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Investigation of Strain State of Slopes

Etude de l'Etat de Déformation des Talus

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SYNOPSIS. The present report deals with the results of the stress-strain analysis of slopes. The analysis was based on the elasto-plastic properties of soil within the scope of the Prager-Drucker's yield theory and on the associative law. The condition of Mises-Schleier-Botkin was adopted as the plasticity surface.

The results of the analysis were verified experimentally by the method of centrifugal modeling and investigation of deformation of homogeneous slopes of cohesive soils. As a result of the experiments the pattern of strain distribution inside the body of slopes of different heights was established and failure surfaces of slopes with the corresponding critical heights determined. The dynamics of collapsing of homogeneous slopes of cohesive soils was investigated. It has been shown that no visible failure occurs on gentle cohesive slopes whose slope inclination is more than 1:1, though there are all indications that the stability has been lost: large deformations, considerable opening of cracks and so on. It is proposed to characterise the strain state by the variation of position of the isostaths of settlements and horizontal displacements.

A comparison of the theoretical analysis and experimental results is presented.

The paper is concluded with a statement that the suggested analysis procedure based on the yield theory can be used for the evaluation of stability of the slopes of earth fill structures.

The analysis procedure can be verified either by the data obtained from field observations or by the experimental results. As a rule, the first method of verification cannot be applied to slopes as under natural conditions their stability and deformation are influenced by a great number of different factors whose individual influence cannot be evaluated. It should be noted that the experiments are difficult to conduct as the deformation of a slope depends on the quantitative values of the stresses much more than on the qualitative similarity of the stress distribution patterns. The development of the centrifugal modelling allowed us to overcome this difficulty and obtain extensive experimental data which can be used for evaluation of the theoretical analysis procedure, the main principles of which are discussed in the second part of the paper.

The experiments were carried out with the help of the centrifugal modelling machine owned by the "Hydroproject" whose rotation radius is 2.5m; it allows to test the models under the acceleration of up to 320 g. The values and distribution patterns of the stress state, developed in the models, were similar to those in dams. The purposes of experiments were to study the processes of deformation and subsequent failure of homogeneous excavated slopes of different inclinations with the increase of their heights and to compare the results of theoretical analyses and experiments to verify the sug-

gested method of analysis.

The investigations were carried out considering the slopes of excavations and cuts on the assumption that the slope foundation was undeformable. A blend of sand and clay resembling sandy loam was used as the model soil. In all the tests the initial density of the blend in the model was 1.70-1.72 g/cm³, the moisture content being 17%. The strength properties of the soil determined by means of a simple strength apparatus were as follows: $\text{tg } \varphi = 0.4$; $C = 0.01$ MPa.

The tests were carried out on several sets of models of slopes from 45 to 90° (vertical slopes). Each model was studied in gradually increasing field of acceleration which corresponded to gradual increase in the height of the prototype. This procedure made it possible to obtain, in the course of the experiments, complete information about the process of slope deformation and to observe the succession of all the stages of its collapse (V.I.Vutsel et al., 1976). Fig. 1 shows the stages of the collapse of a vertical slope in the field of centrifugal forces. At the initial stage under the stresses corresponding to a prototype height of about $H = 3.0$ m, a crack develops at the crest of the slope. Then, at $H = 4.5$ m the crack widens a little and in the lower part of the slope (approximately at 1/3 of its height) a local displacement occurs. With the increase of the stress ($H = 5.2$ m) the crack widens still more and new closer located cracks appears while the zone of local displacement increases. Eventually, when a certain critical height ($H = 6.0$ m) is reached, the cracks at the crest increase considerably in depth and the failure.

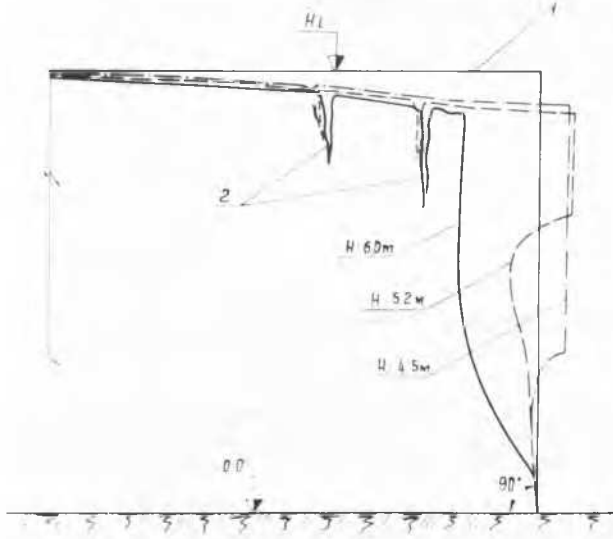


Fig. 1. Collapse of a vertical slope in centrifugal forces field.

1. Initial profile of the slope
2. Cracks

of the slope occurs. It should be noted that the field observation of deformation of steep slopes shows a similar process of their failure. The concentration of shearing stresses in the lower part of the slope seems to account mainly for the beginning of failure in this part of the slope. Experiments proved this supposition. As a result of measuring the settlements and horizontal displacements at all the stages of the model loading, the

patterns of slope deformation were obtained. Fig. 2 shows the distribution of isostaths (lines of equal values) of horizontal deformations of a slope at the moment directly preceding its failure. The concentration of deformations and, subsequently, of stresses in the lower part of the slope, can be easily seen from Fig. 2. It is these stresses that caused the initial failure of the slope.

Experiments with 75° and 60° slopes showed a similar pattern of failure, the only difference being observed in the values of accelerations which caused the failure. To obtain the dependence of the critical height of a stable slope up on its angle of inclination, maximum accelerations causing the failure of slope were measured. This dependence is shown in Fig. 3.

It should be pointed out that while testing relatively gentle slopes ($\alpha=45^\circ$) under high accelerations corresponding to the prototype 20-30 m high, considerable horizontal and vertical displacements as well as a great number of wide and deep cracks were observed.

Similar observations were made during tests on more gentle slopes ($\alpha=30^\circ$). Here again no sliding surfaces were observed though the deformations of the slopes were quite substantial. The above explanations point to the fact that the procedure of the analysis of slopes stability on the basis of the limit equilibrium, employed at present are not satisfactory in some cases from the viewpoint of operational stability evaluation, which means that the analysis based on the limit deformations should be adopted.

This paper recommends employing for the analysis of the stress-strain state of slopes a numerical computation procedure based on the elasto-plastic properties of soil within

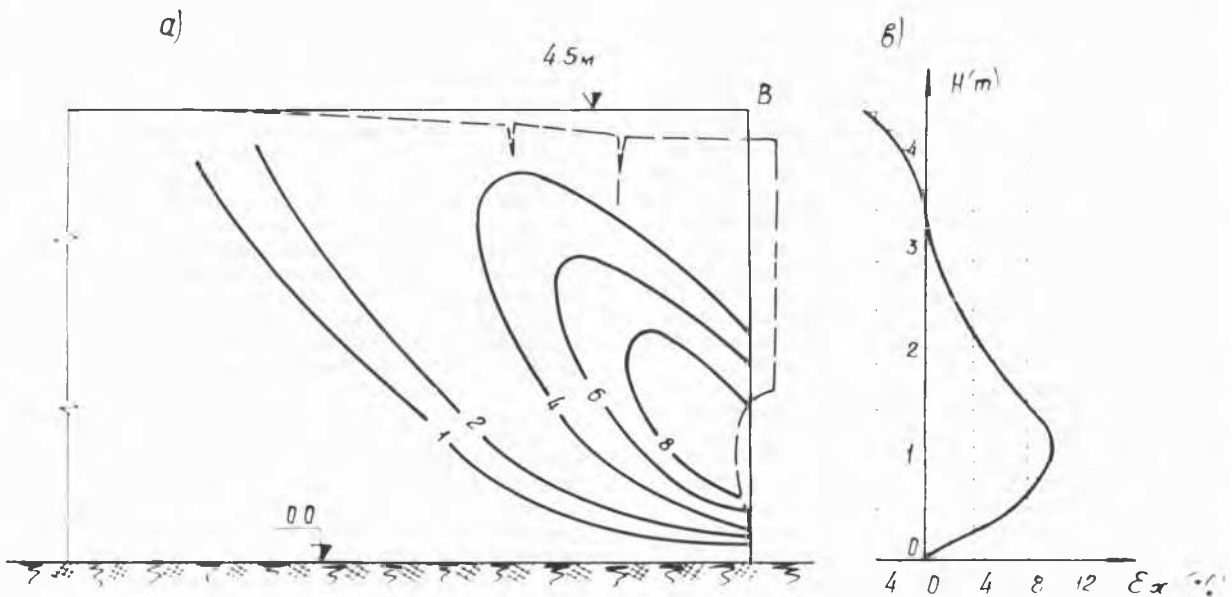


Fig. 2
Horizontal deformations in slope (ϵ_x).
a) Isostaths ϵ_x . b) Horizontal deformations in the plane AB.

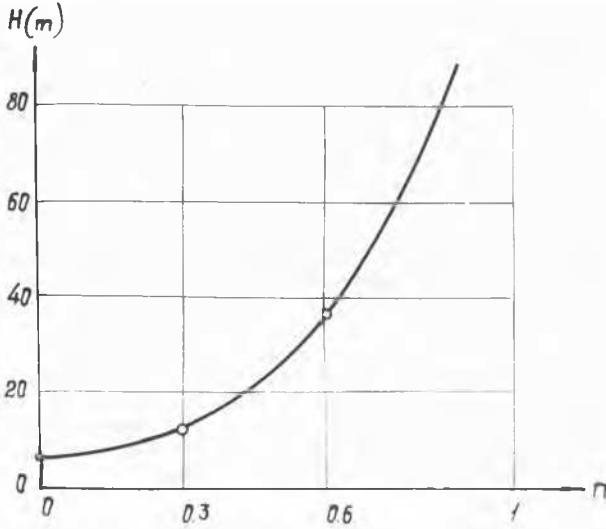


Fig.3. Critical height (H) of a stable slope vs. slope inclinations (l:n).

Prager-Drucker's theory and the associative law. The material used is assumed to be ideally elastic up to the moment of reaching the limit surface.

The analysis procedure is based on the explicit solution of motion equations by the variation-difference method. The procedure of numerical solution of the equations of the theory of elasticity is dealt with in detail elsewhere (V.N.Lombardo,1973). The main points of the analysis procedure are given below.

The interrelation of deformations and displacements is expressed by the equations of Cauchy:

$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \dots (1),$$

where i, j=1,2; ϵ_{ij} -components of deformation tensor; u_i -displacements along coordinate axes; x_i -coordinate axes.

The interrelation between stress and strain is expressed by the Hooke's law:

$$\sigma_{ij} = (K-2\mu)\Delta \delta_{ij} + 2\mu \epsilon_{ij} \dots (2),$$

where σ_{ij} -components of deformation tensor; K-bulk modulus; μ -shear modulus; Δ -first invariant of deformation tensor; δ_{ij} -Kroneker delta-symbol.

To complete the system of equations (1) and (2), the following differential equation of motion should be added to them:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + P_i = \rho \frac{\partial^2 u_i}{\partial t^2} \dots (3),$$

where P_i - components of volume forces; ρ - material density; t- time.

This solution procedure can be applied for the analysis of both static and dynamic loads. In the case of static loads (dead weight) the load is assumed to be gradually increasing and for the more expedite stabilization of the solution the dissipative forces

are introduced.

In the case when the elasto-plastic deformation is taken into account, the algorithm of calculation is the same, excepting the block for stresses calculation.

Total deformation is represented by the sum of the elastic (ϵ_{ij}^e) and plastic (ϵ_{ij}^p) components:

$$\epsilon_{ij} = \epsilon_{ij}^e + \epsilon_{ij}^p \dots (4).$$

Elastic deformations are related to stresses according to the Hooke's law (2).

In order to determine plastic deformations it is necessary to know the limit surface and the law governing the direction of the deformation increments. For the latter purpose the associative law (L.I.Sedov,1974) was applied:

$$d\epsilon_{ij}^p = \frac{\partial f}{\partial \sigma_{ij}} d\lambda \dots (5),$$

where $d\epsilon_{ij}^p$ -increment of components of the plastic deformations tensor; f-yield function; $d\lambda$ - an infinitesimal positive quantity.

The limit surface (function of yield) was determined from the condition of Mises-Shleier-Botkin:

$$f = T + P \cdot k - c \leq 0 \dots (6),$$

where $T = \sqrt{\frac{1}{2} S_{ij} S_{ij}}$ - second invariant of stress deviator;

$S_{ij} = \sigma_{ij} - P \delta_{ij}$ -stress deviator;

$P = \frac{1}{3} \sigma_{ij} \delta_{ij}$ -first invariant of stress tensor;

k and c - functions determined by soil parameters (Malishev M.V.,1969).

$$k = \frac{2 \sqrt{3} \sin \varphi}{3 - \sin \varphi} \dots (7),$$

$$c = \frac{2 \sqrt{3} \cdot C \cdot \cos \varphi}{3 - \sin \varphi}$$

where φ -internal friction angle; c - cohesion.

The following analysis procedure is suggested:

1. From equation (2), on the basis of elastic component of deformation and on the assumption that $d\epsilon^p = 0$, the stresses σ_{ij} are calculated.

2. The values of invariants T_0 and P_0 are determined and condition (6) is verified.

a) In the case when the condition (6) is satisfied, the stresses are determined correctly and the assumption of the increments of plastic deformations being equal to zero is true;

b) In the case when the condition (6) is not satisfied, plastic deformations (5) are expected to increase. The unknown quantity in this expression is $d\lambda$. The associative law (5) together with (4) and (6) allows the following scalar to be computed:

$$d = \frac{3(T_0 + P_0 k - c)}{Kk^2 + 3\mu} \dots (8)$$

as well as the increments of plastic deformations (5).

Elastic deformations are determined from

equation (4) and components σ_{ij} of stress tensor are found from equation (2).

On the basis of the developed procedure, a program for the computer BESM-6 was worked out and calculations for the model tested in the field of centrifugal forces were carried out.

The calculations and tests were carried out on the model of a homogeneous slope of a 45° excavated slope having an undeformable foundation. The model was 30 cm high and was tested under the acceleration of 120 g which corresponds to a 36 m high prototype slope.

The experiment was carried out as follows: the model was subjected to centrifugal acceleration which produced corresponding increase in the dead weight forces; the complete settlement and horizontal displacements of the model were measured. Then the model was unloaded, the residual deformations of the model's crest settlement were measured and the position of the isostaths of residual and horizontal displacements were determined.

Computer calculations of the slopes were carried out by the same procedure as that employed for the experiments. Complete and residual displacements and settlement were determined and the distribution pattern of isostaths was plotted. The initial data for computations were obtained from laboratory tests of the model soil to be $\tan\psi=0.4$, $E=30$ MPa and $\mu=0.3$

Fig.4 shows a comparison of theoretical

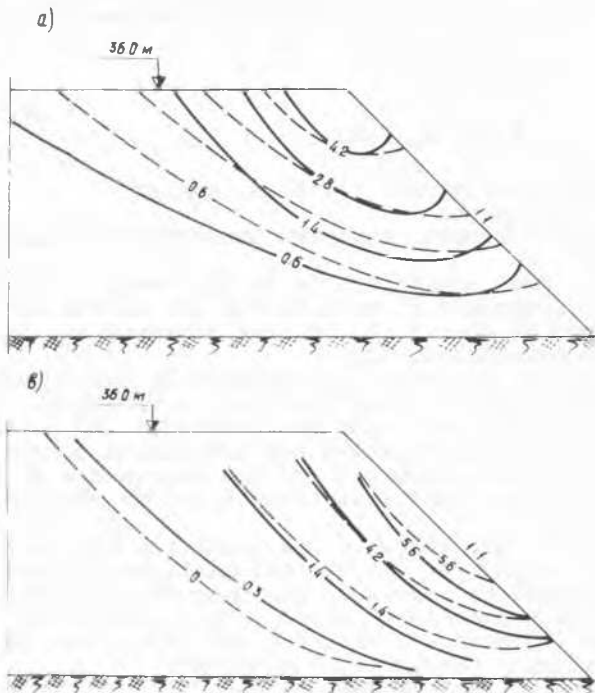


Fig.4. Comparison of theoretical and experimental results. 0.3 - Theoretical isostaths of settlement (a) and of horizontal displacements (b), in % of H. 0.6 - Ex-

perimental isostaths of settlement (a) and of horizontal displacements (b), in % of H.

and experimental isostaths of residual displacements and settlements. The fairly close coincidence of the family of curves testifies to the quite satisfactory results. However, it should be noted that the total experimental settlements differ from the theoretical ones by about 30%. This difference is caused evidently by the two factors: (1) the properties of the soil were determined by means of a direct shear apparatus and not on the basis of triaxial compression tests which, as is universally accepted, give different and more reliable results; (2) the assumption of the elastic soil behavior at the initial stage adopted for the analysis introduces a certain mistake. The comparison performed is only the initial step of verification of the suggested analysis procedure, but even the first results obtained point to the favourable prospects of its application.

CONCLUSIONS

1. Experiments conducted with the help of centrifugal modelling machine made it possible to ascertain the process of failure of the slopes of clayey soils with the increase of their height and the change in their steepness.
2. To determine the stress-strain state of slopes analysis procedure is suggested which takes into consideration the elastoplastic properties of the soil within of the Prager-Drucker theory, the associative law being employed.
3. The comparison of the experimental and theoretical results showed their satisfactory agreement which attests to the favourable prospects of the application of the analysis procedure proposed.

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