Dynamic Decision Procedure of Embankment Construction

Décision Procédé Dynamique sur Construction en Remblai

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SYNOPSIS This paper describes an observational procedure of embankment construction. A probability of slope failure is defined as a function of both ground conditions and dimensions of a designing embankment. It is shown that observation of vertical and horizontal displacements make it possible to predict slope failure. Therefore, these observation give us information to improve the probability density function which represents the uncertainty of ground conditions. Since predictive failure probability can also be reduced by observing the displacements, observation during construction is very available for optimization of successive loading pattern. The decision problem of an optimum consolidation period which is closely related to an increase of undrained shear strength of clay layer is discussed as one example of the practical applications of this dynamic decision method of an embankment.

INTRODUCTION

The present design methods of slope stability involve various kinds of uncertainty, such as inevitable analytical errors in idealization of the complicated real behavior, variation of soil properties and insufficient information due to the limitted numbers of samples.

The reliability design method is very effective to obtain the rational design in which the above mentioned uncertainties are taken into consideration.

The authors already found the following two important points concerning the slope stability problem; (1) The coefficient of variation of the undrained shear strength, c_u, of saturated clay layer does not change during consolidation, (Matsuo, Kuroda, and Asaoka, 1975, Matsuo and Asaoka, 1976) (2) The rapid sliding failure can be predicted on the diagram of the vertical settlement just under an embankment and the horizontal displacement near the toe of the slope. (Matsuo and Kawamura, 1975, Matsuo, 1976)

This paper mainly discusses the following two problems by using these two results ; (1) improvement of the spatial distribution of c_u by the new observed results during construction. (2) The optimization of design of an embankment in which the design can be changed by using the observed results during construction.

When we intend to construct an embankment, there are two important points; one is the decision problem, i.e. so called design itself and the other is the prediction problem of the actual mechanical behavior of an embankment. In the proposed dynamic decision method, these two points are taken into consideration as a whole system.

SOME PRELIMINARY REMARKS

(a) Probability of sliding failure

The stability of an embankment rapidly constructed on a saturated clay layer is generally analyzed by the $\phi_u = 0$ method. Many kinds of errors and uncertainties are inevitable in this conventional design method. Denoting these errors as e, the true safety index F is expressed as follows :

$$F = G + e \tag{1}$$

where G denotes a safety index (so called safety factor) which is calculated by the $\phi_u = 0$ method.

G is formulated as shown in the following equation :

$$G = \frac{R\int_L c_u(\mathbf{x})\ dL}{M_0} \tag{2}$$

where L denotes the length of the slip circle, R the radius of the slip circle, M_0 the overturning moment and $c_u(\mathbf{x})$ the shear strength at the position $\mathbf{x}$. Expressing the hetrogeneity of $c_u(\mathbf{x})$ by Eq.(3), G becomes a random variable which is given by Eq.(4),

$$E[c_u(\mathbf{x})] = \mu\ ,\ \mathrm{Var}\ [c_u(\mathbf{x})] = \sigma^2$$
$$E[c_u(\mathbf{x})c_u(\mathbf{x}+\tau)] = \sigma^2 r(\tau) \tag{3}$$

$$G = \frac{\mu}{\tilde{\mu}} + \varepsilon \tag{4}$$

where μ denotes the spatial mean of $c_u(\mathbf{x})$, σ^2 the spatial variance of $c_u(\mathbf{x})$, $r(\boldsymbol{\tau})$ the autocorrelation coefficient of $c_u(\mathbf{x})$ between two points $\mathbf{x}$ and $\mathbf{x}+\boldsymbol{\tau}$, $\tilde{\mu}$ the equivalent stress on the slip circle caused by the embankment and ε is a random variable with a mean of zero and the following variance :

$$\sigma_\varepsilon^2 = \frac{\sigma^2}{\tilde{\mu}^2 L^2} \iint_L r(\boldsymbol{\tau}) dL\, dL \tag{5}$$

Therefore F of Eq.(1) includes two random variables as shown in the next equation :

$$F = \frac{\mu}{\tilde{\mu}} + \varepsilon + e \tag{6}$$

Thus the probability of failure can be defined as follows :

$$P_f(\theta, \tilde{\mu}) = \text{Prob.}\ (F < 1 \mid \theta, \tilde{\mu}) \tag{7}$$

where θ represents the parameters $(\mu, \sigma^2, r(\boldsymbol{\tau}))$.

(b) Predictive probability of failure

We can not obtain perfect information on θ because of limited numbers of samples, but we can use the probability density function of θ, $\xi(\theta)$, which is obtained by the results of soil exploration. The predictive probability of failure defined in the following equation :

$$P_f(\tilde{\mu}) = \int_\Theta P_f(\theta, \tilde{\mu})\ \xi(\theta) d\theta \tag{8}$$

(c) Cumulative distribution function of $\tilde{\mu}$

In the practical problems, we can not observe the exact value of F during construction. The banking height at $F \fallingdotseq 1$, which is a random variable, can only be observed, and thus we are compelled to use information on θ at $F \fallingdotseq 1$ in order to control the successive construction.

Probability of failure $P_f(\theta, \tilde{\mu})$ can be regarded as the cumulative distribution function of $\tilde{\mu}$ at $F \fallingdotseq 1$. Therefore the probability of failure and its predicted value when an embankment does not fail untill the banking height corresponding to $\tilde{\mu}$ but it will fail during the next construction stage corresponding to $\tilde{\mu} + \Delta\tilde{\mu}$ can be given by Eqs. (9) and (10), respectively.

$$P_f(\theta, \tilde{\mu} + \Delta\tilde{\mu}) - P_f(\theta, \tilde{\mu}) \equiv P_f(\tilde{\mu} \sim \tilde{\mu} + \Delta\tilde{\mu} \mid \theta) \tag{9}$$

$$P_f(\tilde{\mu} + \Delta\tilde{\mu}) - P_f(\tilde{\mu}) \equiv P_f(\tilde{\mu} \sim \tilde{\mu} + \Delta\tilde{\mu}) \tag{10}$$

(d) Transition process of θ due to consolidation

According to our investigations for the transition process of θ, the following relationships are available :

$$T(\theta, t) = (\alpha\mu, \alpha^2\sigma^2, r(\boldsymbol{\tau})) \tag{11}$$

$$\alpha = 1 + \frac{(\Delta p)_m}{(p_0)_m} \beta U(t) \tag{12}$$

where $(p_0)_m$ denotes the spatial mean of preconsolidation pressure, $(\Delta p)_m$ the spatial mean of increased load, $U(t)$ the degree of consolidation for the period t and $T(\theta, t)$ the transition function. Every embankment has a limited width at its top and base and therefore a one-dimensional consolidation condition is not satisfied in general practical problems. This means that the strength does not increase at the whole region along the slip surface. The coefficient β in Eq. (12) represents the reduction of strength due to this influence. Eq.(12) shows that the coefficient of variation of $c_u(\mathbf{x})$ and its spatial autocorrelation coefficient are kept constant before and after consolidation.

PREDICTION OF RAPID FAILURE OF AN EMBANKMENT

The displacement has the close relation with the failure and its measurement is easy. From this point of view we investigate the displacement process of the actual embankments in Japan. Some examples are given in Fig.1 in which ρ is the total vertical settlement at the center just under an embankment and δ is the horizontal displacement near the toe of the slope.

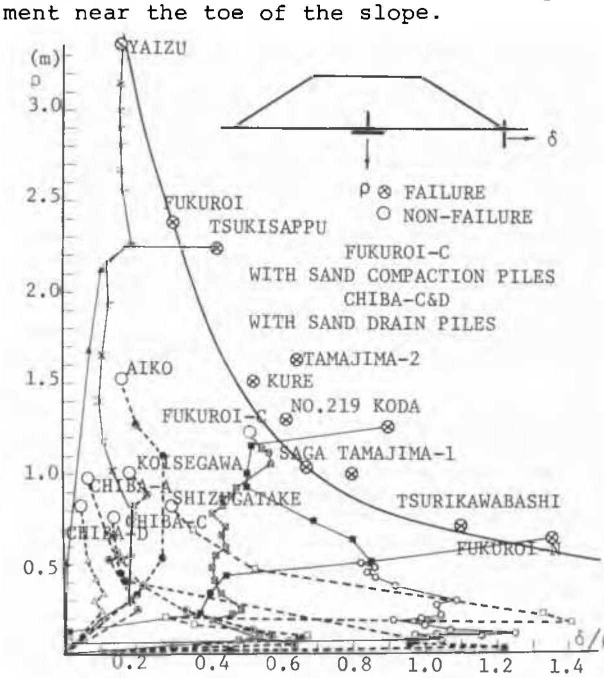


Fig.1($\delta/\rho \sim \rho$)Plots for Prediction of Failure

We should pay attention to the fact that the cross section and the unit weight of each embankment, the soil properties and the thickness of each soft layer and other surroundings are different from each other. It is also important to note that many embankments under different conditions failed near the one curve of this figure. Observing the process of displacement carefully, we become aware that in failure cases, this curve is approached as construction progresses, and on the other hand, in non-failure cases, there is a tendency to be distant from this curve, although it is approached once just after construction. To approach this curve means that the horizontal earth flow is large compared to settlement by consolidation, and to deviate from this curve means that consolidation is predominent compared to the horizontal flow. That is, the embankment becomes safer. Accordingly, we can regard this curve as the failure criterion line and we can predict the failure by plotting the observed settlements and horizontal displacements on this diagram. We can observe the banking height H (i.e. $\tilde{\mu}$ in Eq. (9) and (10)) when the displacement path $(\rho, \delta/\rho)$ approaches to the failure criterion line.

OPTIMIZATION OF CONSTRUCTION PROCESS OF AN EMBANKMENT

The decision method of the consolidation period in which observed displacements during construction is used as additional information is formulated by applying the adaptive control theory. Let us consider an embankment with the constant slope gradient where $\tilde{\mu}$ has a linear relation with the banking height H. After the ground is consolidated for the period t_{n-1} under the fill of H_{n-1} height at the (n-1)th banking stage, the banking at the nth stage is carried out from H_n to $H_n + \Delta H$. If the sign of failure comes out in Fig. 1 during the nth stage, the probability density function $\xi(\theta_n)$ can be improved as follows :

$$\xi(\theta_n | H_n \sim H_n + \Delta H, t_{n-1}) = \frac{P_f(H_n \sim H_n + \Delta H, \theta_n)}{\tilde{P}_f(H_n \sim H_n + \Delta H)} \xi(\theta_n) \qquad (13)$$

where

$$\xi(\theta_n) = \xi(\theta_{n-1} | H_{n-1} \sim H_{n-1} + \Delta H, t_{n-2}) \left| \frac{d\theta_{n-1}}{d\theta_n} \right|$$

$$\theta_n = T(\theta_{n-1}, t_{n-1}) \quad (n = 1,2,\ldots N) \qquad (14)$$

Eqs.(13) and (14) can be easily reduced from Eqs.(9), (10), (11) and (12). This procedure is continued up to the (N-1)th stage without having the failure of fill at any stage. The set of the optimum consolidation periods $(t_1, t_2, \ldots t_{N-1})$ can be determined by minimizing the total Bayes risk. This procedure is shown in Fig. 2.

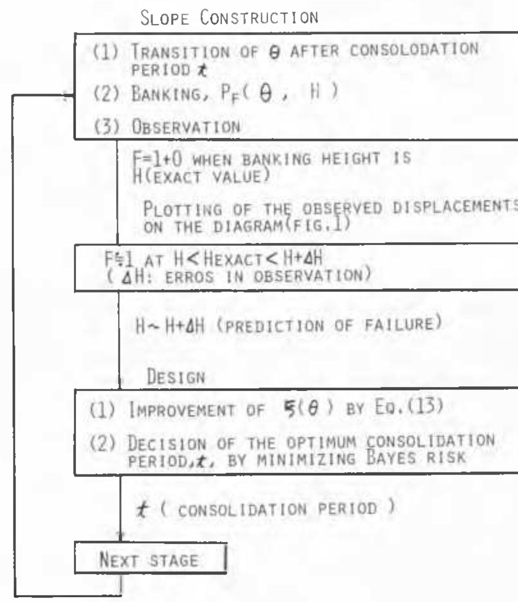


Fig. 2 Dynamic Decision Process

NUMERICAL EXAMPLES

The case of N = 4 is shown as one example in order to help the understanding the dynamic decision process of slope construction. If the total construction period is not limited, the total Bayes risk for the given H_1 is given by the following equation :

$$\begin{aligned} & r(t_1, t_2, t_3) \\ = \; & ct_1 + \sum_{H_1}^{H} \tilde{P}_f (H_2 \sim H_2 + \Delta H \mid H_1 \sim H_1 + \Delta H, t_1) \\ & [\, ct_2 + \sum_{H_2}^{H} \{\tilde{P}_f (H_3 \sim H_3 + \Delta H \mid H_2 \sim H_2 + \Delta H, t_2, H_1 \\ & \sim H_1 + \Delta H, t_1) \; (ct_3 + c_f \int_{\theta_4} \xi(\theta_4 | H_3 \sim H_3 + \\ & \Delta H, t_3, H_2 \sim H_2 + \Delta H, t_2, H_1 + \Delta H, t_1) P_f(\theta_4, \\ & H) d\theta_4)\}] \qquad (15) \end{aligned}$$

where c denotes the unit construction cost per unit construction time and c_f the failure cost.

If the total construction period is limited as $t_1 + t_2 + t_3 = T$ (T : constant), the set of the optimum consolidation periods can be obtained by minimizing the following equation :

$$
\begin{aligned}
& r\,(t_1,\ t_2,\ t_3) \\
= & \sum_{H_2=H_1}^{H} [\tilde{P}_f\,(H_2 \sim H_2 + \Delta H \mid H_1 \sim H_1 + \Delta H,\ t_1) \\
& \sum_{H_3=H_2}^{H} \{\tilde{P}_f (H_3 \sim H_3 + \Delta H \mid H_2 + \Delta H,\ t_2, H_1 \sim H_1 + \\
& \Delta H,\ t_1) \int_{\Theta_4} \xi(\theta_4 | H_1 \sim H_1 + \Delta H, t_1,\ H_2 \sim H_2 + \Delta H, \\
& t_2,\ H_3 \sim H_3 + \Delta H,\ t_3) P_f(\theta_4, H) d\theta_4\}] \quad (16)
\end{aligned}
$$

subject to $t_1 + t_2 + t_3 = T$

The numerical examples of the latter case are given. The conditions used in the calculation and the computed results are shown in Table I and Table II, respectively. (Matsuo and Kuroda, 1974, Matsuo and Asaoka, 1976)

Table I Numerical Conditions

Total Banking Height H : 10 m Construction Period T : 60 days Unit weight of Banking Material : 1.7 t/m^3 Stability Number : 0.182
Relation between Degree of Consolidation and Time (Use of Sand Drain Piles) $U(t) = 1 - \exp(-0.0826t)$ (t : day) (β:0.5
Results of Compression Tests : 30samples Sample Mean : 2.0 t/m^2 Sample Standard Deviation: 0.6 t/m^2
Distribution of c_u : Normal Distribution Distribution of e : Uniform Distribution of Eq.(6) $-0.1 \le e \le +0.1$
Autocorrelation Coefficient : $r(\tau) = \exp(-0.9\|z-z'\|)$: $z-z' = \tau$ z,z': Depth

Table II shows the optimum consolidation process of an embankment under the condition of given final height H = 10 m. The case with the starmark is explained as one example. If the sign of failure is realized on Fig.1 when the banking height reaches to H_1 = 5 ∿ 6 m, the fill should be left as it is for 6 days, as can be seen in Table II. Banking is started again after consolidation for 6 days. If we find the sign of failure again at H_2= 7∿8 m, consolidation for 32.4 days is required. The embankment is completed by repeating the same manner.

CONCLUSIONS

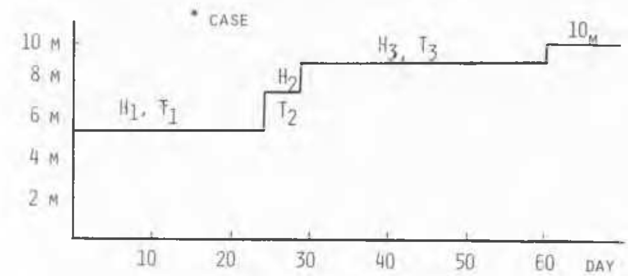
Conclusions obtained from the present study are summarized as follows :

(1) The probability of slope failure is defined as a function of both ground conditions, θ, and equivalent shear stress on the slip surface, $\tilde{\mu}$.

(2) The sliding failure can be predicted by observing the vertical and horizontal displacements (ρ, δ) of the ground.

That is, it is predicted by plotting ρ and δ on the (ρ∿δ/ρ) diagram.

Table II Computed Results



H_1(M)	T_1(DAY)	H_2(M)	T_2(DAY)	H_3(M)	T_3(DAY)
4 - 5	6.0	5 - 6 6 - 7 7 - 8 8 - 9	27.0 27.0 5.4 5.4	6 - 10 7 - 10 8 - 10 9 - 10	27.0 27.0 48.6 48.6
5 - 6*	6.0*	6 - 7* 7 - 8 8 - 9	27.0* 32.4 5.4	7 - 10* 8 - 10 9 - 10	27.0* 21.6 48.6
6 - 7	6.0	7 - 8 8 - 9	32.4 32.4	8 - 10 9 - 10	21.6 21.6
7 - 8	12.0	8 - 9	28.8	9 - 10	19.2

(3) The probability density function, ξ(θ), of the ground conditions, θ, can be improved by observing ρ and δ during construction. This improvement is carried out by the Bayes theorem in which the probability of failure is used to compute conditional and marginal probability functions of $\tilde{\mu}$.

(4) Optimization of observational procedure of slope construction (dynamic decision process) is obtained by minimizing the Bayes risk which is calculated from the improved ξ(θ). As a practical application of the present theory, a multistaged construction problem of an embankment on a saturated clay layer is discussed in which an increase of the undrained shear strength due to consolidation is expected.

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