Isolation of Vibrations by Concrete Core Walls

Isolation des Vibrations par Parois Moulées en Béton

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SYNOPSIS

The isolation effect of concrete core walls to surface waves is studied theoretically by the use of the Finite Element method. By application of an Influence Matrix boundary condition the size of the FE-field to be analysed can greatly be reduced and a large number of FE-calculations, varying the dimensions of the wall and material properties, are economically possible. The same boundary condition assures continuity across the boundaries of the FE-field. The results show that the vibration isolating effect of plane concrete core walls does not depend on the geometrical shape of the wall but only on the area of the cross section.

INTRODUCTION

In soil dynamic problems different types of elastic waves are involved, among which the so-called Rayleigh wave is one of the most important. It is a special type of elastic wave, which is coupled to the free surface of a half-space and consists of a transversal and a longitudinal wave component. The functions of the horizontal and the vertical displacement amplitudes with depth in a homogeneous half-space are shown in fig.1. The wave length of the Rayleigh wave is slightly smaller than that of the shear wave.

A wave source on the surface of the half-space will generate both body waves and Rayleigh waves. By reason of the much higher geometrical damping of the body waves the major part of the wave energy near the surface will be transmitted by the Rayleigh wave, except for the region very close to the source. Thus, if a building or any shock-sensitive installation has to be isolated from ground vibrations, it is necessary mainly to reduce the amplitude of the Rayleigh wave components. This can be done by open or slurry filled trenches or other wave-diffracting obstacles.

WOODS (1968) reported on measurements of the isolating effect of open trenches near to the source (active isolation) and in the far-field (passive isolation). The latter represents the Rayleigh wave isolation.

He observed a reduction of the vertical Rayleigh wave component to 25% at a trench depth of slightly more than one Rayleigh wave length λ_R. DOLLING (1970) conducted experiments with slurry filled trenches, getting satisfactory results without separating near-field and far-field isolation.

However, trenches - even slurry filled - are not stable over a long period without continuous maintenance and are therefore inappropriate as permanent isolating measures in practice. Concrete core walls are stable structures and can be expected to have a vibration-isolating effect. However, their effectiveness is not well known until now.

An investigation of the isolating effect of concrete core walls to plane steady-state Rayleigh waves, travelling perpendicular to the long axis of the wall is reported here. The analysis is performed theoretically as a plane strain problem by using the Finite Element method.

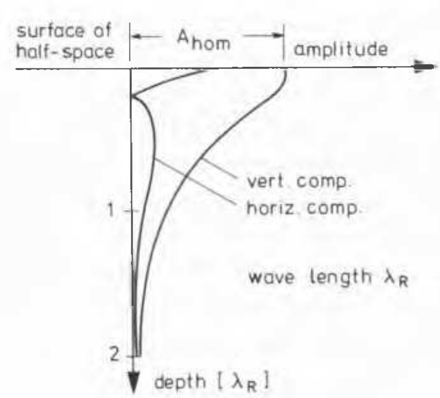

FIG.1 : RAYLEIGH WAVE : AMPLITUDE-FUNCTIONS WITH DEPTH ; HOM. HALF-SPACE

BOUNDARY CONDITIONS

In investigations of steady-state dynamic problems by this method one of the main problems is the application of appropriate conditions at the boundaries of the FE-field. For the Rayleigh wave in the homogeneous half-space, of which the theoretical amplitude functions are shown in fig.1, an exact boundary condition has been published by LYSMER/KUHLEMEYER (1969). This will be referred to as Rayleigh wave boundary-condition (RW-BC). However, near the wave source or an inhomogeneity within the half-space the theoretical amplitude distribution of the two components with depth does not exist. It will develop with the horizontal distance from the wave source, so that the RW-BC can be applied only after a distance of some wave lengths from the source. The FE-field to be analysed has, therefore, to be much larger than the actual area of interest only to fulfil the conditions for satisfactory application of the RW-BC.

LYSMER/WAAS (1972) developed a boundary-condition for all wave types, which is applicable also at short distance from any wave source providing good energy absorption. As this boundary condition depends on the presence of a rigid base at the lower boundary of an elastic layer, it is not applicable to the case of a free unlimited half-space.

In the analysis reported here, a new method is used to solve large steady-state wave-field problems, which has been outlined in detail by HAUPT (1976). It results in a boundary condition which bypasses the problems described above and is accurate for all wave types and amplitude functions regardless of the distance between the wave source and the boundary. The procedure is based on the establishment of a matrix transmitting the influence of one FE-field to an adjacent one. This influence matrix boundary condition (EM-BC) provides, in addition, the means for a considerable reduction of computer time. It will be described briefly here.

The total FE-field to be analysed is subdivided into three fields, one inner field F_I and two identical outer fields F_{II} (see fig.2a). The fields F_I and F_{II} are connected by the boundary S_{I-II}. Let the number of nodal points on S_{I-II} be m. For a plane strain condition this yields 2m degrees of freedom on S_{I-II}. Here the material is assumed to be linear visco-elastic and the time function to be harmonic:

$$\tilde{u}(x,t) = u(x)e^{i\omega t} \qquad (1)$$

where ω is the circular frequency and $i=\sqrt{-1}$. For F_{II} the structural matrix $[A]_{II}$ can be assembled by classical procedure, following for example ZIENKIEWICZ (1971). For the given material and time function it contains, in addition to the stiffness matrix, $[K]_{II}$, the damping matrix, $[C]_{II}$, and the mass matrix, $[M]_{II}$:

$$[A]_{II} = [K]_{II} + i[C]_{II} + [M]_{II} \ . \qquad (2)$$

Following the normal FE-procedure, a system of linear equations for F_{II} will be obtained:

$$[A]_{II} \{u\}_{II} = \{b\}_{II} \qquad (3)$$

where $\{u\}_{II}$ represents the displacement vector and $\{b\}_{II}$ the load vector. Now in each of the 2 m degrees of freedom on S_{I-II} a force of amplitude "1" is assumed to be acting separately. Each force represents one load vector. They are all collected in a load matrix $[B]_{II}$ with 2 m columns, each column containing one "1" and elsewhere zeros.

Equation (3) thus becomes the matrix equation

$$[A]_{II} [U]_{II} = [B]_{II} \ . \qquad (4)$$

The influence matrix of F_{II} with respect to F_I can then be determined by eq.(5).

$$[B]^T_{II} [U]_{II} = [EM]_{I-II} \qquad (5)$$

By inverting the influence matrix one obtains the interaction matrix:

$$[EM]^{-1}_{I-II} = [R]_{II-I} \qquad (6)$$

This matrix contains the total reaction of F_{II} to a given displacement vector $\{u\}_{I-II}$ on S_{I-II}. For the analysis of F_I it is simply necessary to superimpose $[R]_{II-I}$ to the structural matrix of F_I, yielding the system of linear equations for F_I:

$$([A]_I + [R]_{II-I})\{u\}_I = \{b\}_I \qquad (7)$$

Solving (7) the displacements $\{u\}_I$ within F_I will be such, as if the total system (F_I+F_{II}) has been calculated, although only F_I has been analysed. Hence $[R]_{II-I}$ simulates exactly F_{II} with respect to F_I. Now $\{u\}_I$ is known and the displacements in F_{II} can easily be calculated from:

$$\{u\}_{II} = [U]_{II} [R]_{II-I} \{u\}_{I-II} \qquad (8)$$

$[R]_{II-I}$ in fact represents the numerical solution of an integral equation problem.

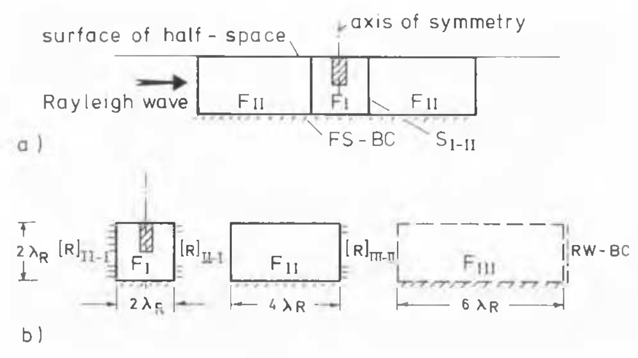

FIG.2 a): TOTAL FE-FIELD AND SUBDIVISION
b): SYSTEM OF FE-FIELDS F_I - F_{III}

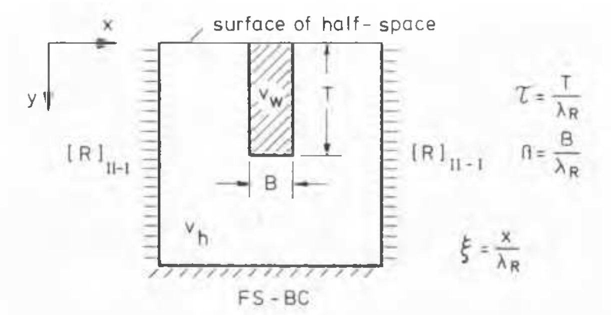

FIG.3 : PLANE OBSTACLE IN HOMOGENEOUS HALF-SPACE

SYSTEM

Once $[R]_{II-I}$ has been established, it is necessary to analyse only F_I by FE-technique for each different case to be calculated. The displacements within the two outer fields then are obtained by simple matrix multiplication using (8).

In the present problem the dimensions of the inner field F_I have been chosen to be $2\lambda_R$ X $2\lambda_R$ and those of the fields F_{II} to be $4\lambda_R$ X $2\lambda_R$ (see fig.2 b). This results in 1528 and 3008 complex unknowns for F_I and F_{II} respectively. By using the influence matrix procedure a reduction of computer time down to about 25% is achieved.

At the lower boundary of all fields the full-space boundary condition (FS-BC) reported by LYSMER/KUHLEMEYER (1969) has been applied. At the outer vertical boundaries of the fields F_{II} a further EM-BC is active, derived from a third field F_{III} with a length of $6\lambda_R$ and the RW-BC developed by LYSMER/KUHLEMEYER (1969) applied at the right (for the right F_{III}) and the left vertical boundary (for the left F_{III}) respectively (s. fig.2b). These latter fields are not included in the analysis of the different cases but they serve only to provide absolutely perfect conditions at the boundaries of the total field considered.

At the left boundary of the left field F_{II}, stresses are acting in a way that a steady-state Rayleigh wave in its theoretical shape (see fig.1) is generated, which propagates to the right across the fields F_{II}, F_I, F_{II}, and is thereby distorted by the obstacle in F_I (see fig.2a).

The inhomogeneity within the otherwise homogeneous half-space consists of a plane rectangular obstacle, extending from the surface into the half-space. In plane view it is situated normal to the direction of the propagation of the waves. The depth of the lower boundary of the obstacle below the surface normalized over the Rayleigh wave length is $\tau = T/\lambda_R$, and the thickness is $\beta = B/\lambda_R$ (see fig.3).

The main topic of the investigation was the influence of the shape of the wall on the isolation effect. Therefore the dimensions of the wall have been varied independently, i.e. β from 0.1 to 1.2 and τ from 0.1 to infinity. Most of the calculations have been performed with materials representing a concrete core wall in a sandy ground. This series has been called series A. For the material properties see table I. In this table the subscript w refers to the wall material and the subscript h to the half-space material. G is the shear modulus, ρ the density, v the velocity of wave propagation and δ the logarithmic decrement with respect to distance. Poisson's ratio has been chosen to be equal, $\nu = 0.25$, for all materials.

To get an idea of the influence of the material properties on the isolation effect, three more series, B,C,D, have been conducted, for which the material properties are also listed in table I. In all 137 calculations have been performed, each of them involving 7448 complex unknowns. This could only be achieved by using the influence matrix procedure.

Series	$\frac{G_w}{G_h}$	$\frac{\rho_w}{\rho_h}$	$\frac{v_w}{v_h}$	$\frac{\delta_w}{\delta_h}$
A	34.29	1.37	5.0	5.0
B	25.0	1.0	5.0	5.0
C	1.0	1.37	0.854	0.854
D	0.25	1.0	0.5	0.5

TABLE I RATIOS OF MATERIAL PROPERTIES

RESULTS

In fig.4 the central part of $6\lambda_R$ by $2\lambda_R$ of the FE-field is presented as a computer plot. The grid is deformed by the Rayleigh wave propagating from the left to the right. A reduction of the wave amplitude behind the wall can be observed as also the cumulation of amplitude caused by reflection in front of it. The wall itself is only slightly deformed.

The propagation of the waves across the FE-field of fig.4 and their distortion by different types of walls can be seen much better in a movie fild which has also been generated by computer.

Fig.5 shows two typical results of series A. The normalized amplitude of the vertical component of the Rayleigh wave at the surface of the half-space is presented over the total length of the system. The value of $\gamma(\xi)$ is defined as the ratio of the amplitude in the case of distortion by the obstacle, A_{inhom}, to the amplitude in the case without any obstacle, A_{hom}.

$$\gamma(\xi) = \frac{A_{inhom}}{A_{hom}} \qquad (9)$$

The geometrical parameters of the two cases shown in fig.5 are $\tau = 0.4$, $\beta = 1.0$ (fig.5a) and $\tau = 1.0$, $\beta = 0.4$ (fig. 5b).

In front of the wall a typical reflection scheme is detectable in both curves: the horizontal distance between two peaks is exactly $\lambda_R/2$. At the upper surface of the obstacles the amplitudes are very low, and almost uniform over the width in the case of the deep wall ($\tau = 1.0$), but non-uniform in the case of the shallow one ($\tau = 0.4$). The shape of the latter amplitude curve - with a minimum almost at the mid-point - shows that this obstacle behaves mainly like a rigid body, vibrating in a rocking mode at the surface. Behind the walls the amplitudes are more or less constant having a value of $\gamma(\xi) = 0.5$ to 0.6.

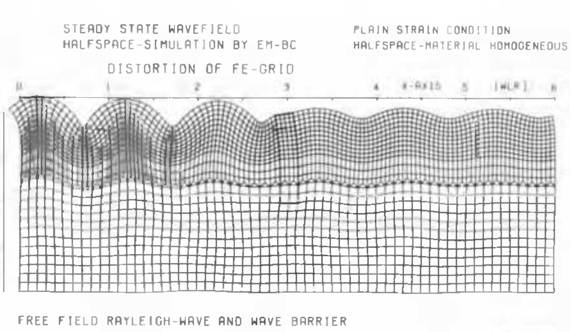

FIG.4 : DISTORTED FE-GRID ; RAYLEIGH WAVE FROM THE LEFT ; $\tau = 1.0$, $\beta = 0.2$

In the further analysis a so-called amplitude reduction factor, A_R, has been computed, which is an avarage value of the normalized vertical amplitude at the surface behind the obstacle:

$$A_R = \frac{1}{4} \int_{\xi=6}^{\xi=10} \gamma(\xi) \, d\xi \qquad (10)$$

In fig.6 this value is plotted as a function of the depth of the wall, τ , and the thickness, β , for series A. Each point represents one FE-computation. As the FE-field has the vertical dimension of two Rayleigh wave lengths with the FS-BC applied at the lower boundary, the cases with $\tau = 2.0$ represent in reality the cases with infinite depth.

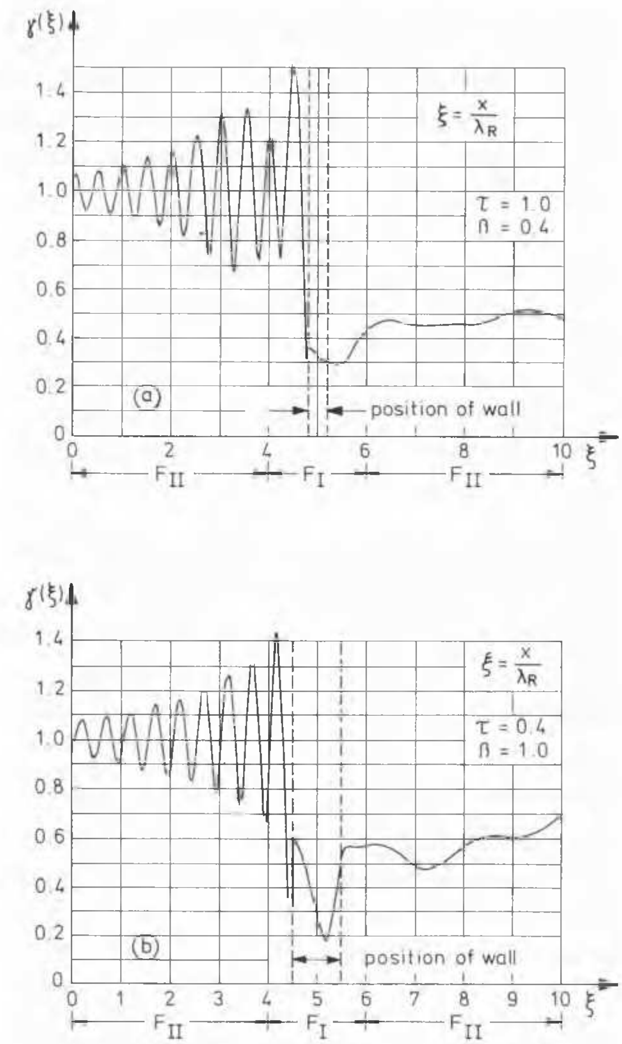

FIG.5 : NORMALIZED VERTICAL AMPLITUDE AT THE SURFACE OF THE HALF-SPACE

a) $\tau = 1.0$, $\beta = 0.4$
b) $\tau = 0.4$ $\beta = 1.0$

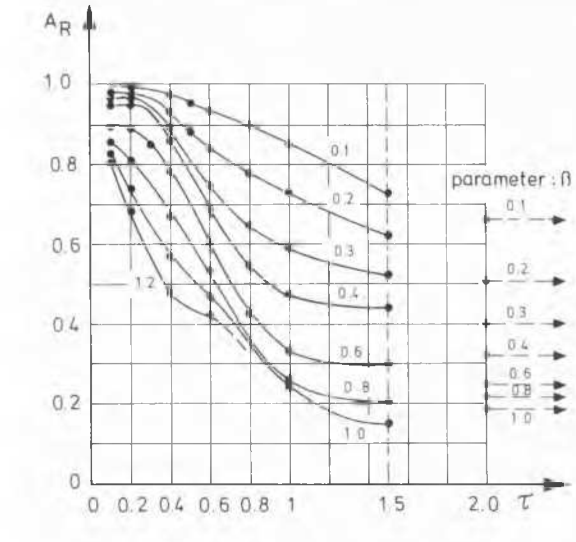

FIG.6 : A_R DEPENDING ON NORMALIZED PARAMETERS β AND τ

It may be seen from fig.6 that thin deep walls can produce the same amplitude reduction factor as shallow wide ones. For obstacles with a thickness of β = 0.6 or more, no significant decrease of A_R seems to occur, if τ exceeds the value of about 1.2 . Obviously, the lower limit of the amplitude reduction factor achievable is about 0.15 for the geometrical system and the material properties described above.

As it has been stated above, the same A_R may be obtained by walls having totally different dimensions. Thus A_R has been related to the dimensionless area of the cross section of the wall:

$$\alpha = \tau \cdot \beta = \frac{T \cdot B}{\lambda_R^2} \qquad (11)$$

In fig.7 the relationship between A_R and α is presented for series A. All values of A_R resulting from the computations are included in a narrow band reaching from A_R=1 at α = 0 to $A_R \sim 0.2$ at α ≥ 1.4 . By comparing the symbols, it may be observed that the vibration isolating effect of the concrete core wall does not depend on its geometrical shape but only on the area of the cross section.

In fig.8 the relation between A_R and α is presented for series B. Here the density of the wall material is equal to that of the half-space material, whereas the shear modulus is chosen to be 25 times higher, yielding a ratio of wave velocities equal to 5.0 It can be seen that there is almost no difference between these results and those ob-

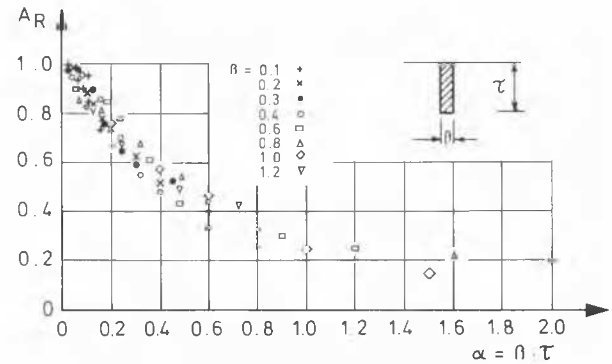

FIG.7 : A_R DEPENDING ON NORMALIZED AREA OF CROSS SECTION, α, SERIES A

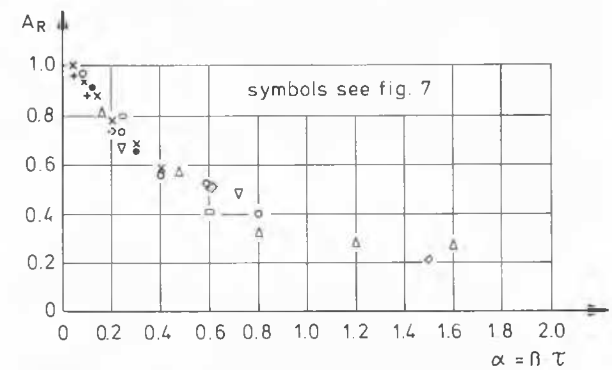

FIG.8 : A_R DEPENDING ON NORMALIZED AREA OF CROSS SECTION, α, SERIES B

tained for series A. This agrees well with the results for series C, presented in fig.9.

In this series the shear moduli of both materials are equal, but the density of the wall-material is that of series A. As, in this case, A_R is always very close to unity, it can be concluded that the density of the wall material is of almost no significance for the isolation effect, at least within the practical range of variation.

As in the case of a rigid wall, a partial reflection of the incoming wave can also be expected for a wall of a very soft material. Therefore, in series D the shear modulus of the wall material has been chosen to be a quarter of that of the half-space material. The results of this series are shown in fig.10. It can be observed that there is no close relation between A_R and α, as was found in the series A and B. For large values of α the limiting value of A_R seems to be about 0.7 . For values of α up to 0.6, A_R may diminish to 0.5, but this does not hold for all wall-shapes. In practice the wall cannot be chosen very soft without losing its character as a time stable isolating measure. From fig.10 it can be established, that a soft wall material does not have a satisfactory isolating effect except for some special wall dimensions.

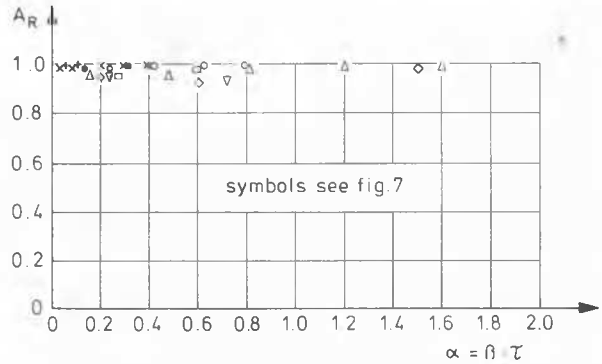

FIG.9 : A_R DEPENDING ON NORMALIZED AREA OF CROSS SECTION, α, SERIES C

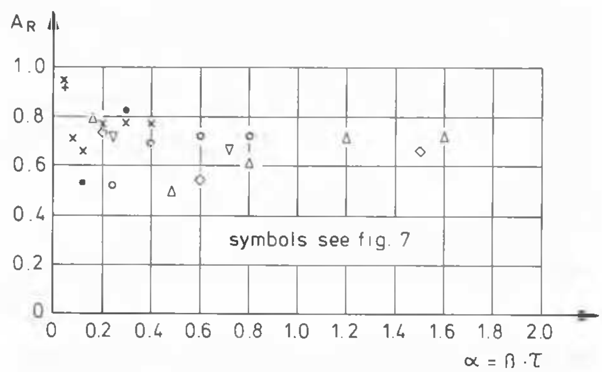

FIG.10 : A_R DEPENDING ON NORMALIZED AREA OF CROSS SECTION, α, SERIES D

There are different physical reasons for the isolating effect of the stiff wall:

1) there is a partial reflection at the interface of the two materials depending only on the material properties

2) thin deep walls are stimulated to vibration by the Rayleigh wave at the surface and transmit the energy down into the halfspace. Their lower part acts like a wave source, spreading out body waves

3) wide shallow obstacles, due to their rigidity, prevent the propagation of the Rayleigh wave, which is coupled to the surface, thereby transforming the Rayleigh wave partially into body waves

4) all obstacles are stimulated to vibrations by the incoming wave, consequently reducing its energy. This effect is small, as may be seen from the results of series C.

For a soft material only the first reason is valid. Thus, less isolation effectiveness must be expected for such walls.

It should be noted that this analysis has been performed assuming a half space with constant shear modulus, i.e. constant wave velocity. In practice there is usually an increase of wave velocity with depth. The results reported herein will, however, be applicable, if there is not a significant change of wave velocity in the ground.

CONCLUSION

The problem of isolating buildings against ground vibrations by means of concrete core walls has been investigated. These walls, which are longtime stable and need no maintenance as against open or slurry-filled trenches may have a sufficient isolating efficiency. The results reported above permit the design of vibration isolating measures corresponding to isolation requirements, available space, cost etc. Soft wall materials (compared to the half-space material) do not seem to have satisfactory isolating effect but further studies are necessary concerning this problem.

The application of the influence matrix procedure permits the calculation of a large number of cases, thus enabling the use of the FE-method as an investigation method which yields general relations instead of special case results.

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