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Bodies Falling Down on Different Slopes—Dynamic Study

La Chute de Corps sur Différents Talus—Etude Dynamique

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YPNDSIS.— When a body falls down a slope, it might run down:

- Keeping the contact with it, or it can separate itself from it at a given place jumping up and bouncing during the travel until it comes to a stop.
- The movement might be: sliding, rolling, or a combination of both movements, and in any case:
 - Without, or little friction, and with friction.
- It is studied the variation of the kinetic energy adquired by the body (on traslation and rotation); the potencial energy (height) and the energy which is lost because of the friction and collision during its fall.
- Different types of slope are considered: circular, cycloid, and general, and, over them, masses punctual, spherical, revolution, etc.
- A related computer program provides with a general design method.

INTRODUCTION

The necessary conditions for a slope to be steady are studied in the treatises of soil mechanics and geotecnics. According to their geometrical characteristics: heights, gradients, etc. and to their mechanical characteristics - density, cohesion, friction angles, etc. the said stability is defined and it is even affected by a determined safety factor.

Yet, this stability is referred to the slope section, or to the embankment section, considered macroscopicly, as a whole. It is evident that even in slopes that we could call "steady", isolated landslips of small and medium volume can turn up, sliding, rolling and bounding on it. Depending on the circumstances, the above mentioned landslips could be stopped in their way down, as well as they could go to (the foot of) its base or even further.

According to their volume these landslips -- could be considered of little importance; -- yet, they are very dangerous for the neighbouring constructions. We are particularly referring to communication lines - railways, roads, etc., in the lay-out of which it is very usual, nowadays, to construct slopes with dimensions from 30 to 40 meters or even more. The falling down of stones on a freeway or a railway track may cause a serious accident, specially if there is a lot of traffic and travelling at a great speed.

To prevent them from accidents, the slopes are protected in different ways, among which

we can point out the following ones:

- a) by reducing the slope angle.
- b) by protecting them with walls or reinforced plates (anchored or not).
- c) by leaving intermediate berms or side ditches near the edge.
- d) by covering it with gunite so as to reduce the erosion.
- e) by disposing an adequate drainage.
- f) by putting a mesh on the slope, hanged, from a beam in coronation and/or anchored in intermediate points by means of anchors. On this mesh we can shoot-concrete.
- g) by constructing walls on both sides of the railway or the road, so that the landslips would be stopped by them. Or even better, by making artificial tunnels with protection enough to resist landslips.

We see that for projecting in a most convenient and economical way, we must have at least an approaching idea of:

- The course to be followed by the landslips, according to the geometrical and mechanical characteristics, as much for the mass that will fall down as for the ground profile to be followed by the mass, specifying the place where the landslips will stop; and:

- The kinetic energy, that is, the speed it will have in each point, specially before reaching the area to be preserved, so as to calculate and estimate the adequate protection to be made in case the mass would not stop before reaching the mentioned area.

As there is no doubt about the great complexity of the problem, we shall start by showing

it in a very easy way, introducing very few parameters, then, we will introduce some variations of these parameters so as to approach the fact. Finally we will show some ways to solve the most general cases.

We shall give some formulas of easy application for slopes of geometrical section expressed analytically, and for any section of -- slope, on the basis of separating it in consecutive elements made up of rectilinear segments and circumferential arches.

And also, with the help of a computer, we -- shall see how to apply the above mentioned -- statement to a slope defined by the dimension of different points, being sufficiently close. The computer plots the speeds adquired in each point and the trajectories it follows in the air, in case the mass would withdraw from the slope and then fall on it bounding and sliding down.

THE PROBLEM

(I) Let suppose it is a question of a punctual mass m which slides down smoothly on any section; then, we know the speed V in -- each point is only a function of the -- difference of level y existing between -- the starting point and the considered -- one, the value of which is: $V^2 = 2gy$ (1) If the starting speed was V_0 , the adquired one would be: $V^2 = V_0^2 + 2gy$ (2)

(II) In case there was some friction μ along -- the section and being the latter a constant, the speed would be smaller. The increase of the kinetic energy would be similar to the reduction of the potential energy m.g. diminished in the friction work. As this work along the arch element ds is equal to $\mu.N.ds$ being N the result of the forces which act perpendicular to the profile. (See Fig.1 below)

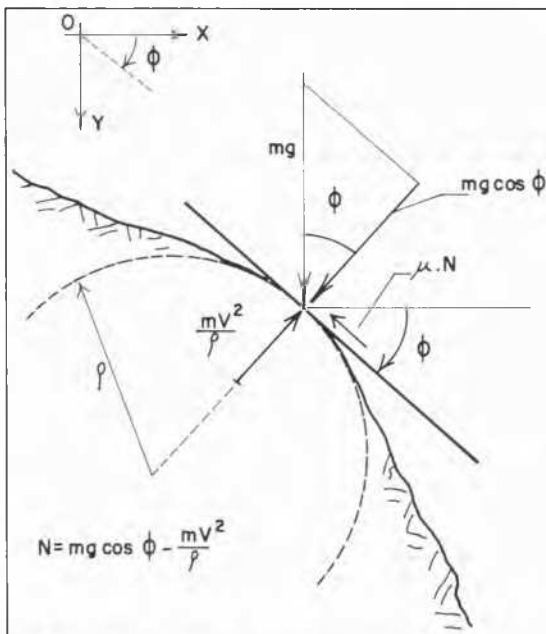


Fig 1 Forces acting on a section.

But, N consists of two parts:

- 1) Component of the weight $m.g.$ normal to the surface; let θ be the angle of the slope, then its value will be: $m.g.\cos\theta$.
- 2) The normal component of the centrifugal force, that is $\frac{m.V^2}{\rho}$, being ρ the radius of curvature of the section (function, in general, of θ).

Then: $N = m.g.\cos\theta - \frac{m.V^2}{\rho}$
(Let ρ have plus sign convex sections and minus sign in concavo sections)

Replacing it, the result will appear in a -- differential form (conservation of energy).

Thus: $d(\frac{m.V^2}{2}) = m.g.dy - \mu(m.g.\cos\theta - \frac{m.V^2}{\rho})ds$

Where: $dy = ds.\sin\theta$ and by definition $\rho = ds/d\theta$
Operating, it results:
 $V.dV/d\theta - \mu V^2 = g(\sin\theta - \mu\cos\theta)$ (3)

This expression relates V and θ , being g a -- function of θ

In case μ is a constant, this differential -- equation is immediately integrated (formerly multiplying both terms by $\exp(-2\mu\theta)$ then, -- after simplifying, it results:

$$V^2 = V_0^2 \exp(-2\mu(\theta - \theta_0)) + 2.g. \exp(2\mu\theta) \int_{\theta_0}^{\theta} g \exp(-2\mu\theta).(\sin\theta - \mu\cos\theta) d\theta \quad (4)$$

APPLICATIONS

We will consider 2 ways of applying the obtained results:

- 1) Departing from the analytic expression of the curve in the $\rho = \rho(\theta)$ form.

If we have it in the cartesian form $y=y(x)$ we pass it to the previous one, knowing that $y' = dy/dx$; $\tan\theta = y'$; $\rho = (1+y'^2)^{3/2}/y''$

Example 1: Cycloid in inverted position

In this case $\rho = -4r \cos\theta$ (let ρ be negative, as it is a concavo section);

r = radius of the circumference roulette. Replacing it in (4) $V^2 = -8g.r.\exp(2\theta)(I_1 - I_2)$ being

$$I_1 = \int_{\pi/2}^{\theta} \exp(-2\mu\theta). \sin\theta d\theta$$

$$I_2 = \int_{\pi/2}^{\theta} \exp(-2\mu\theta). \cos\theta d\theta$$

Calculating the integrals and simplifying:

$$V^2 = 4.g.r/(1+\mu^2) (\exp(-(\pi-2\theta)\mu) - 1 + (1-\mu^2).\cos^2\theta + 2\mu\sin\theta.\cos\theta)$$

In the lowest point and corresponding to $\theta = 0$ it results:

$$V^2 = 4.g.r/(1+\mu^2) .(\exp(-\mu\theta) - \mu^2)$$

It would come to rest in case it is proved $\mu^2 \exp(\mu\theta) = 1$ that is, for $\mu = 0,47454$

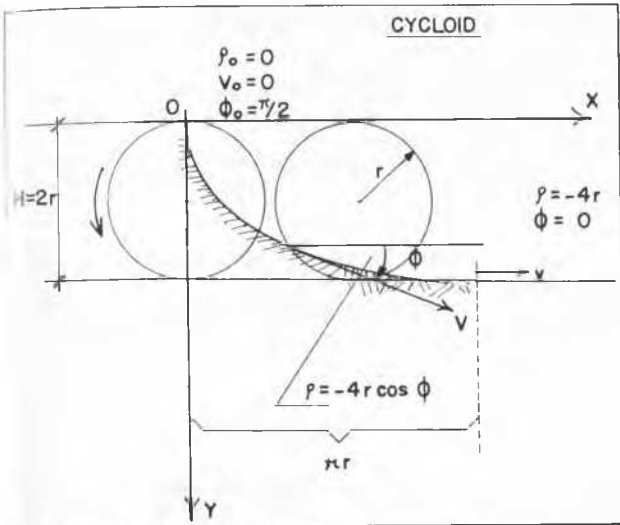


Fig 2. Cycloid in inverted position.

Example 2: Any profil defined by the dimensions

z_j ($j = 0, 1, 2 \dots$) function of the x_j ; being equal the increases of the x_j that is:

$$\Delta x_{j+1} - \Delta x_j = \Delta x$$

This expression can be transformed - changing the integrals by, in the approaching:

$$V_1^2 = V_0^2 \exp(-2\gamma(\theta_1 - \theta_0)) + 2 \cdot g \cdot \exp(2\gamma\theta_1)$$

$$\sum_{j=1}^i \beta_j \exp(-2\gamma\theta_j) \cdot (\sin\theta_j - \gamma \cos\theta_j) \Delta \theta_j$$

The θ_j and β_j values can be obtained by finite differences, of

$$\tan\theta_j = y_j' \approx (z_{j-1} - z_{j+1}) / 2\Delta x$$

$$y_j'' = -(z_{j+1} + 2 \cdot z_j - z_{j-1}) / (\Delta x)^2$$

$$\beta_j = (1 + y_j''^2)^{3/2} / y_j'$$

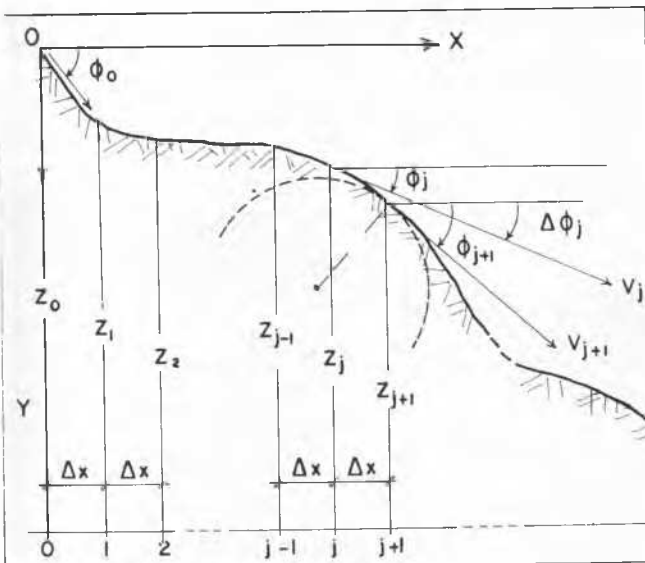


Fig 3. Finite differences.

COMPUTER PROGRAM

Applying the last example, we have produced a computer program to calculate and plot the - total energy, kinetic energy, potential energy, speed, travel time and trajectory of the falling body.

The condition for the latter to separate itself from the slope is $-V^2/g \cdot \cos\theta < 0$ ($\gamma < 0$ in concavo sections).

The program interpolates the exact point of separation, and the speed vector at that - point. From there on, the exact equation of the trajectory is: (x, z)

$\Delta z = \tan\theta_0 \cdot \Delta x - (g \cdot \Delta x^2) / (2V_0^2 \cdot \cos^2\theta_0)$ being θ_0 the angle of the speed vector with a horizontal - line, and V_0 its magnitude. When the trajectory comes to an intersection with the profile, once again the exact point of intersection is computed, so as the speed vector and the incidence angle i .

The friction coefficient γ , which may be introduced in the program as a function of x , - is no longer important if the block is not - rotating.

According to the theories of Newton, Routh, - Hertz, Saint-Venant, Boussinesq et al. the - reflection angle r , is computed from $\tan r = \tan i / \epsilon$ where $0 < \epsilon < 1$ is the restitution - factor.

If the block rolls, the equation becomes - $\tan r = (\tan i + (1 + \epsilon) \cdot \gamma) / \epsilon$

The rotation speed, for central symmetric bodies is $\omega = V/R$ being R its radius.

If we neglect friction, we may see the influence of rotation in reducing the kinetic energy.

The decrement of potential energy equals now; $1/2 m V^2 + 1/2 I \omega^2$ I being the inertia - moment.

or $1/2 m V^2 + 1/2 m \omega^2 \cdot \beta_1^2 = 1/2 m V^2 (1 + \beta_1^2 / R^2)$ where $\beta_1 = I/m$

So the speed of a spherical block rolling is reduced to $V^2 = 10/7 g \cdot h$ instead of $2 \cdot g \cdot h$, if we take into account that for a sphere - $\beta_1^2 = 2/5 R^2$

The effect is similar to a reduction in g to $7 m/s^2$

For a cylindric block $\beta_1^2 = R^2/2$, and $V^2 = 4 \cdot g \cdot h/3$

If we wish to take variable friction into - account we substitute it into the differential equation. Making use of the Runge-Kutta method the solution is once again found, as we will - show in example n.4.

APPLICATIONS

Example 3: Block sliding and bounding along a profile

Fig. 4 shows the trajectory (T) of a block sliding and bounding on a profile.

Plots of kinetic energy (KE), speed (S), travel time (TT) and total energy (TE) are included.

The friction coefficient μ was taken as 0,4 in the first slope, 0,45 in the first horizontal line, 0,5 in the second slope and 0,7 at the base.

The restitution coefficient ϵ was assumed to be 0,55.

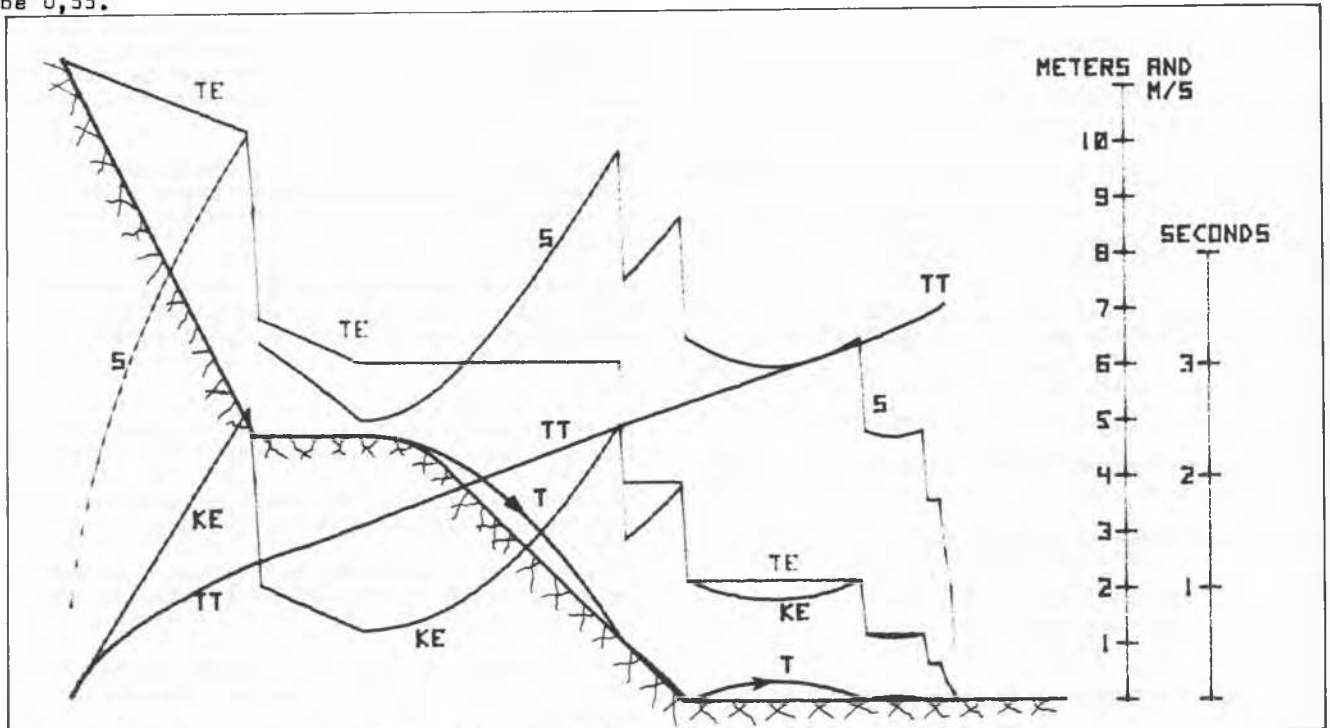


Fig 4. Block sliding and bounding along a profile.

Example 4: Block sliding along an inverted cycloid.

Fig. 5 shows the kinetic energy for a block sliding along an inverted cycloid in three cases (A) no friction

- (B) friction coefficient = 0,47454/K
- (C) friction coefficient = 0,47454

Where $K = 1 + 0.2V$

The decisive influence of taking into account the speed is emphasized.

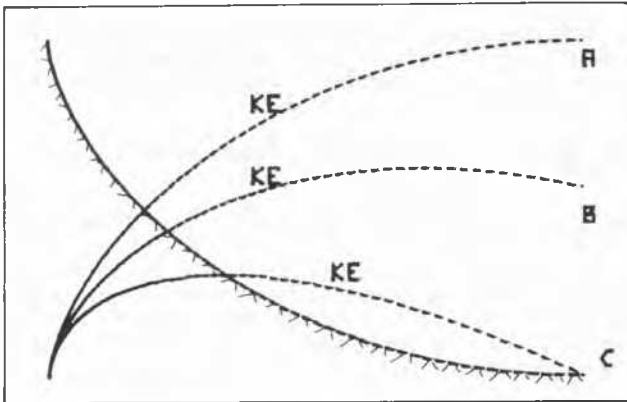


Fig 5. Influence of taking into account the speed when considering the friction effect.

CONCLUSIONS

- The computer program may be used as a tool for:
 - deducing the intrinsic characteristics of -- blocks from observation and timing of fall.
 - compute, for a given profile, the probabilistic distribution of reach and energy of blocks

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