress similarity can be established between a model with a geometrically reduced scale of 1/N and a prototype, by giving the acceleration of N times the gravitational acceleration (g) to the model. But many problems arise when the centrifuge is used in model experiments on the bearing capacity of soils. Among them three are considered to be most important.

The first problem comes from the structure of the centrifuge. In the centrifuge, the acceleration acts in the radial direction and its magnitude increases in proportion to the distance from center of rotation. In the case of prototype, the uniform acceleration acts perpendicularly to the soil surface. Examination by the use of Kötter's equation showed that difference between bearing capacity of the centrifuge model and that of the prototype was less than 10% (Yamaguchi et al. 1975). The second problem is the influence of the particle size of soil on the bearing capacity. When a model is subjected to the acceleration of N times the gravitational acceleration, the size of each particle in the model becomes in effect N times as large as the original size. To study the influence of the particle size, a series of bearing capacity tests was conducted for glass ballotini of six different gradings (Fig.1). Load intensity-settlement curves for glass ballotini of B_1 and B_2 are shown in Fig.2. It may be found that bearing capacities show practically no difference, although average particle sizes are very much different from each other. In addition, the difference between bearing capacities obtained for glass ballotini of A_2 and C_2 was only 15%, although the uniformity coefficient of C_2 was approximately three times as large as that of A_2. From these

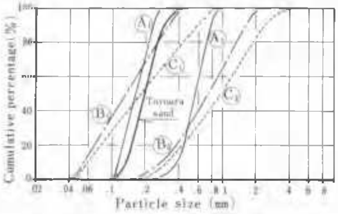


Fig.1 Grading curves of glass ballotini and Toyoura sand

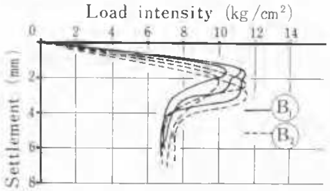


Fig.2 Load intensity-settlement curves for glass ballotini

results the authors conclude that there exist no significant influence of the centrifuge on the bearing capacity in terms of the particle size.

Although no satisfactory solutions have yet been derived for the last problem about the deformation similarity, the authors consider that the bearing capacity obtained for a model footing with the breadth of B under a centrifugal acceleration of Ng can represent the bearing capacity for a footing with the breadth of N·B which is a size for practical use. In this paper, B_N denotes the breadth of the model footing when it is subjected to the acceleration of Ng, and the model footing with B_N of larger than 90 cm shall be specified as a "wide" footing.

(B) Test procedure A series of plane strain shear tests was performed on Toyoura sand for obtaining the stress-strain relationship for various values of initial void ratio. The shear tests were conducted under ambient pressure ranging between 0.5 and 5.0 kg/cm^2.

A model ground consisted of compacted layers of Toyoura sand. The sand was compacted by a vibrator for 1 minute per each layer of 1 cm. The average properties of the model ground are given in Table I. Bearing capacity tests were performed for model footings with three different breadths (B) of 2, 3 and 4 cm by taking the depths of embedment (D_f) as 0, 0.5B and 1.0B. The experiments were conducted both in the centrifuge field with acceleration of 10,

Table I. Average properties of model ground

Unight weight,	γ_t (g/cm^3)	1.62
Water content,	w (%)	0.12
Porosity,	n (%)	40
Relative density,	D_r (%)	87

Table II. Specification on the centrifuge

Radius of rotation	118.0 cm
Range of rotation	100 ∿ 600 r.p.m.
Centrifugal acceleration	10 ∿ 300 g

20 and 40g and in the gravitational field with acceleration of 1g. General information on the centrifuge are summarized in Table II. A stronger emphasis was placed on the experiments for the rough base condition in which sand particles were glued to the footing base. But some tests were also conducted for the smooth base condition in which the footing base was lubricated with rubber membrane and silicone grease.

DISCUSSIONS

In the plane strain shear tests under ambient pressures (σ_3) of less than 4 kg/cm^2, a unique relationship between the angle of shearing resistance (ϕ') and the shearing strain ($\bar{\gamma}$) was obtained as shown in Fig. 3.

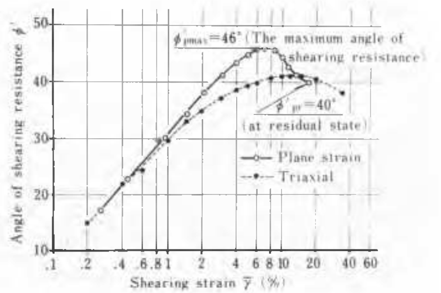


Fig.3 Unique relationship between ϕ' and $\bar{\gamma}$ for Toyoura sand

The maximum mean normal stress in the specimen under σ_3 of 4 kg/cm^2 is considered to be about 12 kg/cm^2. According to the result of preliminary computation by Yamaguchi et al. (1975), the mean normal stress exceeding this magnitude was to appear only in the vicinity of a footing in the present experiments. The use of the above mentioned unique relationship for bearing capacity computation may be therefore justified. For the case of B_N=60 cm, the load intensity-settlement relationships for the rough base footing and for the smooth base footing are compared in Fig. 4, and in Fig.5 slip lines observed by an X-ray apparatus are shown for the two cases.

It is confirmed based on these results that the smooth footing certainly gives symmetrical wedges beneath it and yields smaller bearing capacity than the rough footing as theories predict. In Fig.4, the measured bearing capacities scatter very little for the smooth footings, giving the value about 65% of the bearing capacity for the rough footing on an average. For the case of B_N=120 cm, however, the bearing capacity of smooth footings was about 80% of that for the rough footings.

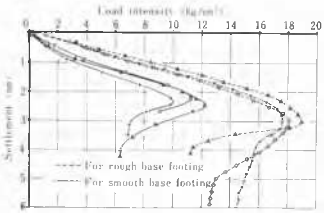
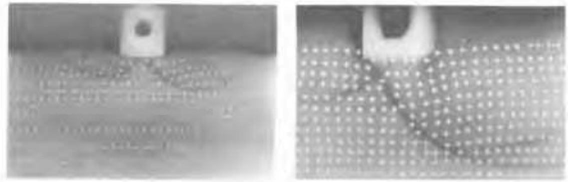


Fig.4 Load intensity and settlement (B_N=60 cm, D_f/B=0)



(a) smooth footing (b) rough footing

Fig.5 Slip lines observed by an X-ray apparatus

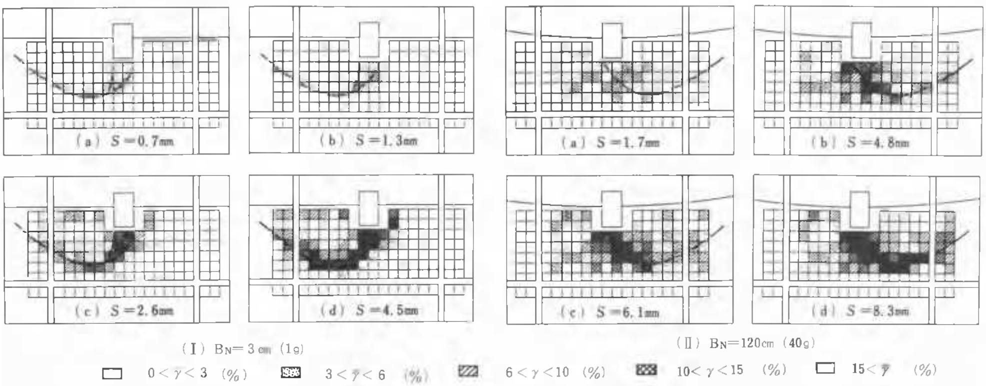


Fig.6 Distribution of measured shearing strains in models and location of the final slip line (D_f/B=1)

Observed shear strain distribution at four stages of the footing settlement are shown in Fig.6 for the footings of B_N=3 cm and 120 cm with D_f/B=1. The location of the observed final slip line is also given in the figure.

It is clearly demonstrated that shear strains develop along the slip line and that at the peak load(S=1.3 mm for B_N=3 cm, S=4.8 mm for B_N=120 cm) the magnitude of shear strains along the slip line differs considerably for each series of tests. For the footing with B_N=3 cm the shear strain $\bar{\gamma}$ in almost all the regions along the slip line remain at around 5%, while the maximum value of shear strain is around 6∿7% at the peak. In the case of the footing with B_N=120 cm, the shearing strains are generally large and show considerable variation along the slip line. By combining the ϕ'-$\bar{\gamma}$ relationship for the plane strain tests in Fig.3 with the observed shear strain distribution in Fig.6, the mobilized angle of shearing resistance along the slip line $\phi'(\bar{\gamma})$ can be obtained. The average values of $\phi'(\bar{\gamma})$ are shown in Table III as $\bar{\phi}'(\bar{\gamma})$. The value of $\bar{\phi}'(\bar{\gamma})$ corresponding to Fig.6(I)(b) for B_N=3 cm is 45° which is very close to ϕ'_{pmax} but that corresponding to Fig.6(II)(b) for B_N=120 cm is 41.5° which is much smaller than ϕ'_{pmax} and is approximately equal to ϕ'_{pr}. This may explain why bearing capacity formulae generally overestimate the bearing capacity of actual foundation soils if ϕ'_{pmax} is put into use (Table III).

In Fig.7 a bearing capacity factor $N_{\gamma q}$ is plotted against a parameter $\gamma B_N/E_q$. The factor $N_{\gamma q}$ is a non-dimensionalized parameter defined by an expression

$$N_{\gamma q} = \frac{q_{max}}{\gamma B_N/2}$$

where q_{max} and γ denote the bearing capacity and the unit weight of sand respectively. The parameter $\gamma B_N/E_q$ is the one proposed by de Beer (1965 A), where E_q denotes Young's modulus of the parent rock of the sand used in tests. For the same type of sand, E_q can be taken as unity. It is shown in Fig.7 that $N_{\gamma q}$ generally decreases with the increase in the parameter $\gamma B_N/E_q$, i.e. with the increase in the B_N value. This phenomenon is called the scale effect of the footing. It is considered that the increase in shearing strains along the slip line with the increase in the footing breadth gives rise to the scale effect. The average value of the mobilized angle of shearing resistance along the slip line $\bar{\phi}'(\bar{\gamma})$ decreases with the increase in B_N value as is shown in Table III, which will explain why the scale effect arises. But it can be seen from Fig.7 that the scale effect vanishes at around the B_N value of 90 cm, and

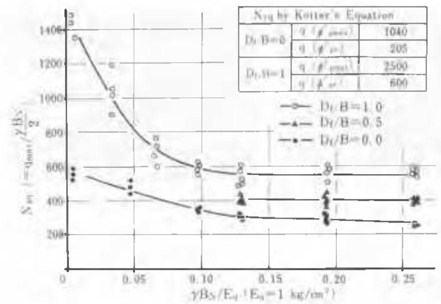


Fig.7 Relationship between $N_{\gamma q}$ and $\gamma B_N/E_q$ for Toyoura sand

that Kötter's solution based on the ϕ'_{pr} yields very reasonable values for "wide" footing. This can be understood by considering the fact that the mobilized angle of shearing resistance assumes almost the same value, which is approximately equal to ϕ'_{pr}, for shearing strains larger than 16∿18%. The investigation on the distribution of shear strains along the slip line for the case of $D_f/B=0$, which is not given here, shows that it does not vary so much with the B_N value as in the case of $D_f/B=1$. This may be because the confinement is less for the case of D_f/B =0 than for the case of $D_f/B=1$ and it will account for the fact that the scale effect becomes more marked as the ratio D_f/B increases.

It has been known that the moisture in the air and interlocking effect gives a small amount of apparent cohesion to sands. For determining this cohesion, the critical height of sand mass was investigated, which gave about 4 g/cm² of the cohesion for Toyoura sand. Since this small cohesion might give rise to the scale effect up to the B_N values of 30 cm, a series of additional bearing capacity tests was performed for glass ballotini, for which no cohesion could be detected. As is shown in Fig.8, the scale effect appears even in these tests. This fact seems to justify the above discussion of the scale effect due to mobilization of the angle of shearing resistance.

In Table III, measured bearing capacities are

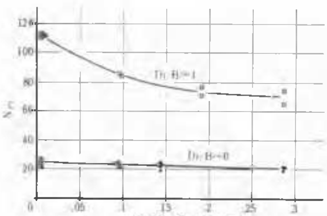


Fig.8 Relationship between $N_{\gamma q}$ and $\gamma B_N/E_q$ for glass ballotini

Table III. Comparison of measured bearing capacity with Kötter's theoretical value

D_f/B	B_N (cm)	q_{max} (kg/cm²) ; measured	$\bar{\phi}'(\bar{\gamma})$ (degree)	q; Kötter (kg/cm²) ϕ'_{pmax}	$\phi'(\bar{\gamma})$
0	3	1.36	44.5	2.48	0.61
	60	17.6	43.7	49.6	18.8
	120	22.3	43.0	99.3	23.0
1	3	3.95	45.0	6.15	0.82
	80	37.6	42.1	164	—
	120	52.0	41.5	247	49.0

compared with theoretical values obtained from Kötter's equations using ϕ'_{pmax} and $\phi'(\bar{\gamma})$ at the peak load. It is concluded that the use of $\phi'(\bar{\gamma})$ in the theories gives very reasonable values of the bearing capacity for the footing with the size of practical use.

CONCLUSIONS

(1) There seems to be no significant influence of the centrifuge on the bearing capacity tests in terms of the particle size.

(2) The smooth base footing gives symmetrical wedges beneath the footing and yields the bearing capacity of about 65∿80% of that by the rough base footing.

(3) The scale effect exists with respect to the bearing capacity factor $N_{\gamma q}$ but it vanishes at around 90 cm of the B_N value. The scale effect becomes more marked as the ratio D_f/B increases.

(4) The average value of the mobilized angle of shearing resistance decreases with the increase in the footing breadth. The extent of the decrease increases with the increase in the ratio D_f/B.

(5) The scale effect of the footing can be more reasonably explained by the variation of shearing strains along the final slip line.

(6) The bearing capacity of dense sand under a footing with a size for practical use can be reasonably obtained by incorporating the mobilized angle of shearing resistance along the final slip line into Kötter's bearing capacity theory.

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