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# Line Loads on Cohesive Slopes

## Pentes Argileuses Linéairement Chargées

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**SYNOPSIS** The undrained stability of cohesive slopes subjected to surcharge line loads of various lengths is evaluated by means of two- and three-dimensional analyses. Results show that the magnitude of the surcharge load causing failure is significantly affected by three-dimensional end effects when: 1 - The slope is close to failure due to gravity only; and 2 - The length of the surcharge load is short compared to the height of the slope. Stability charts, suitable for design practice, are presented.

### INTRODUCTION

The stability of earth and earth-supported structures is of major importance in the design of many geotechnical projects. One class of problems of interest is the undrained stability of cohesive slopes ( $c = c_u$  and  $\phi_u = 0$ ) subjected to surcharge loads. Structures "rapidly" built near the crest of cohesive slopes and heavy equipment (e.g., draglines and railways) produce such loadings. Conventional methods of stability analysis are restricted to two-dimensional (plane strain) modes of failure and the effect of surcharge loads on stability is usually taken into consideration by replacing the surcharge with an equivalent line load of infinite extent.

This article considers the undrained stability of cohesive slopes subjected to line loads of various lengths. Two-dimensional (plane strain) solutions corresponding to a line load of infinite extent are first obtained. Three-dimensional stability analyses of slopes subjected to a line load of finite length are then made and the results compared to two-dimensional solutions in order to ascertain the influence of end effects on the magnitude of the line load causing failure. Results are presented in the form of charts for performing undrained stability analyses.

### LINE LOADS OF INFINITE EXTENT

Fig. 1 shows a cohesive slope of height  $H$  making an angle  $\beta$  with the horizontal. The saturated clay possesses a total unit weight  $\gamma$  and a cohesion (undrained strength)  $c$ . In addition to gravity forces, the slope is subjected to an infinite line load of intensity  $p^0$  which acts at an edge distance  $A$  from the crest of the slope.

The stability of this slope due to gravity only, i.e., with no surcharge, was analyzed by Taylor (1937) using the circular arc method. The shear surface is assumed to be a cylinder of infinite length and the mechanism of failure consists of a rigid body rotation of the cylinder about its axis  $O - O$  ( $z$ -axis, Fig. 1).

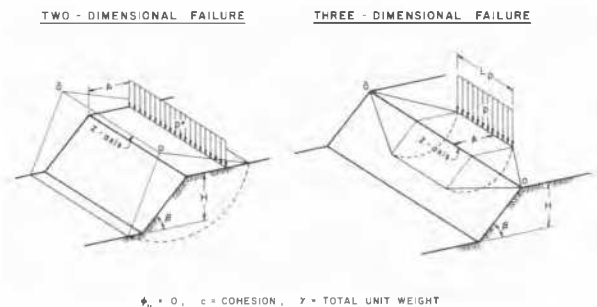


Figure 1 Cohesive Slopes Subjected to Line Loads

For a slope subjected to a surcharge load, failure can take place according to one of two modes: a) Bearing capacity, where the shear surface does not intersect the slope and, therefore, is limited to the neighborhood of the load (bearing capacity theories for horizontal ground surfaces are applicable to such modes of failure); and, b) Slope failure, where the critical shear surface extends beyond the crest and hence involves part of the slope. In the case of a line load, bearing capacity considerations are meaningless because they lead to a zero value of the load. Hence, for the treatment of line loads presented herein, only slope failures were considered.

The effect of a surcharge line load  $p^0$  on the overall stability of a slope was treated by the circular arc method as an extension of Taylor's approach in the case of gravity alone. Different locations of the  $z$ -axis and different radii of the shear circle were assumed, and the value of  $p_{CR}^0$  required to equate the driving and resisting moments for a unit length of the slope was computed numerically. The process was repeated until a minimum  $p_{CR}^0$  was obtained. Details regarding the location of the critical shear surfaces and the method of solution are given by Azzouz and Baligh, 1976.

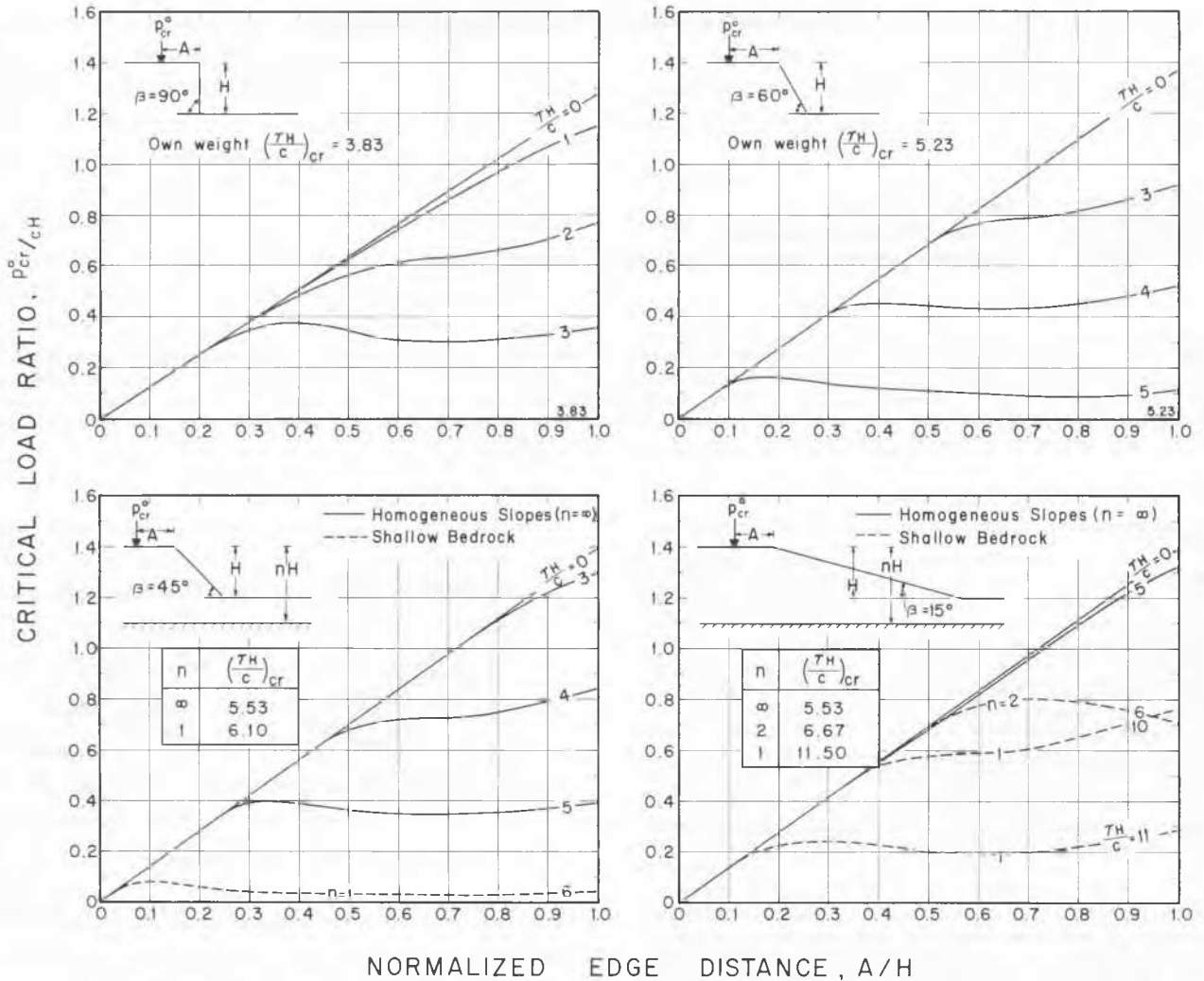


Figure 2. Stability Charts for Line Loads of Infinite Extent

Fig. 2 presents the dependence of the critical load ratio,  $p_{cr}^0/cH$ , on the normalized edge distance  $A/H$  for different values of  $\gamma H/c$  and for slope angles between 90 and 15 degrees. Also given are the values of  $(\gamma H/c)_{cr}$  which correspond to Taylor's solution for slope failure due to its own weight alone. The results for steep slopes ( $\beta \geq 45^\circ$ ) show that:

- (1) For small values of  $\gamma H/c$  compared to  $(\gamma H/c)_{cr}$ ,  $p_{cr}^0/cH$  increases linearly with  $A/H$ . This linearity basically means that for given values of  $\beta$ ,  $\gamma$ ,  $c$ , and  $A$ , the critical load is independent of  $H$  if the slope has a high factor of safety for self-weight only.
- (2) For large values of  $\gamma H/c$ , the surcharge load is small. In the limit, when  $\gamma H/c$  equals  $(\gamma H/c)_{cr}$ ,  $p_{cr}^0/cH$  equals zero.
- (3) For intermediate values of  $\gamma H/c$ , three distinct types of behavior can be depicted: a) When the load is near the crest, i.e.,  $A/H$  is small ( $\leq 0.3$ ), the

effect of gravity is negligible and  $p_{cr}^0$  is very close to the case of  $\gamma H/c = 0$ ; b) for values of  $A/H$  between 0.3 and 0.8 to 0.9, gravity becomes important and decreases  $p_{cr}^0/cH$  from the value given by the  $\gamma H/c = 0$  envelope; and c) When the edge distance is large ( $A/H > 0.8 - 0.9$ ),  $p_{cr}^0/cH$  increases with  $A/H$  because of a greater increase in the resisting moment. This is a result of considering slope failures only, as discussed earlier.

(4) The effect of the slope inclination  $\beta$  can be seen by comparing the results obtained for different values of  $\beta$ . For given values of  $\gamma H/c$  and  $A/H$ ,  $p_{cr}^0/cH$  decreases as  $\beta$  increases, i.e., the steeper the slope, the more important the gravity effects, and, hence, the smaller the surcharge load  $p_{cr}^0$  that can be supported by the slope. This decrease in  $p_{cr}^0/cH$  with  $\beta$  is also more pronounced for large values of  $\gamma H/c$ .

For a flat slope (e.g.,  $\beta = 15^\circ$ ),  $p_{cr}^0/cH$  is signif-

icantly affected by the location of the bedrock, as expressed by the parameter  $n$ . For given values of  $\gamma H/c$  and  $\beta$ , the slope becomes more stable as the depth to bedrock decreases (smaller values of  $n$ ). This results in larger values of  $p_{cr}^o/cH$ .

LINE LOADS OF FINITE LENGTH

A three-dimensional analysis is required to determine the stability of slopes subjected to line loads of finite length. This was performed herein on the basis of the formulation presented by Baligh and Azzouz (1975). The geometry of the slope and the soil properties are assumed to be constant along the axis of the slope. Instead of the infinitely long cylindrical shear surface used in plane strain solutions, two different types of shear surface are considered. The first consists of a cone attached to a cylinder as shown in Fig. 1, whereas the second is an ellipsoid attached to a cylinder. The length of the cylinder is equal to that of the line load,  $L_p$ .

The search for the critical shear surface is performed numerically along the same lines as in the plane strain analysis. The location of the z-axis,

the cylinder radius and the length of the cone (or ellipsoid) are assumed and the corresponding value of the load required to cause failure of the slope,  $p_{cr}$ , is obtained. After several trials, the minimum value of  $p_{cr}$  is determined. Details regarding the location and geometry of the critical shear surfaces as well as the method of solution are given by Azzouz and Baligh, 1976.

The three-dimensional end effects are illustrated herein by means of the influence ratio  $f_L$  shown in Fig. 3 and defined by the expression:

$$p_{cr} = f_L p_{cr}^o \dots \dots \dots (1)$$

Given  $\beta, \gamma H/c$  and  $A/H$ , the value of the critical line load  $p_{cr}$  acting along a length  $L_p$  can therefore be obtained by first determining  $p_{cr}^o/cH$  from Fig. 2 and then evaluating  $f_L$  from the plots presented in Fig. 3. The results show that:

- (1) The magnitude of  $f_L$  decreases with increasing  $L_p/H$  for fixed values of  $\beta, \gamma H/c$  and  $A/H$ . When  $L_p/H$  is very large,  $f_L$  equals unity (plane strain conditions).

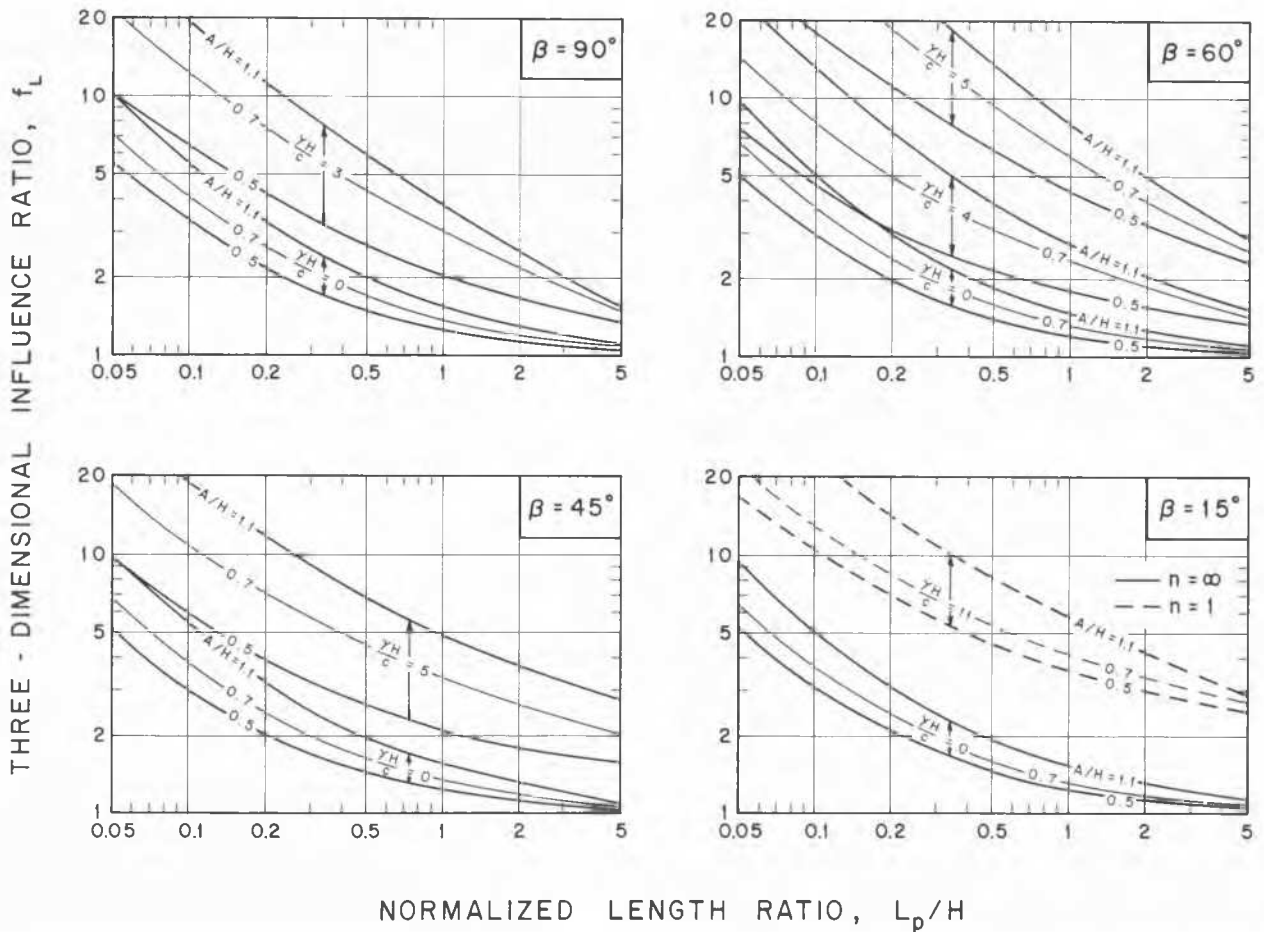


Figure 3. End Effects for Line Loads of Finite Length

(2) Three-dimensional effects can be very significant. This is illustrated by  $f_L$  values in excess of 20. Moreover, the largest  $f_L$  values for a given geometry are associated with slopes which are close to failure due to gravity alone, i.e., where  $\gamma H/c$  is close to the critical value  $(\gamma H/c)_{cr}$ .

(3) The case of a concentrated load, corresponding to very small values of  $L_p/H$ , provides meaningless results. The most critical condition for this case was found to correspond to a failure surface having zero area, i.e., a bearing capacity type of failure with zero load.

(4) For intermediate values of  $L_p/H$ , the magnitude of  $f_L$  is generally affected by  $\beta$ ,  $\gamma H/c$  and  $A/H$ . For a given slope angle  $\beta$ , the value of  $f_L$  increases with  $\gamma H/c$  and  $A/H$ . However, the dependence of  $f_L$  on the slope angle  $\beta$  is not significant for slopes having high factors of safety under gravity forces, e.g., when  $\gamma H/c$  is small compared to  $(\gamma H/c)_{cr}$ . Furthermore, for values of  $A/H$  smaller than 0.5,  $f_L$  is essentially independent of both  $\gamma H/c$  and  $\beta$ . (Azzouz and Baligh, 1976).

#### SUMMARY AND CONCLUSIONS.

1. Charts are presented for the undrained stability of a surcharge line load of infinite length acting on slopes of constant strength  $c$  and unit weight having a geometry as defined in Fig. 1. Fig. 2 plots the normalized line load causing failure  $p_{cr}^o/cH$  vs. the normalized edge distance  $A/H$  as a function of  $\gamma H/c$  for several slope angles and depths to bedrock.

2. Three-dimensional analyses were performed to determine the magnitude  $p_{cr}^o$  of a surcharge line load of finite length  $L_p$  which will cause failure of cohesive slopes. The results are expressed by the parameter  $f_L = p_{cr}^o/p_{cr}^o$ , which is plotted in Fig. 3 vs. the normalized length ratio  $L_p/H$  as a function of  $\gamma H/c$  and  $A/H$  for several slope angles and depths to bedrock.

3. The results presented herein are approximate upper-bound solutions because they are based on extensions of the circular arc method of analysis. Moreover, they exclude bearing capacity modes of failure, i.e., when the critical failure surface does not extend beyond the crest of the slope.

4. Three-dimensional end effects, as expressed by the parameter  $f_L$ , become significant whenever  $L_p/H$  is less than unity and are especially important when the factor of safety of the slope due to self weight forces only is close to unity. Values of  $f_L$  in excess of 5 to 10 would be expected for many practical situations.

#### ACKNOWLEDGEMENT.

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